Relating arithmetical techniques of proportion to geometry: The case of Indonesian textbooks

Dyana Wijayanti

Relating arithmetical techniques of proportion to geometry: The case of Indonesian textbooks

Dyana Wijayanti

University of Copenhagen, Science Education Department, Copenhagen, Denmark, dyana.wijayanti@ind.ku.dk

The purpose of this study is to investigate how textbooks introduce and treat the theme of proportion in geometry (similarity) and arithmetic (ratio and proportion), and how these themes are linked to each other in the books. To pursue this aim, we use the anthropological theory of the didactic. Considering 6 common Indonesian textbooks in use, we describe how proportion is explained and appears in examples and exercises, using an explicit reference model of the mathematical organizations of both themes. We also identify how the proportion themes of the geometry and arithmetic domains are linked. Our results show that the explanation in two domains has different approach, but basically they are mathematically related.

Keywords: Proportion, arithmetic, geometry, textbooks, Indonesia.

INTRODUCTION

Trend of fragmentation of the school curriculum makes students visit a theme after the other, but somehow the rational for visiting them or the way they are related to each other does not become clear. Chevallard (2012) defined this trend as "visiting monument" in which students are encouraged to admire and enjoy without knowing about its *raisons d'être*, now or in the past. Thus, relation among themes is necessary, particularly for an important theme such as proportion. This theme appears in many different domains, such as arithmetic and geometry. When it goes across domain, the relationship is important because it is easier to relate a theme within a domain than across domains or across grades. Making those links could help students having the experience that mathematics is a connected body of knowledge not just a collection of tricks that teachers play out to satisfy the children.

The notion of "proportional reasoning" is often used to indicate what is needed to fully operate with a variety of mathematical objects and models such as scales, probabilities, percentage, rates, trigonometry, plane geometry, linearity, fractions, etc. The so-called "missing value problems" are a common way to introduce proportion problems to the students. For example, if it requires two hours of work to make 3 puppets, how many hours are needed to make 25 puppets? Here we can find three values in the task (2, 3, and 25) and one missing value to be found. In general, proportion is considered as a relationship between quantities. More abstractly, proportion is about two n-tuples of quantities related in the same way to each other (constant ratio), for instance the ingredients in two recipes of the same cake or the sides in two similar triangles.

The mathematical notion of proportion is antique and goes back as far as to Euclid (about 300 BC). In book V, definition VI it is said that magnitudes having the same ratio are called proportional (Fitzpatrick, 2008). In book VI, Euclid also defines the geometric notion of similarity. It is also said that similar figures are those which have their corresponding angles equal and the corresponding sides about the equal angles proportional. This definition is still used in the elementary curriculum, although it adopts a more modern formulation. In particular, according to Miyakawa and Winsløw (2009), it can be said that Euclid's notion of proportion is static (about a property of given quantities) rather than dynamic (in terms of a functional relationship between variables). The dynamic definition is common in the algebraic domain which has become dominant in scholarly mathematics over the past four centuries and has been conceived in terms of linear functions between real numbers.

In the Indonesian school, the proportion theme in arithmetic is introduced as ratio equality: given four numbers a, b, c and d, the equality $(a, b) \sim (c, d)$ indicates that *a* and *b* are in the same proportion as *c* and *d*. At the beginning of the seventh grade, students start to work with missing value problems. However, proportion also appears in the geometry and statistics domain and mostly in upper secondary school, in the introduction of functions.

The fact that proportion is found not only in arithmetic, but also in geometry, makes it interesting to investigate how textbooks introduce and treat the theme of proportion in geometry (similarity) and arithmetic (ratio & proportion), and especially how these themes are linked to each other. To formulate and investigate this phenomenon with more precision, we use the anthropological theory of the didactic (ATD) by (Chevallard, 1999, 2002) in particular the notion of *praxeology*. It is not our aim to address the issue of how proportion in arithmetic and geometry should be taught in lower secondary education (in Indonesia), or how it should be treated in textbooks. However, we expect that our research into the approaches chosen in textbooks will contribute to this discussion and give it more precision.

PROBLEM BACKGROUND

The research about how teachers use resources for teaching is increasing. Gueudet and Trouche (2012) point out that documentation work where teachers interact with resources, selects them and work on them is a central in teachers' professional activity. Considering textbooks as an example of resources, this study implies that the textbooks are not only seen by teacher as a text to follow, but also it can be used as a resources for teacher learning. However, Pepin (2012) argues that it is still less clear what kind of textbooks that can help teacher learning. In her study she used tool analysis for reflection and feedback to help teachers develop further understanding and enrichment of mathematical tasks. Pepin's study inspired me to use another tool to analyse task in order to enrich teachers' point of view of quality materials for teaching.

A textbooks analysis developed by González-Martín, Giraldo, and Souto (2013) who consider the case of the introduction of real numbers in Brazilian textbooks and found an unintegrated mathematical organization as knowledge to be taught. Furthermore, Hersant (2005) conducted a historical study of how the arithmetic of proportion appears in the French compulsory education from 1884 to 1988. Based on an elaborate reference model, she demonstrated how the teaching approach is changing over time. Finally, García (2005) proposed a reference epistemological model in terms of a sequence of *praxeologies* to study linear systems and linear functions, and he used it to identify proportional relations, both in arithmetic and in the study of functions, showing a quite poor coherence between the two as they appeared in school mathematics. Our proposal to look at proportions as they are treated in the arithmetical and geometrical domain has to be considered as an extension of Hersant's and Garcia's work relying on González-Martin and colleagues' methodology.

THEORETICAL APPROACH

According to ATD, knowledge is produced, communicated, learned and used in institutions, and depends on them. The relationship between knowledge in scholarly institutions and school institutions is described using the notion of *didactic transposition* (Chevallard & Bosch, 2014). Even though our study does not aim at analysing the full transposition problem, the notion of reference model enables us to analyse specific textbook transpositions of scholarly knowledge in a wider framework, without assuming this scholarly knowledge as a universal or fixed measure.

In ATD, mathematical activity is identified into two blocks. First, there is *practical block (praxis)* which is made of the type of task and technique to solve. Second, there is *knowledge block (logos)* formed by technology and theory to explain and justify the *praxis*. We refer to Barbé, Bosch, Espinoza, and Gascón (2005) for more details on these notions. The aim of this paper (and my thesis) is to develop a textbook analysis method based on ATD.

Our analysis of textbooks mainly considers *mathematical praxeology*, but the textbook itself is also a rich resource for developing *didactic praxeology*. Specifically, textbooks contain a wealth of explicit mathematical tasks, demonstration of techniques using specific technologies, and also *theoretical discourse* which explains, relates and justifies the technologies.

Reference model

Proportion appears as themes (collections of mathematical *praxeologies* that are unified by a common technology) with different theoretical frameworks (sectors) that appear in different school mathematical domains; we consider here mainly the domains of arithmetic and geometry, while related themes appear also in other domains, especially algebra and probability. We now present parts of our reference model to analyse the two themes separately. We mainly use a categorization of techniques which are sufficient to explain *praxis* (as the types of tasks are also evident) in this model.

Proportion in the arithmetic theme is defined as follows, for pairs of numbers: $(x_1, x_2) \sim (y_1, y_2)$ if $\frac{x_1}{x_2} = \frac{y_1}{y_2}$. More generally, $(x_1, ..., x_n) \sim (y_1, ..., y_n)$ if $(x_i, x_j) \sim (y_i, y_j)$ for all $i, j = 1, ..., n$. There are three main types of task that are categorized:

T_1^{Ar}: given $(x_1, ..., x_n)$ and $(y_1, ..., y_n)$ decide if $(x_1, ..., x_n) \sim (y_1, ..., y_n)$

T_2^{Ar}: given $(x_1, ..., x_n)$ and $(y_1, ..., y_n)$ compare $\frac{x_i}{x_j}$ for $i = 1, ...,n$

T_3^{Ar}: given $(x_1, ..., x_n)$, y_1 find $y_2, ..., y_n$ so that $(x_1, ..., x_n) \sim (y_1, ..., y_n)$

The actual tasks in textbooks are usually given through the description of a daily situation, such as using scales on a map, buying and selling goods. technique that is used for T_1^{Ar} is to calculate the ratios of related terms and conclude about the proportionality. In T_2^{Ar}, the technique consists in comparing ratio of corresponding numbers or magnitudes. For T_3^{Ar}, the detail technique can be categorised as proposed by Hersant (2005). We illustrate these types of tasks with typical textbooks task:

T_1^{Ar}: The price of 2 kg rice in shop 'A' is Rp. 5.000,- and the price of 5 kg rice in shop 'B' is Rp. 12.500,- . Do both shops have the same price of rice?

T_2^{Ar}: In Bu Ina's grocery, the price of a package containing 2 kg of sugar is Rp. 9.400,-and the price of a package containing 5 kg of sugar is Rp. 22.750,-. Which package is cheaper?

T_3^{Ar}: Two students can carry 15 books. How many books can 8 students carry?

In geometry, proportion is connected to the notion of "similarity". Two polygons of the same kind (triangles, quadrilateral, etc.) are defined as similar if the corresponding angles have the same measures and the ratio of the lengths of corresponding sides are equal. Common types of tasks in the textbooks treatment of similarity include the following which are closely related to T_1^{Ar} and T_3^{Ar} respectively:

T_1^{G}: Given two polygons with the same angles and also given the side lengths of two polygons, decide if the polygons are similar.

T_3^{G}: Given similar figures with corresponding sides $(x_1, ..., x_n)$ and $(y_1, ..., y_n)$ with $x_1, ..., x_n$ and y_1 known. Find the unknown sides $y_2, ..., y_n$.

There are some textbooks examples of the types of tasks T_1^{G} and T_3^{G}, together with corresponding techniques:

T_1^{G}: One rectangle is 12 cm in length and 8 cm in width. Another rectangle is 6 cm in length and 4 cm in width. Are they the similar?

T_3^{G}: Quadrilaterals *ABCD* and *LMNO* are similar. The side of *ABCD* are 6 cm, 10 cm, 12 cm and 14 cm in length, respectively. The shortest side of *LMNO* is 9 cm. Find the length of other side of *LMNO*.

To analyse the *praxeologies* proposed in the textbooks, we will mainly use the categorization of types of tasks explained above (the basic reference model).

CONTEXT AND METHODOLOGY

Indonesia is a big country with 252 million people inhabitant. It is a big market for publishing companies and makes among them are competing with each other. Furthermore, the fact that Indonesia still lacks of teachers and some of them have a weak educational background makes them dependent on the textbooks. Thus, it is the reason why the quality of textbooks should be controlled or become a priority for the ministry of education.

The proportion theme in arithmetic (Ar) is introduced in grade 7. In Indonesian, proportion is called "perbandingan" which literally means proportion. Proportion in geometry (G) is introduced in grade 9 as "kesebangunan dan kekongruenan" (similarity and congruence). We study two themes in three different textbooks for each of the two grade levels. We refer to the textbooks as shown in Table 1 and 2.

Code	textbook title	Authors, year of publication
Ar_1	Book for studying mathematics 1	Wagiyo, Surati, and Supradiarini (2008)
Ar_2	Contextual teaching and learning mathematics	Wintarti and colleagues (2008)
Ar_3	Mathematics 1: concept and application	Nuharini and Wahyuni (2008)

Table 1: Lower secondary textbooks grade 7, analysed in this paper

Code	textbook title	Authors, year of publication
G_1	Book for studying mathematics 3	Wagiyo, Mulyono, and Susanto (2008)
G_2	Contextual teaching and learning mathematics	Sulaiman and colleagues (2008)
G_3	Easy way to learn mathematics 3	Agus (2007)

Table 2: Lower secondary textbooks grade 9, analysed in this paper

To analyze the data, we studied how the above mentioned themes in the arithmetic and geometry domain were introduced in the textbooks (that is in the main text, rather than in the collection of exercises). We considered how proportion is introduced through examples (*praxis*), with explanations of techniques and use of theoretical justifications (*logos*). This way we identified the main elements of the mathematical organization to be developed by students in both themes, according to the textbooks. Furthermore, we were also interested in how much student autonomy was foreseen in solving tasks; for this, we considered the variety of tasks that are proposed for student work (in the exercise sections of the textbook) and also their degree of similarity to examples given in the main text. Obviously, the analysis of exercises as tasks is incomplete because there are no given techniques, unlike in the working examples. Furthermore, we identified how the proportion themes in arithmetic and geometry are linked, based on the characteristics of mathematical organization and the presence or absence of explicit cross references in the text.

RESULTS AND DISCUSSION

How proportion is introduced in the arithmetic theme

We first provide some informal remarks about the theoretical structure of the relevant chapters. The title of proportion theme was different in each textbook: application of algebra (Ar_1), ratio and proportion (Ar_2), proportion and daily life arithmetic (Ar_3).

There were three subchapters in the Ar_1: 1. linear equations with one variable; 2. problem solving in daily life arithmetic; 3. proportion. Between these three subchapters we find connecting sentences such as: "we often use algebra to solve economic activity' and 'in the daily activity a lot of things correspond to proportion" (Wagiyo, Surati, & Supradiarini, 2008, p. 108 & 115). Furthermore, the authors also relate proportion to fractions with a brief sentence. In Ar_3, there were two main subchapters: daily life arithmetic and proportion, and there was no explicit connection between them. However, the authors use fractions as a tool to introduce proportion: "in the last chapter you already learned that a fraction can be considered as a proportion of two numbers" (Nuharini & Wahyuni, 2008, p. 152). In contrast to Ar_1 and Ar_3, the authors in Ar_2 did not relate proportion with linear equations and daily life arithmetics. They directly discussed a new terminology called *'ratio'* to introduce proportion. Due to the focus of research, we will only consider the proportion subchapters.

To explain what proportion is, the authors tried to avoid formal explanations. For example:

$$\frac{\text{the price of one book}}{\text{the price of fives books}} = \frac{500}{2500} = \frac{1}{5}$$

> Comparison between the amount of the books and the prices give the same numbers, so that the amount of the book and the price are proportional (Wagiyo, Surati, & Supradiarini, 2008, p. 120).

The authors used daily life situations to introduce proportion, for example, selling and buying price. Furthermore, the technologies that cover proportion in these three textbooks were very similar. For further explanation, the following table shows the number of examples in three textbooks, given to the two figures of task T_1^{Ar} and T_3^{Ar}.

Type of task	Ar_1	Ar_2	Ar_3
T_1^{Ar}	2	1	1
T_3^{Ar}	5	4	7

Table 3: Number of examples in the main text

From the table above, we noticed that the textbooks have more T_3^{Ar} than T_1^{Ar}. Also, we found a considerable number of techniques related to T_3^{Ar}. For example: reduction of unit, multiplication by ratio, proportion, graph and etc. (for further study, we refer to Hersant, 2005, techniques).

Task type T_2^{Ar} appeared in the two textbooks (Ar_1 and Ar_2), but it was only as an exercise. In the exercise section, we also found a variation of T_3^{Ar} that is symbolised as $T_3^{Ar'}$. The example of this type of task is:

> $T_3^{Ar'}$: A contractor estimates a bridge to be completed within 108 day, if it is done by 42 workers. After working 45 days, the work is stopped for 9 days for some reasons. Determine how many workers, which must be added to finish on times (Nuharini & Wahyuni, 2008, p. 159).

To solve this type of task requires considerate student autonomy. They should master in mathematics modelling before they apply technique from T_3^{Ar}.

We can point out four important observations from the discussion above: 1. proportion is located in one subchapter with a connecting sentence to the previous subchapter and there is a large variation in the titles of chapter where proportion is explained; 2. the authors tend to use informal examples (daily activity case) to introduce the notions of proportion; 3. there are two types of task that are provided by textbooks in the main texts (T_1^{Ar} and T_3^{Ar}); 4. to work with exercises, students sometimes require to develop new variations of techniques demonstrated in the main task (T_3^{Ar}).

How proportion is introduced in the geometry theme

Proportion was introduced as similarity in the geometry domain. Similarity was always explained together with congruence. Due to the focus of research, we will only consider the similarity theme. We found connecting sentences in G1 and G3 which support students to relate arithmetic techniques to geometry techniques. G1 tries to connect similarity with proportion using scale , whereas this theme is also discussed in arithmetic domain. For further explanation, see the example bellow:

> We already studied about scale in the seventh grade. Pictures that are of same scale can be found by magnification or reduction. So that scaled picture has the same proportion as the real picture. We can say that the scaled picture and real picture are similar (Wagiyo, Mulyomo, & Susanto, 2008, p. 7)

However, G3 used plane geometry to introduce similarity and there was no connecting sentence in G2.:

> We already learned about triangles, rectangles, squares, trapezoids and kites. In this chapter, we will discuss about the similarity of those figures (Agus, 2007, p. 1).

To explain similarity, the authors of three textbooks used more formal definitions such as:

> Two figures are similar if the corresponding angles have the same measure and the ratio of the leghts of corresponding side is equal (Sulaiman et al., 2008, p. 10).

We counted the number of examples of the two types of task T_1^G and T_3^G:

Type of task	G_1	G_2	G_3
T_1^G	6	3	4
T_3^G	4	1	4

Table 4: Number of examples in the main text

From Table 4, we can see that the number of T_3^G was smaller than T_1^G, because in many examples T_3^G is explicitly given that the two figures are similar. Therefore, students only need to compute the unknown side. On the other hand, student need to consider the property of similar figures to solve T_1^G. Furthermore, there is also T_4^G whereas students need to develop their own technique:

> T_4^G: A rectangular frame of photographs is 40 cm × 60 cm, and a rectangular photograph is 30 cm × 40 cm. Are the frame and the photograph similar? Suppose we modify the size of the frame so that the frame and the photograph are similar. What is the size? (Sulaiman et al., 2008, p. 8).

Students need to elaborate new techniques to solve task T_4^G because they face a new type of task which is different from the example.

From the above, we can conclude four things: 1. similarity is introduced together with congruence; 2. to explain similarity, the authors use formal definitions; 3. there are two types that appear as examples in the textbooks (T_1^G and T_3^G); 4. students sometimes require autonomy to develop new variations of techniques demonstrated in the main task T_4^G in the exercise.

How the arithmetic and geometry themes are connected to each other

Based on the discussion, we can see the relation between two themes, especially the use of arithmetic techniques as a part of geometry techniques. There are two types of tasks in arithmetic (T_1^{Ar} and T_3^{Ar}) which correspond to geometry type of task (T_1^G and T_3^G).

There is a relation between arithmetic and geometry themes, but we consider it to be relatively weak. From the data, there is only one textbook (G_1) in the 9th grade which refers explicitly to 7th grade proportion. The other textbooks discuss about plane geometry (G_3) and there is not even a connecting sentence in G_2.

CONCLUSION

We have identified the main types of tasks related to proportion in the domain arithmetic and geometry as they appear in the textbooks. The introduction of proportion in arithmetic theme is more informal than for the geometric theme. For example, in arithmetic the authors use daily life activity as a tool to explain what proportion is. While in geometry, the definition of similarity is based on formal definition. However, we also found that both themes have varied types of tasks, including tasks with a considerable variety as concerns the student autonomy required: from tasks that require techniques to be reproduced from an example to main tasks that require a technique to be developed independently by the students.

We found that textbooks establish two mathematically related types of tasks in each of two themes within different domains, and found that the explicit link between these types of tasks is relatively weak. Making this link is seen explicitely by students could help them experience mathematics as connecting body knowledge. Furthermore, we point out that methodology of this study can contribute as a new approach to analyse textbooks or a new approach to choose a textbook.

REFERENCES

Agus, N. A. (2007). *Mudah belajar matematika 3: untuk kelas IX sekolah menengah pertama/madrasah tsanawiyah* [Easy way to learn mathematics 3: for grade 9]. Retrieved from http://bse.kemdikbud.go.id/

Barbé, J., Bosch, M., Espinoza, L., & Gascón, J. (2005). Didactic restrictions on the teacher's practice: The case of limits of functions in Spanish high schools. In C. Laborde, M.-J. Perrin-Glorian, & A. Sierpinska (Eds.), *Beyond the apparent banality of the mathematics classroom* (pp. 235–268). Dordrecht, The Netherlands: Springer.

Chevallard, Y. (1999). L'analyse des pratiques enseignantes en théorie anthropologique du didactique. *Recherches en didactique des mathématiques, 19*(2), 221–265.

Chevallard, Y. (2002). Organiser l'étude. Structures et fonctions. In J.-L. Dorier, M. Artaud, M.Artigue, R. Berthelot, & R. Floris (Eds.), *Actes de la 11ème Ecole d'Eté de Didactique des Mathématiques* (pp. 3–22). Grenoble, France: La Pensée Sauvage.

Chevallard, Y. (2015). Teaching Mathematics in tomorrow's society: a case for an oncoming counter paradigm. In S. J. Cho (Ed.), *The proceedings of the 12th International Congress on Mathematical Education: Intellectual and attitudinal challenges*. 8–15 July 2012, COEX, Seoul, Korea (pp. 173-187). New York, NY: Springer.

Chevallard, Y., & Bosch, M. (2014). Didactic Transposition in Mathematics Education. In *Encyclopedia of Mathematics Education* (pp. 170–174). Dordrecht, The Netherlands: Springer.

Fitzpatrick, R. (2008). *Euclid's Elements of Geometry*. Retrieved from http://microblog.routed.net/wp-content/uploads/2008/01/elements.pdf

García, F. J. G. (2005). *La modelización como herramienta de articulación de la matemática escolar. De la proporcionalidad a las relaciones funcionales* (Doctoral dissertation). Universidad de Jaén, Spain.

González-Martín, A. S., Giraldo, V., & Souto, A. M. (2013). The introduction of real numbers in secondary education: an institutional analysis of textbooks. *Research* *in Mathematics Education, 15*(3), 230–248. doi: 10.1080/14794802.2013.803778

Gueudet, G., & Trouche, L. (2012). Teachers' work with resources: Documentational geneses and professional geneses. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From Text to 'Lived' Resources* (pp. 23–41). Dordrecht, The Netherlands: Springer.

Hersant, M. (2005). La proportionnalité dans l'enseignement obligatoire en France, d'hier à aujourd'hui. *Repères IREM, 59*, 5–41.

Miyakawa, T., & Winsløw, C. (2009). Didactical designs for students' proportional reasoning: an "open approach" lesson and a "fundamental situation". *Educational Studies in Mathematics, 72*(2), 199–218. doi: 10.1007/s10649-009-9188-y

Nuharini, D., & Wahyuni, T. (2008). *Matematika 1: konsep dan aplikasinya: untuk kelas VI SMP/MTs I* [Mathematics 1: concept and application for grade 7]. Retrieved from http://bse.kemdikbud.go.id/

Pepin, B. (2012). Task Analysis as "Catalytic Tool" for Feedback and Teacher Learning: Working with Teachers on Mathematics Curriculum Materials. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From Text to 'Lived' Resources* (pp. 123–142). Dordrecht, The Netherlands: Springer.

Sulaiman, R., Eko, T. Y. S., Nusantara, T., Kusrini, I., & Wintarti, A. (2008). *Contextual teaching and learning matematika: sekolah menengah pertama/ madrasah tsanawiyah kelas IX* [Contextual teaching and learning mathematics: grade 9] (4th ed.). Retrieved from http://bse.kemdikbud.go.id/

Wagiyo, A., Mulyono, S., & Susanto. (2008). *Pegangan belajar matematika 3: untuk SMP/MTs kelas IX* [Book for studying mathematics 3: for grade 9]. Retrieved from http://bse.kemdikbud.go.id/

Wagiyo, A., Surati, S., & Supradiarini, I. (2008). *Pegangan belajar matematika 1: untuk SMP/MTs kelas VII* [Book for studying mathematics 1: for grade 7]. Retrieved from http://bse.kemdikbud.go.id/

Wintarti, A., Harta, I., Rahaju, E. B., Wijayanti, P., Sulaiman, R., Maesuri, S., . . . Budiarto, M. T. (2008). *Contextual teaching and learning matematika: sekolah menengah pertama/ madrasah tsanawiyah Kelas VII* [Contextual teaching and learning mathematics: grade 7] (4th ed.). Retrieved from http://bse.kemdikbud.go.id/