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Teaching actions conducting mathematical whole class discussions

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The aim of this paper is to understand teacher practice conducting whole class mathematical discussions. Teaching practice is seen as an activity based on a motive and carried out through a set of actions. The study is carried out in a grade 9 classroom of an experienced teacher. Data is gathered by classroom observations (video and audio recorded), and by discussions with the teacher. Data analysis is carried out based on a model about teachers' actions. The results show that a central challenging action embodies all discussion segments and that teacher's actions are deeply related to students' knowledge about the situation being discussed.

Keywords: Teachers' practice, teachers' actions, mathematical discussions.

INTRODUCTION

Nowadays, inquiry based or exploratory learning is regarded as a fruitful learning environment in school mathematics. It differs from other environments by the ways in which challenging tasks are used and whole class discussions are conducted. However, proposing such tasks and providing students' opportunities to present and discuss their reasoning poses teachers many challenges (Stein, Engle, Smith, & Hughes, 2008). Organizing and conducting whole class discussions is particularly important to students' learning, being an important feature of teachers' professional practice.

In an exploratory setting, working on a task typically involves the students working on a task individually or in small groups (Ponte, 2005). That is not the case of the task presented in this paper, as it is proposed, discussed and solved within a discussion, making this a rather special whole class discussion. With this study, by analysing this particular whole class discussion, we aim to understand teachers' practice during whole

class discussions, focusing on the teachers' actions and on the mathematical processes involved.

TEACHER'S PRACTICE IN WHOLE CLASS DISCUSSIONS

Teachers' practice may be characterized as the activity developed by the teacher (Jaworski & Potari, 2009) that unfolds in actions established according to an action plan (Schoenfeld, 2000). In traditional classes the teacher control is very high and students' interventions are very limited. In contrast, exploratory classes have many inquiry and divergent moments. In such learning environment, conducting whole class mathematics discussions is not only challenging for the teacher, but essential to students' learning.

Features of teachers' practice during whole class discussions have been highlighted by several authors. Wood (1999) states the relevance of involving students in presenting their solutions and in discussing those of their colleagues. Potari and Jaworski (2002) refer that the level of challenge of teachers' questioning during whole class discussion is an important feature of teachers' practice. Stein, Engle, Smith and Hughes (2008) point out that an essential feature of teachers' practice is to shape students' incomplete or poorly phrased ideas into more precise and powerful mathematical ideas. They argue that a productive mathematics discussion (i) is supported by students' thinking, and (ii) provides important mathematical ideas. They also present a model to prepare and conduct mathematics discussions that include actions of anticipating likely students' responses, monitoring students' responses, selecting students to present their responses, sequencing students' responses, and making connections between students' responses and key mathematical ideas. More recently, Cengiz, Kline, and Grant (2011) identify as main teachers' actions in whole class discussions aiming to extend students

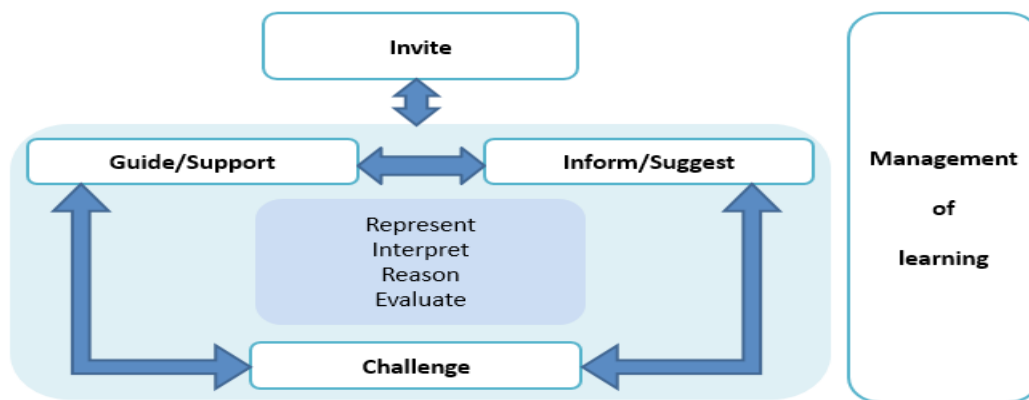


Figure 1: Framework to analyze teachers' actions (adapted from Ponte et al., 2013)

thinking: (i) eliciting actions that aims to elicit students' methods, (ii) supporting actions that aims to support students' conceptual understanding; and (iii) extending actions that aims to extend students' thinking. With a similar intent, Scherrer and Stein (2013) present a guide to analyze teachers actions (moves) during whole class discussion of cognitively demanding tasks that includes (i) begin a discussion, (ii) elaborate or deepen students' knowledge by furthering the discussion, (iii) elicit information, and (iv) other moves, like providing information and thinking aloud.

Teachers' actions during whole class discussions can be distinguished between actions directly related to mathematical topics and processes and actions that mainly relate to management of learning. Ponte, Mata-Pereira, and Quaresma (2013) identify four main categories related to mathematical aspects: (i) inviting actions – leading students to engage in the discussion, (ii) guiding/supporting actions – conducting students along the discussion in an implicit or explicit way in order to continue the discussion; (iii) informing/suggesting actions – introducing information, providing an argument or validating students' interventions; and (iv) challenging actions – leading students to add information, provide an argument or evaluate an argument or a solution. Guiding/supporting, informing/suggesting, and challenging actions, are the main support to develop whole class mathematical discussions, and involve key mathematical processes such as (i) representing – provide, revoice, use, change a representation (including procedures), (ii) interpreting – interpret a statement or idea, make connections, (iii) reasoning – raise a question about a claim or justification, generalize a procedure, a concept or a property, justify, provide an argument, and (iv) evaluating – make judgments about a method or solution, compare different methods. A model that

relates teachers' actions in whole class mathematical discussions and these mathematical processes is presented on Figure 1.

RESEARCH METHODOLOGY

This study follows an interpretative and qualitative methodology. It is developed in a grade 9 class of a teacher with 13 years of experience. According to the teacher, the class has a very productive working environment and fruitful whole class discussions are possible because some of the students have a high performance and all of them are usually eager to participate. Data collection was conducted by the first author in the classroom and used direct observation (with audio and video recording of class) and document collection. Data analysis is based on the categories of the model presented in Figure 1, and is done with the support of NVivo software.

The situation analysed is a whole class discussion that aims to introduce how to solve incomplete 2nd degree equations generically represented as $ax^2 + bx = 0$. Before this situation, the students solved and discussed a problem that, implicitly, involves the zero product rule, but that was not explored with the aim of generalizing this rule to solve incomplete 2nd degree equations. Moreover, the teacher reminded how to solve incomplete 2nd degree equations as $ax^2 = 0$ and $ax^2 + c = 0$, that students had worked on in the previous lesson.

THE WHOLE CLASS DISCUSSION

We present some of the segments of the whole class discussion in order to analyse the teacher's actions to promote students' interpretation of the issues, reasoning and evaluation of solutions.

The first invitation

The teacher begins by writing down on the blackboard the equation $4x^2 - 12x = 0$ and, right after, she invites the students to solve this equation:

Teacher: . . . You have this equation, OK? An incomplete one, we are dismissing c . Who want to try to solve this equation with me? Just with what you already know. You already know how to solve 1st degree equations, how to solve some 2nd degree equations. I could tell you: Let's try to solve this equation. Irina, what would you do?

Irina: I would... Probably, I would get minus 12 to plus 12 in the other side [of the equal sign].

Teacher: So, I will write down [on the blackboard], $4x^2$ equals $12x$.

Irina: Exactly.

Teacher: It is true, this equation [$4x^2 - 12x = 0$] is equivalent to this one [$4x^2 = 12x$].

In this introduction, the teacher begins by inviting students to solve an equation. She also prompts students' answers by challenging them to make connections with their previous knowledge about equations. As Irina begins solving the equation, the teacher informs students by representing on the blackboard what she said. As this is the first time that students are solving this sort of equation, the teacher also suggests that Irina is going in the right direction by stating that both equations are equivalent. This suggesting action by the teacher leads Irina to keep going on her proposal to solve the equation:

Irina: Then, I would do the same procedure as usual. I would do squared x equals $12x$ divided by 4.

Teacher: [Writes down $x^2 = 12x/4$] Perfectly equivalent. Five stars. [Waits but Irina stands quiet.] Now the question is, when I am solving an equation, what is the aim?

Students: Find the value of the unknown.

Teacher: [Questioning Irina] Did you found the value of the unknown?

Once more, the teacher represents on the blackboard what Irina said and suggests that she is moving forward on her solving process. As Irina seems not to know how to continue, the teacher decides to high-

light the aim of solving an equation, aiming to guide students on moving forward.

In this segment, the teacher's actions are marked by challenging the students on making connections between previous knowledge and this new situation. As students begin solving the equation, the teacher mostly uses guiding and suggesting actions to lead students to achieve the envisioned aim of using previous knowledge. Despite the aim of making connections, these guiding and suggesting actions are mostly representing actions, allowing students to stay focused on the solving process.

Overcoming misconceptions

As Irina does not know how to continue the solving process, the teacher selects Hugo from the students who want to participate:

Hugo: x^2 less x equals zero less 4 plus 12.

Teacher: Is that possible?

Students: No.

Teacher: Why? He would do this [writes down on the blackboard $x^2 - x = 12 - 4$]

Despite suggesting before that Irina was going in the right direction, the teacher does not suggest that Hugo is presenting an invalid proposal. Instead, she challenges the class to interpret Hugo's intervention and to further justify that interpretation. The teacher represents Hugo's proposal on the blackboard alongside with Irina's. Then, the teacher upturns the validation of Hugo's answer and, to do so, she suggests the students to establish connections with the monomial operations:

Teacher: Is this possible? This expression [$4x^2 - 12x = 0$] is equivalent to this one [Hugo's equation]? Remember that this is like a whole, a monomial, I cannot disconnect the coefficient from the literal part. I do not have something like this Hugo.

Towards this, Hugo presents another way to solve the equation, letting no time to his colleagues to justify why his first proposal was invalid:

Hugo: So... What if I add the x from 12 to x^2 ?

Teacher: So, first of all, can I add this two small monomials [x^2 and $12x$]?

Students: No!
 Guilherme: You can.

Once more, the teacher challenges the students to interpret Hugo's statement, aiming to clarify the properties of operations with monomials. Most students seem to know that one cannot add two monomials with different degrees, but Guilherme states the opposite. So, the teacher promotes the analysis of this mistake, indicating that he can continue the solving process:

Teacher: So, do it.
 Guilherme: $4x^2$ is the same as having $4x$ times $4x$.
 Teacher: Is it?
 Students: No.
 Teacher: Guilherme asked if $4x^2$ is the same as having $4x$ times $4x$.
 Students: No.
 Teacher: Why not?

As Hugo, Guilherme proposes an invalid procedure, so, the teacher keeps challenging students to interpret his proposal and, to support students, she revoices Guilherme's statement. Again, the teacher challenges the students to justify their answer, but Guilherme realizes his mistake and corrects it right away:

Guilherme: But we could get, for example, $2x$ times $2x$, $4x^2$. And then, we would get $2x$ times $2x$ less $12x$ equals zero.
 Teacher: OK, to that point it is OK. I will write down for you. $2x$ times $2x$, less $12x$ equals zero. It is also equivalent, OK?

With this new statement, the teacher suggests that Guilherme is doing a right procedure, arguing that this equation is equivalent to the previous ones. She also informs students about Guilherme's proposal by representing it on the blackboard. Then, despite being asked to keep going on his solving process, the student does not know what to do. So, in order to guide Guilherme, the teacher revisits the aim of solving an equation, which allows him to continue:

Guilherme: Not to stand again $2x$ times $2x$, after one would get... I do not know if I could divide by k . $2x$ less $12x$ equals zero divided by $2x$.

Teacher: This one here [$2x$] is multiplying the whole expression?

Students: No.
 Teacher: When I divide, when I move to here dividing by $2x$, it means I divided the whole expression.

As the new proposal of Guilherme is not valid, the teacher suggests a reinterpretation of his statement and recalls the use of an equivalence principle of solving equations in order to show that he cannot do the procedure he is proposing.

In this segment, the students did several wrong procedures, highlighting that they do not have yet a mastery of monomial and polynomial properties. These wrong procedures led the teacher to challenge students to make connections with polynomial operations. As students have some knowledge about this topic, the teacher keeps using challenging actions instead of suggesting and guiding actions as she did in the previous segment. Nevertheless, when needed, the teacher uses suggesting and some guiding actions to move the discussion forward. All these actions are mostly around interpreting and reasoning about that interpretation.

An unexpected proposal

As before with Irina, Guilherme cannot move forward on solving the equation, so, the teacher invites another student to continue:

Teacher: Maria.
 Maria: Teacher, getting back to that one [referring to $x^2 = 12x/4$].
 Teacher: Getting back to this [pointing out the equation on the blackboard].
 Maria: Is it possible to do x^2 divided by x equals 12 divided by 4 ?
 Teacher: Oh, Maria did this... Have you seen what she has done?
 Students: Yes.
 Teacher: I am going to do what she has done, this x [on the left side of the equal sign]... She wrote this, I will continue [writes down $x^2/x = 12/4$].

As Maria begins where Irina stopped, the teacher guides students to focus their attention by revoicing and pointing out what Maria is referring to. As Maria presented her proposal, the teacher informs students

by representing it on the blackboard. Before moving forward, a student states that Maria cannot separate 12 from x in $12x$, which leads to a segment of the discussion very similar to the segment *Overcoming misconceptions*. After clarifying some monomial properties, the teacher asks Maria to proceed with her way to solve the equation:

Teacher: And now what? Maria, I stopped here, what's next?

Student: x equals 3.

Teacher: In the other lesson we didn't get to this point. Now I am curious. Look here. If I would go this way, x^2 divided by x ...

Students: It's x .

Teacher: Equals...

Students: Three!

Maria: So, it is possible!

Maria does not get to answer as there is a colleague that anticipates the response. The teacher asks again the question, suggesting students to develop the representation of the equation made by Maria. At this point, several students are following Maria's reasoning and completed solving the equation with no further support from the teacher. Thereby, Maria concludes that her way to solve the equation is valid. Nevertheless, the teacher explores the situation a little bit further by challenging the students to interpret the situation and then to justify their interpretation:

Teacher: Is it possible?

Students: It is.

Teacher: How can I see if the solution is correct?

Student: Replacing.

As a student readily answers how to justify that the value they found is a solution of that equation, the teacher suggests most of the computational procedures needed:

Teacher: So, let's do it slowly. Do it with me, slowly. Jorge, can you do it slowly? Here is 3 [referring to x]. x^2 ...

Lourenço: It's 9.

Teacher: 9 times 4...

Lourenço: 36.

Teacher: 36. 3 times 12...

Students: 36.

Teacher: 36 less 36...

Students: Is going to be zero.

This segment develops around Maria's proposal that emerge from the teacher's first challenge to solve the equation through making connections with previous knowledge. As students engage in the discussion about Maria's proposal, the teacher mostly suggests procedures that lead students to find a solution of the equation and, later, to justify that solution. In between, the teacher challenges students to justify that the value found is an equation's solution, which students easily do.

Comparing solutions

At this point, a student, by trial and error, figures out that zero could also be a solution, which led the teacher to introduce a discussion aiming the generalization of the zero product rule. This segment of the discussion had a first section where students factorized the proposed equation and a second section where the students figured out its solutions. Then, the teacher proposed some problems involving the zero product rule to students to solve in small groups. Previously to the discussion of those problems, the teacher recalls Maria's solving process:

Teacher: Earlier, I'll call it now "Maria's process". By Maria's process how many solutions have we found?

Student: That has one [solution].

Teacher: One solution. But when we did it by factorizing, we got two. My question is... We saw, we confirmed, that actually this equation has two solutions. The question is, what happened in this process, why zero does not emerge as a solution? This would be... x equals zero or x equals 3, two possibilities. Here [in Maria's process] only emerges x equals 3. Suddenly it looks like a solution has been hidden. Luís readily said that there was another despite this one... Why doesn't the solution show up here? [No student reacts.] Why did I found here x equals 3 and there I found out two solutions, zero and 3?

The teacher begins by saying what is going to be discussed, challenging students to evaluate the two processes of solving the equation. Mário says that solving using the zero product rule is the way to "obtain all possibilities", so the teacher guides the students to interpret Maria's process. But the students centre their attention on zero, referring to zero on the sec-

ond member of the equation $4x^2 - 12x = 0$. At some point, Mário asks the following question:

Mário: Teacher, a question. Is zero the second solution in equation that... c equals zero [referring to c in $ax^2 + bx + c = 0$]

Teacher: That is a very good question. But my question... That is really a good question, it is very relevant. But my question here is why doesn't zero appear here? Before we go to that one. Why doesn't it appear here? And there it appeared.

Mário's intervention represents a very interesting generalization about zero being a solution of any 2nd degree equation like $ax^2 + bx = 0$. The teacher decides not to explore it but to move back to Maria's solving process, challenging students to get some justification on the lack of one of the solutions in Maria's process. A student gets back to the zero rule process' solution but the teacher guides students again to focus on Maria's process. Hélder intervenes:

Hélder: Because one cannot divide by zero.

Student: Yes you can!

Teacher: [Writes a huge 2/0] Question to the class...

Student: What a huge 2!

Teacher: It really is to be seen. Hélder said, here does not appear zero as a solution because Maria, when did this step, divides by x . And, if I place here a zero, what Hélder is saying is that it doesn't make sense to place here a zero. My question is, why? This equals to what?

Student: Zero.

João: Undetermined.

As Hélder's answer to the teacher prior challenge lead to a disagreement, the teacher challenges students to justify if Helder's statement leads to a justification or not. This discussion leads the students to conclude that it is not possible to divide by zero, as Madalena states ("Even so, no number can be divided by zero") and the teacher supports reinforcing ("No number can be divided by zero").

As the evaluation of the solving processes relies on the reason why Maria's process leads to just one solution, and that reason is justified, the teacher ends this whole class discussion.

In this last segment of the discussion, the teacher main action is challenging students to evaluate both solving processes. To promote students' activity towards evaluation aim, the teacher guides the students to focus on Maria's process and challenges them to justify the single solution it yields. In this segment there are few informing actions, and teacher's challenging actions lead to many students' reactions.

CONCLUSION

In this whole class discussion, we identify a variety of teachers' actions. We note that each segment is structured by a main action. As this class discussion aims to introduce new knowledge, one could expect that central actions would be mostly guiding, subtly or explicitly leading students to a solving procedure using the zero product rule. Nevertheless, attending this class characteristics, the teacher begins all four segments with a challenging action leading to quite unpredictable students' responses. This teachers' choice provided rich learning opportunities that would probably not occur if guiding would be the central action in each segment. We also note that, albeit marked by a main challenging action, all segments differ in their nature. Teachers' actions seem to depend on students' knowledge about the situation, as challenging actions are followed by other challenging actions when the students have the tools to embrace those actions and are followed by guiding or suggesting actions when the mobilization of students' knowledge needs further support.

Regarding mathematical processes, the teachers' central actions mostly rely on interpreting processes. Nevertheless, more demanding processes also occur as in *Comparing solutions*, where challenging evaluating actions assume great importance. As with the actions themselves, the mathematical processes addressed also depend on students' knowledge. New situations to students require a focus in representing and interpreting processes, while segments where students have some previous knowledge to rely on, reasoning and evaluating processes are more likely to occur, as in *Overcoming misconceptions* or *Comparing solutions*.

In summary, the model used to analyse teachers' actions (Figure 1), extending the previous models of Stein and colleagues (2008) and Cengiz, Kline and Grant (2001), contributed to better understand

teachers' practice. Also, an interesting relationship between teachers' actions, mathematical processes and the situation itself emerged from the analysis with this model, which suggests the need of further research of teachers' practice during whole class discussions focusing on these features.

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