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# Conducting mathematical discussions as a feature of teachers' professional practice

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*We analyse teacher's actions during whole class discussions in exploratory classes (in which students are asked to design their own strategies) and their relation to students' learning. Data is collected through participant observation, with videotaping of lessons. The results show that the exploratory approach favours the emergence of disagreements among students and their formulation of generalizations and justifications provided that the teacher intertwines guiding and suggesting actions and makes appropriate challenging actions at key points. In such whole class discussions, the teacher has to make important decisions in relation to problematic situations raised by students' difficulties or unforeseen responses as well as by the need to figure out productive ways of continuing a discussion.*

**Keywords:** Teacher practice, mathematical discussions, communication, reasoning.

## INTRODUCTION

An exploratory approach to mathematics teaching seeks to propose students situations where they have to deal with tasks for which they do not have an immediate solution method or in which a new representation, concept or procedure may be useful. This approach creates opportunities for students to build or deepen their understanding of concepts, representations, procedures, and mathematical ideas. The students are called to play an active role in interpreting the questions proposed, in representing the information given and in designing and implementing solving strategies which they are called to present and justify to the whole class. This teaching approach is based in the fundamental distinction between task (the objective to be achieved) and activity (the work to be done to achieve this goal) (Christiansen & Walther, 1986). The work on exploratory classes develops usually in three phases (Ponte, 2005): (i) presenting and

interpreting the task; (ii) carrying out the task individually, in pairs, or in small groups; and (iii) presenting and discussing results and doing a final synthesis.

In this study, we focus our attention in the work of the teacher in leading whole class discussions, in which students present and justify their solutions and question the solutions of their colleagues. We do not seek to establish a normative framework, saying what the teacher “must” do, but rather to analyze the phenomena that take place in the classroom, in order to understand the situations that occur and the actions that the teacher can do to promote students' learning. As students carry out exploratory work, the diversity of situations that may arise is very large and depends on the age level of the students, their mathematics ability, the culture of the classroom, and the mathematical topics under study. In addition, one must keep in mind the influence of other factors such as teachers' and students' concerns about assessments, school guidelines on curriculum management, textbooks and other resources available, physical conditions of the room, etc. In this way, our study has essentially an analytical stance, aiming to examine the diversity of actions that the teacher is called to undertake in whole class discussion moments and their relation to student learning.

## THE DYNAMICS OF DISCUSSION MOMENTS IN THE CLASSROOM

Teachers' practices have an important influence on students' learning (Ponte & Chapman, 2006). An important aspect of such practices is the nature of the tasks that the teacher proposes to their students. If a task only requires students to select and apply a solution method that they already know, they have just to identify and carry out this method. By contrast, a task with challenging features (Ponte, 2005) or involving a high cognitive demand (Stein, Remillard, & Smith,

2007; Stein & Smith, 1998) may lead to a diversity of strategies that can be compared and evaluated, resulting in interesting classroom discussions.

Another aspect that frames teachers' practices is the nature of the classroom communication (Bishop & Goffree, 1986; Franke, Kazemi, & Battey, 2007). A fundamental aspect of communication are the questions posed by the teacher. Among these, inquiry questions that admit a range of legitimate responses are particularly useful. In addition, another important feature of classroom communication is the process of negotiation of mathematical meaning (Bishop & Goffree, 1986), leading students to make new connections among mathematics ideas, and helping the teacher to recognize their sometimes unforeseen points of view. Franke, Kazemi, and Battey (2007) stress the importance of processes that support students' language development, like revoicing. Whole class discussions provide opportunities for particular forms of communication, such as explanations and argument and are attracting a growing interest of mathematics education researchers (Bartolini-Bussi, 1996; Cengiz, Kline, & Grant; 2011; Fraivillig, Murphy, & Fuson, 1999; McCrone, 2005; Scherrer & Stein, 2013; Sherin, 2002; Stein, Engle, Smith, & Hughes, 2008; Wood, 1999).

The teacher role is to prepare the moment of discussion, taking into account the work carried out by the students and the class time available. In order to do this, Stein, Engle, Smith and Hughes (2008) highlight the importance of anticipating how students might think, to monitor their work, to gather relevant information, to select aspects to note during the discussion, to sequence the students' interventions and to establish connections among the different solutions during the discussion. A preparation made under these conditions is an important support for conducting a discussion. However, the actual development of a discussion involves other issues beyond the establishment of connections. Many of these issues cannot be fully predicted prior to the discussion, but create problems that the teacher must be prepared to face. As Sherin (2002) indicates, the teacher needs to be able to balance aspects relating to mathematics knowledge, which requires filtering ideas focusing students' attention in fundamental ideas, and aspects related to mathematical processes that require a frequent attention.

Seeking to identify situations of particularly productive discussions, both Potari and Jaworski (2002) and McCrone (2005) emphasize the value of challenging students mathematically. Wood (1999) underlines the potential of exploring disagreements among students, as teachers lead them to justify their positions and encourage other students to join the discussion. Fraivillig, Murphy and Fuson (1999) and, subsequently, Cengiz, Kline and Grant (2011) developed a framework of analysis for the teacher's actions in conducting mathematical discussions that distinguishes three main types of actions: (i) eliciting actions, to lead students to present their methods, (ii) supporting actions, to promote their conceptual understanding, and (iii) extending actions, to widen or deepen students' thinking. In another study, Scherrer and Stein (2013) developed an intervention to support teachers in analyzing whole class discussions based in four main coding categories of moves: (i) those that begin a discussion; (ii) those that further the discussion by elaborating or deepening students' knowledge; (iii) those that elicit information; and (iv) other moves.

With a similar intent, Ponte, Mata-Pereira and Quaresma (2013) developed a framework that assumes that the teacher performs actions directly related to the topics and the mathematical processes as well as actions that have to do with management of learning (Figure 1). Focusing their attention on actions related to the mathematical aspects, they point out that inviting actions are used to start a discussion and guiding actions allow leading students on solving a task through questions or observations that implicitly point the way forward. In informing/suggesting actions the teacher introduces information, presents arguments or validates students' answers. Finally, challenging actions seek to lead students to produce new mathematical knowledge. In informing/suggesting, guiding, and challenging actions it is possible to identify fundamental aspects of mathematical processes such as (i) representing (constructing, using, or transforming a representation), (ii) interpreting, including the establishment of connections, (iii) reasoning, including formulating a strategy to achieve a goal, producing a statement, generalizing procedure and justifying, and (iv) evaluating, making judgments about a concept, representation, or solution. A generalization may concern a definition, a statement or a procedure and a justification may be informal and related to the context of the situation or more formal as is the hallmark of mathematical work.

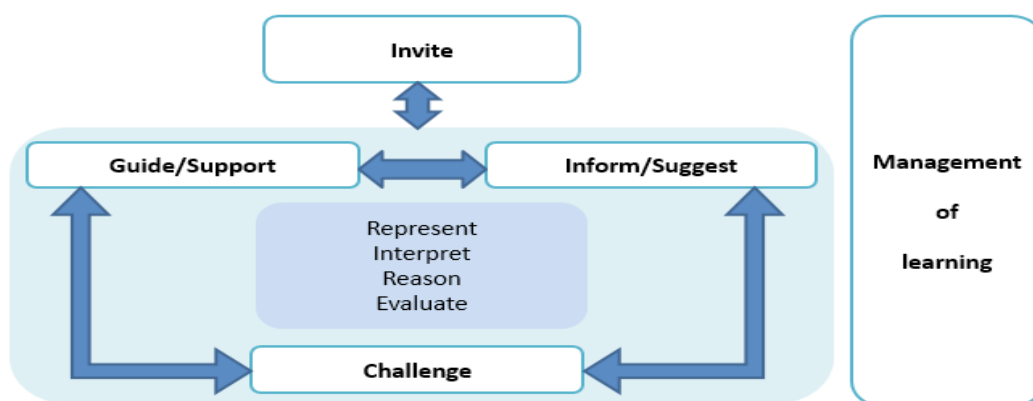


Figure 1: Framework to analyse teachers' actions (adapted from Ponte et al., 2013)

## RESEARCH METHODOLOGY

This study follows a qualitative and interpretive approach (Denzin & Lincoln, 1989) using participant observation. Both authors assumed the role of teachers (striving to follow an exploratory approach) and researchers – as one conducted the class, the other acted as a participant observer. The grade 6 class, with 19 students, is in a rural elementary public school, in a deprived area. The students, usually, show little commitment to school activity and do not get themselves much involved in working in the mathematics class. The study involves five 90-minute lessons, in which students carried out several tasks presented in three worksheets. The first worksheet included diagnostic questions on comparing, ordering, adding and subtracting rational numbers, the second aimed to introduce the multiplication of a natural number by a fraction and the multiplication of two fractions, and the third was intended to develop the notion of operator in the context of problem solving. After the introduction of the task, the students began by working in pairs and the teacher monitored their work, helping them to move on, when necessary, but striving to not provide direct responses to the questions stated in the task. Finally, there was a whole class discussion, in a register of dialogical communication (Ponte, 2005).

The classes were recorded on video and the whole class discussions were integrally transcribed. Data analysis began by identifying the segments in the discussion of the solution of each task, coding the teacher's actions according to the categories shown in Figure 1. Then, we sought to establish relationships between these actions and specific events as regards interpretations, representations, and reasoning made by the students. For this paper, we selected two epi-

sodes that illustrate several aspects of these relationships.

## DEALING WITH STUDENTS' DIFFICULTIES

In this episode we find two rather common situations in the classroom: (i) students with difficulty in understanding a written mathematical question and, (ii) students with difficulty in expressing their thinking. We show a first situation with several guiding and suggesting actions but where a challenging action proves to be critical and a second situation in which guiding and suggesting assume an identical role and there is a very low level of challenging. This takes place as students work on a task in a mathematical context (Figure 2) that asks to evaluate the validity of a statement involving two fractions. This task is a problem that requires the students to figure out that they either must find counterexamples or justify that the statement is always true.

**Task 1.**  $\frac{2}{4}$  is larger than  $\frac{1}{3}$ ,  $\frac{4}{5}$  is larger than  $\frac{3}{4}$ . Do you think it is possible to make the following statement: "If we want to compare two fractions and we verify that one of them has the numerator and the denominator larger than the other fraction, we can immediately conclude that this is the larger fraction"? Justify your answer.

Figure 2: True or false? (task based on Lin & Tsai, 2012)

As students began working individually, they immediately show difficulty in understanding what is asked in the question and the teacher realizes the need to promote a whole class interpretation of the statement and in helping students find a solution strategy:

Teacher: The two cases are true. (...) OK, this and this [ $\frac{2}{4} < \frac{1}{3}$  and  $\frac{4}{5} < \frac{3}{4}$ ] are true. May I

always say that whenever the numerator and the denominator of a fraction are larger than the numerator and the denominator of the other fraction, then [the fraction] which has larger numerator and denominator is always larger than the second [fraction]? Does this always happen?

A student: No...

Teacher: How can you know if it always happens or not?

Daniel: Doing more fractions...

Teacher: Finding more examples... It may be a good suggestion from Daniel...

The teacher recalls the main aspects of the statement ("this and this are true") and then makes a more general statement ("when the numerator and the denominator of a fraction are larger than..."). The students realize that the statement is true in some cases but have difficulty in knowing what to do to know whether, in general, the statement is true or not. The teacher makes an inquiry question ("How can you know if it happens always or not?") and this leads Daniel to suggest a promising strategy. The teacher supports this idea and she revoices it formally in more appropriate terms.

In this first segment, as the students show difficulty in finding a strategy to answer the question, the first intervention of the teacher helps them to interpret the statement and is a guiding action, which is followed by an inquiry question a challenging action. The final intervention, supporting the proposal of Daniel, is a suggesting action. The emphasis of the teacher's intervention is in interpreting (the task and its different elements) as a basis to support students' reasoning as the aim is knowing and justifying whether a given statement is mathematically valid or not.

In the sequence of this exchange, during the whole class discussion of the results, Guilherme, presents a counterexample for the statement, identifying fractions that satisfy the given conditions ( $\frac{2}{4}$  and  $\frac{3}{16}$ ) but which do not verify the inequality. To compare these fractions he converts them in percent. The teacher considers interesting that the whole class observe this solution, but some students do not understand it:

Teacher: So, Guilherme found... a way to transform fractions in percent and... He

found that  $\frac{2}{4}$  is 50%, right? And what did you find about  $\frac{2}{4}$ ,  $\frac{2}{4}$  was how much? Some of you discovered... Changed the fraction into a decimal...

Jaime: It was 0.5.

Teacher: Oh! It was 0.5. And so, 0.5 in percent is...

Guilherme: It is 50%.

Teacher: Oh! It is 50%. Oh, so that means that he arrived to the same conclusion as you did, but using a different representation . . . So Guilherme prefers to work with percent . . . So, he discovered that  $\frac{2}{4}$  is 50% and  $\frac{3}{16}$  is 18,75%... Why have you done this, Guilherme?

The teacher leads the students to compare the representations used by Guilherme (percent) and by the remaining students (decimal). With a final question, the teacher seeks to guide Guilherme, supporting him in explaining his solution.

In this second segment the teacher asks a student to present his solution, which she found to have a remarkable originality, but is faced with the problem that the student has great difficulty in explaining his reasoning. As many students are confused, the teacher's actions alternate between guiding and suggesting, with no challenging actions. There is much attention to representations and their transformations (converting between decimals and percent) but the focus of the teacher's interventions is in interpreting, revoicing the student's statements in a more understandable and correct way, in order to allow an interpretation and understanding of the other students in the class.

## CHALLENGING STUDENTS

Next we show a situation that begins with a teacher challenge which is then followed by inviting, guiding and informing/suggesting actions, which lead the students to establish a first generalization connecting multiplication of an integer by a fraction with successive addition of fractions and a second generalization that highlights an understanding of equivalent fractions. It takes place when students work on the task shown in Figure 3 that asks for the value corresponding to seven repetitions of a certain magnitude in a contextualized situation. The students had not yet learned to multiply a whole number by a fraction. It was expected that they would solve the task through

repeated addition, perhaps proposing by themselves a definition for the multiplication of a whole number by a fraction.

**Task 2.** In the grade 6 class of the school Wide Horizon, the teacher made the following problem: "Every morning, Raquel drinks  $\frac{1}{4}$  of liter of milk. How much milk does she drink in a week?" You must solve the problem yourself and justify your answer.

**Figure 3:** Task involving the multiplication of a natural number by a rational number

Two students solved the task using repeated addition of seven equal fractions. However, in their solution they wrongly indicate that  $\frac{1}{4} + \frac{1}{4}$  is equal to  $\frac{2}{8}$ . During the whole class discussion, in a first segment, the teacher decides then to question how much is  $\frac{1}{4} + \frac{1}{4}$ . The students indicate several answers, some correct, such as  $\frac{2}{4}$  and 0.50, and some incorrect such as  $\frac{1}{8}$  and  $\frac{2}{8}$ . To guide the students in distinguishing among correct and incorrect answers, the teacher draws a pictorial representation (a rectangle divided in four equal parts) and asks again the students what will be the response.

Daniel, who had already presented a response to the question  $\frac{1}{4} + \frac{1}{4}$  as  $\frac{2}{4}$  and as a decimal (0.50), suggests a new answer, using the equivalent fraction  $\frac{4}{8}$ . The teacher notes that the student is thinking in fractions equivalent to, decides to validate his solution and asks for a justification (a challenging action):

Teacher: Exactly,  $\frac{4}{8}$  would be also an answer. Why? Why is  $\frac{4}{8}$  equal to  $\frac{2}{4}$ ?...  
 Guilherme: Because it is 0.50.

Guilherme's justification is based on a change of representation. At this moment teacher decides to take the opportunity to recall equivalent fractions, emphasizing the relationship that exists among  $\frac{2}{4}$ ,  $\frac{4}{8}$  and  $\frac{1}{2}$ , and this leads to a new discussion segment.

Driven by the intervention of Guilherme and the suggestion of the teacher, Edgar suggests another equivalent fraction. The following dialog takes place:

Edgar: Oh! Teacher, I know another... 8 divided by 16 also does it!  
 Teacher: Also does it...  $\frac{8}{16}$  also does it... Very good... Any other that also does it?

As two students (Juliana and Edgar), in the last class, had also made an interesting discovery related to this issue, the teacher encourages them to indicate it. Juliana corresponds to this invitation, stating a generalization:

Juliana: A number divided by its double will always yield its half."

The teacher challenges then the students to give more fractions equivalent to  $\frac{1}{2}$ , and they correspond in an enthusiastic way:

Teacher: Very well...  $\frac{2}{4}$  is equal to  $\frac{1}{2}$  that is equal to  $\frac{8}{16}$ ... And I want another one!  
 Rui: So, now 16 divided by 32...  
 Teacher:  $\frac{16}{32}$ . And I want still another one...  
 Students: 32 by 64.  
 Teacher: Ah... Very good,  $\frac{32}{64}$ . Still another...?  
 Students: 64 and 128...

Other students join the discussion and suggest more fractions equivalent to  $\frac{1}{2}$ . The teacher supports this enthusiasm, revoices their suggestions using a correct fraction language and challenging them to find other fractions that satisfy the same condition.

In summary, at the beginning of this episode several students show that they do not recall the procedure to add two fractions with the same denominator. The teacher seeks to lead them to understand the rule to add two unit fractions, using for that purpose a pictorial representation. When all agreed that  $\frac{1}{4} + \frac{1}{4} = \frac{2}{4}$ , and assuming the opportunity provided by the fact that different correct responses were already provided, the teacher begun challenging the students to provide justifications regarding equivalent fractions and to find further equivalent fractions. In this episode, the teacher's most important actions are challenging, although one recognizes inviting, guiding and informing/suggesting actions as well. Starting from a simple procedural question, the teacher ends up leading an inquiry-oriented reasoning, with the establishment and use of a generalization to produce equivalent fractions.

## CONCLUSION

This paper shows how teachers' actions may unfold during whole class discussions conducted within an exploratory approach. In the first episode the teacher

seeks to support students in interpreting a written mathematical statement and leads those who solved a question correctly to explain it to their colleagues. The teacher provides some challenge but uses mainly guiding and suggesting actions, without indicating the solution to the students. Drawing on counter-examples, the teacher seeks to make the elements available to the whole class so that the students can figure out that the statement is false. In the second episode, after some work on pictorial representations to figure out a correct answer, the teacher challenges the students to present more answers, seeking the emergence of disagreements and, in response, the students produce a sequence of equivalent fractions. That is, in both episodes challenging is a critical action (Potari & Jaworski, 2002; McCrone, 2005) but needs to be underpinned by other types of actions. The way the teacher intertwines guiding and suggestion actions and makes appropriate challenging actions at key points is critical to foster students' involvement and to achieve the learning goals, notably, (i) when students present promising conjectures, (ii) when there is room for important justifications, and (iii) in situations that may prompt fruitful conjectures from the students. In addition, both episodes show how the teacher may promote the interconnection of representing and interpreting and create opportunities to foster students' reasoning, notably asking them for generalizations and justifications (Ponte, Mata-Pereira, & Quaresma, 2013).

These whole class discussion moments provide many opportunities for interpreting statements and using representations (Bishop & Goffree, 1986), for improving the students' language revoicing their claims (Franke, Kazemi, & Battey, 2007), for establishing disagreements (Wood, 1999), and formulating generalizations and justifications (Lannin, Ellis, & Elliot, 2011). However, whole class discussion moments also create many problems for the teacher, requiring the ability to deal with unforeseen situations and to notice opportunities for promoting students' learning (Scherrer & Stein, 2013). The discussion episodes presented in this paper include many moments in which the teacher needs to make decisions with respect to different situations, which are constituted as problems that she has to deal with in the course of the action. Some of these problems have to do with students' difficulties in understanding what they can do in a proposed task or in interpreting some aspect of a solution provided by another student.

Other problems arise from unexpected responses from students, sometimes correct and other times incorrect. There are also problems which arise from the students' difficulty in explaining their reasoning. Finally, other problems arise from the need to manage, in a productive way, the range of students' responses and in keeping an appropriate pace for the classroom work. That is, besides planning the discussions and anticipating possible students' difficulties (Stein et al., 2008), the teacher must be ready to make important decisions in relation to problematic situations raised by students' difficulties in understanding the tasks, in figuring out strategies, in expressing themselves and by unforeseen students responses. In addition, at many points the teacher needs to figure out what is the most productive way of continuing a discussion. Such problems that conducting whole class discussions raise to teachers' practice creates an important agenda for research concerning mathematics teacher professional development.

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