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# Interdiscursivity and developing mathematical discourse for teaching

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*This paper aims at further elaborating on a redefined theory of mathematical knowledge for teaching in terms of participation in discourse. This redefined theory of mathematical discourse for teaching is used to analyze data from mentoring sessions in initial teacher education. The results indicate that the mentor teacher, who is a more central participant in the mathematical discourse for teaching, is able to more dynamically switch between the different discourses related to mathematics teaching than the pre-service teachers—referred to as interdiscursivity.*

**Keywords:** MKT, discourse, teacher education, mentoring dialogues.

## INTRODUCTION

Learning is often defined as a process of acquiring knowledge; this metaphor of acquisition is popular in education research (Sfard, 1998). Following this metaphor, an aim of education is for students to learn something (i.e. acquire knowledge), and the teacher's responsibility is to ensure that students gain the required knowledge. In order to handle the tasks that are involved in this work of teaching mathematics, teachers need a particular kind of professional knowledge. Ball, Thames and Phelps (2008) have proposed a theory of mathematical knowledge for teaching (MKT) that has become widely used. Ball and colleagues define MKT as "the mathematical knowledge needed to carry out the work of teaching mathematics" (p. 395). Following Shulman's (1986) theory of teacher knowledge, they describe several subdomains of MKT. These subdomains of knowledge are seen as distinguishable and measurable. Measuring particular subdomains of MKT is, however, challenging. When analyzing the connection between teachers' responses to multiple-choice MKT items and additional written reflections, Fauskanger and Mosvold (2013) found that

teachers use knowledge from different subdomains of MKT when responding to items that were designed to measure one particular subdomain only. Sticking with the metaphor of acquisition, a quandary emerges: How can teachers use knowledge from different subdomains of MKT when responding to an item, if these subdomains are distinct and items have been developed to measure distinct subdomains only?

Instead of adhering to the acquisition metaphor to knowledge and learning, I follow the participation metaphor (Sfard, 1998; Skott, 2013) in this paper when I look at mathematical knowledge for teaching from a discourse perspective. In doing this, I adopt Sfard's (2008) commognitive framework and follow Cooper's (2014) suggestion of redefining MKT into "mathematical discourse for teaching". Cooper argued that such a redefinition of the MKT framework is useful for interpreting data from professional development of teachers. My aim is to extend this redefined framework further and apply it in analysis of example data from initial teacher education. With this theoretical framework, I will argue, the quandary with the distinct subdomains of MKT becomes obsolete.

## THEORETICAL FRAMEWORK

Before Shulman (1986) presented his ideas about teachers' professional knowledge, classroom research often had a focus on observable behavior of teachers and/or students. His main contribution was to focus researchers' attention on the role of teachers' knowledge. Shulman, who adhered to an acquisition metaphor, suggested that the knowledge required by teachers could be divided into several distinct categories. Among the most well-known are subject matter knowledge and pedagogical content knowledge, and these two categories are often presented as import-

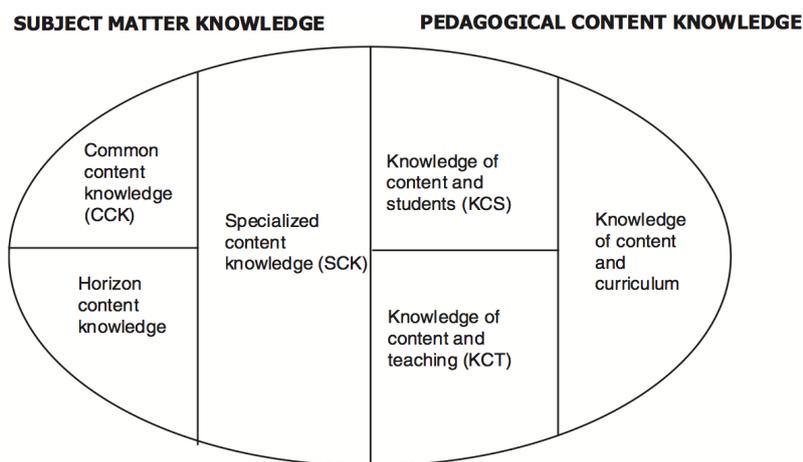


Figure 1: “The egg”—depicting MKT and its subdomains (Ball et al., 2008, p. 403)

ant aspects of teacher knowledge (e.g., Baumert et al., 2010). When Ball and colleagues (2008) presented their theory of MKT, they continued to scrutinize Shulman’s initial categories. Subject matter knowledge and pedagogical content knowledge were divided into subdomains, and this model was depicted in a figure that has later been referred to with nicknames as “the egg” or “the oval” (Figure 1).

Sfard’s (2008) theory represents a different view. A main idea is that cognition and communication are inseparable, and she combines the two terms into a new theoretical term: “commognition”. She defines thinking as “an individualized version of (interpersonal) communication” (Sfard, 2008, p. 81). Knowledge is related to participation in discourse – not acquisition of an objectified entity – and learning is seen as a permanent change in discourse. This change can be either on an object level (where new words are introduced) or on a meta-level (where the rules of discourse change). For the researcher, then, the study of communication and participation in discourse(s) becomes pertinent.

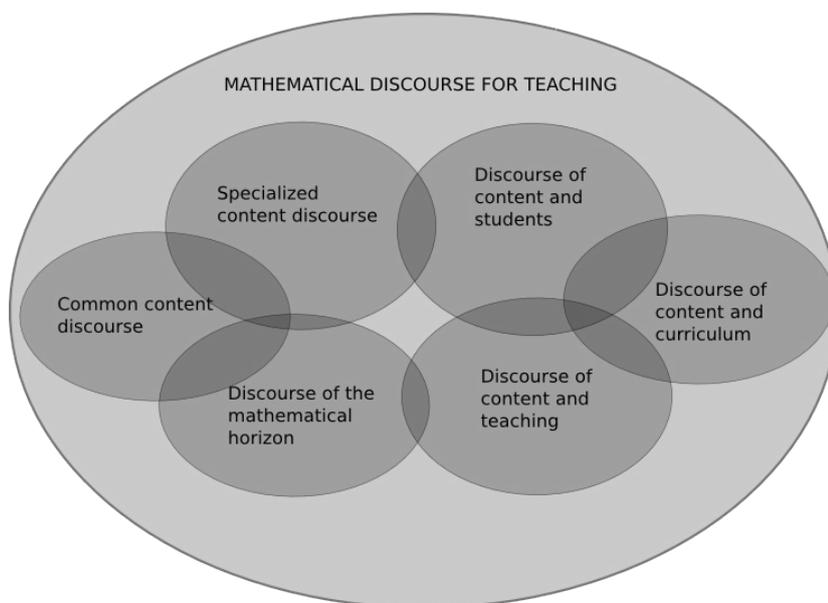
Cooper (2014) applied Sfard’s theory when he attempted to translate the MKT model into a discursive framework. He suggested redefining MKT as “mathematical discourse for teaching” (p. 338), and he substituted subject matter knowledge with a mathematical content discourse; pedagogical content knowledge was replaced with what he referred to as “pedagogical discourse for teaching” (ibid.). Building upon the ideas of Sfard (2008), he suggested that the following features could identify these discourses: 1) main words that appear in the discourse, 2) visual mediators that are commonly used in the discourse, 3) routines that are

distinctive to the discourse, and 4) endorsed narratives. In this paper, I attempt to take Cooper’s reinterpretation of MKT one step further and introduce the subdomains of MKT. I envisage this as a revised model consisting of several partly overlapping discourses (Figure 2).

Instead of following Cooper’s approach and investigate words, mediators, routines and narratives in a discourse, I focus on participation in different discourses. I build upon the theories of Lave and Wenger (1991). They focused on how learners move from being peripheral participants to full or central participants in communities of practice. Instead of discussing communities of practice, however, I focus on communities of discourse (c.f., Sfard, 2008). I follow Sfard’s (2008) definition of discourse as: “The different types of communication, and thus of commognition, that draw some individuals together while excluding some others” (p. 91). Instead of investigating how teachers increase their MKT, I attempt to study the process in which (pre-service) teachers move towards full participation in the mathematical discourse for teaching.

**METHODS**

In this paper, I do not investigate the complete process of becoming full participants in the mathematical discourse for teaching. Instead, I use data from mentoring sessions between three pre-service teachers and their mentor teacher as well as classroom data as an exemplification of one part of the process. The data material was collected as part of a larger project: Teachers as Students (TasS). An overall aim of this project was to investigate how pre-service teachers develop knowledge, skills and competence for teach-



**Figure 2:** A tentative model of mathematical discourse for teaching

ing. In this paper, I use a discursive approach to investigate pre-service teachers' mathematical discourse for teaching.

In Norway, initial teacher education is organized as a four-year bachelor program. National curriculum guidelines require a total of 100 days of teaching practice, and teaching practice is supposed to be integrated in all subjects. The participants in this study were in the second year of their teacher education, and the data material was gathered in connection with a period of teaching practice. As example data, I use transcripts from two mentoring dialogues between a group of pre-service mathematics teachers and their mentor teacher before and after one lesson that was taught by the pre-service teachers. The group consisted of three pre-service teachers: Fiona, Rachel and Harry (all pseudonyms). The mentor teacher—who is a practicing teacher—is simply referred to as this in order to avoid confusion.

Recordings from the mentoring dialogues were transcribed verbatim, and these transcripts provided the foundation for my analyses. The analyses in this paper are meant to illustrate a possible use of an extended version of Cooper's (2014) redefinition of MKT into mathematical discourse for teaching as a framework, and I focus in particular on how the participants draw upon different discourses related to mathematics teaching in the mentoring dialogues.

## RESULTS AND DISCUSSIONS

In the following, some excerpts from the data material will be presented as an illustration. A main emphasis will be on the mentoring dialogues before and after a mathematics lesson in 8<sup>th</sup> grade, but data from the actual lesson will be included in order to contextualize the mentoring discussions. The goal of the lesson was to enhance the students' understanding of the equal sign.

### Pre-mentoring dialogue

When the three pre-service teachers met with their mentor teacher for a last mentoring session before teaching the lesson, their main focus was on discussing the lesson plan. They had discussed with the mentor teacher the day before, and they had agreed that they would have to adjust the level of the content they were going to present. In order to make it easier and more understandable for the students, they decided to start with a realistic problem. The context of the problem is that Fiona wants to go shopping, and she has to figure out how much money she could spend—given the various expenses that would be charged every month. With this problem as a starting point, Fiona explains how they are going to introduce the equal sign:

20. Fiona: And ask about how we are going to get that box (a), and what they did. Yeah, they subtract (b), and then you kind of take away 3500 from this side and that side, in order to balance [it]. And then we get to the

equal sign (c), and then kind of having scales and things like that, that kind of becomes a theme then, and work with the equal sign to really make them understand that it has to be balanced on both sides (d).

As we can see, Fiona has already included the unknown – the “box” – in this problem (a), but she is thinking of the equal sign as a main theme for the lesson (c). In this utterance, she navigates in a pedagogical content discourse. She describes how they will present the content for the students (d), and it can thus be interpreted as if she mainly draws upon a discourse of content and teaching. We can see that she anticipates a certain student response already (b), but her main focus is on the discourse of content and teaching. As a response to this, the mentor teacher goes into another discourse when she argues that they might be going too far too soon:

21. MT: Yes, what the equal sign really means (e). But isolating the box on one side, then you have really gotten far (f). Having something isolated on one side and move things, then you are really up there on the algorithm level right away (g). You might not have to talk about that at all, but I think they will see it when you say: “Okay, what’s missing here?” They kind of see what is missing (h), and that is what the unknown is. It is something that is missing, and that you’ll try to figure out. And then there are many ways of figuring it out. I mean, there are many ways of finding the unknown without rearranging and getting one box isolated, which is really the last part [in the process] perhaps. So try to hang in there as long as possible, only focusing on the understanding of what it is (i), and then one of the goals for the lesson is to understand the equal sign. Because then it is easier, and then you know what you want them to learn from the lesson. Being practical about it.

In the beginning of her utterance, the mentor teacher goes into a discourse of mathematical content. She starts by pointing at the true meaning of the equal sign (e), before shifting her focus to the mathematical horizon (f). Focusing on the mathematical implications of their choices in relation to the mathematical location of where the students are currently work-

ing, she claims that they have already moved to an algorithm level (g). I interpret this as an example of how the mentor teacher draws upon her experience from participating in a discourse on mathematical content in general and a discourse of the mathematical horizon in particular. The mentor teacher then draws upon her experience from the classroom and shifts into a discourse of content and students when she says that, “they kind of see what is missing” (h). Then she shifts again and moves into a discourse of content and teaching when she makes suggestions about how the pre-service teachers might present it to the students (i). The pre-service teachers still appear to be more peripheral participants in these discourses, and Fiona responds to the mentor teacher by drawing upon her experiences from the previous period of teaching practice instead (j):

22. Fiona: The equal sign is, in our last period of teaching practice (j), the equal sign and the understanding of an unknown was inseparable.

The mentor teacher follows up by confirming that the equal sign and the unknown constitute a sensible goal for the lesson. After this, they continue to discuss different aspects of the lesson plan until the mentoring session ends after 18 minutes.

### **Classroom discourse**

In the lesson, the pre-service teachers start by presenting themselves, since this is the first time they are in that particular class in this period of teaching practice. After a round of presentations, Fiona starts teaching the lesson. “I have a problem that I want you to help me solve,” she says. Then she explains that she wants to go shopping, but as a student in teacher education, there are certain expenses she needs to take into consideration. On the blackboard, she writes down the amount of money she gets from loans every month (6700 NOK), and then she writes down all the expenses below. The question is, “how much money is left for shopping?” After having given the students some time to think, one of the students presents 1200 NOK as an answer to the question. They spend some time discussing this before Fiona presents another similar problem: “It is the national day, and we have 200 NOK in our pocket. We want to buy ice cream, and an ice cream costs 20 NOK. How can we write down an expression that helps us calculate the number of ice creams we can buy?” The students come up with

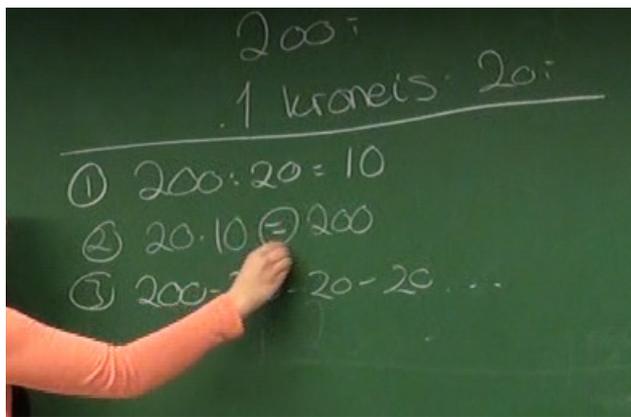


Figure 3: Buying ice cream

different alternatives, and Fiona writes them down on the blackboard. She circles in the equal sign and asks if anyone knows the name of this (see Figure 3).

When a student responds, “is equal”, Fiona continues by asking what the equal sign actually means. A student responds: “what comes behind it is often the answer”. Fiona hesitates a bit before answering that it is partly correct, but that this is something they will come back to later in the week.

### Post-mentoring dialogue

The day after having taught the lesson, the pre-service teachers meet with the mentor teacher for another mentoring session. The mentor teacher provides them with feedback, and they discuss different aspects of what they have observed in the lesson.

45. MT: And then you came to: What does the equal sign mean?
46. Fiona: Mhm
47. MT: “Behind it comes the answer”, said one of the students. And then another one said: “add together and insert [an] equal [sign] later”
48. Fiona: Yes, and then I thought...
49. MT: Andrew (name of a student) said it.
50. Fiona: (laughs)
51. MT: And you didn’t make much out of it really, and this was perhaps, you could have taken it further and shown: What does it become then? But you decided not to make much of it, and in this situation I think it was okay.
52. Fiona: Yes, I didn’t really≈
53. Harry: ≈I was thinking, when we were working with the cards, then he had the ex-

pression which is equal to, and the expression which is equal to, that’s what I meant.

54. Fiona: Yes, I was thinking≈
55. MT: ≈“If you have fifteen plus five equals twenty, and then minus three equals seventeen”, that’s how he said it.
56. Fiona: Mhm

The mentor teacher starts by referring to the question asked by Fiona in the lesson (45). The mentor teacher then follows up by presenting some of her observations from how students responded in the classroom (47). All of the pre-service teachers as well as the mentor teacher experienced the classroom discourse, but we see how the mentor teacher is the more central participant in the mathematical discourse for teaching (51) whereas Fiona and her fellow pre-service teachers are more peripheral participants. It can be argued that this relates to their different roles as mentor teacher and students in pre-service teacher education, but I argue that the differences can also be seen in how the participants navigate between different discourses. When evaluating Fiona’s response, the mentor teacher mainly draws upon a discourse of content and teaching, but then the focus shifts:

57. MT: Is this a correct use of the equal sign? (k) Because it is a very common thing to do for students, and it is a wrong use of the equal sign (l).

In this follow-up question (57), the mentor-teacher first draws upon a subject matter discourse (k), and then she combines this with a comment from a discourse of content and students (l): “it is a very common thing to do for students”. In the following, Harry tries to go back to the classroom discourse, but the mentor teacher interrupts him and continues:

58. Harry: He had≈
59. MT: ≈especially when it is multiplication and everything (m), then many [students] often add directly to the answer (n), and then the balance is really all wrong.

After having briefly drawn upon her experience from participating in a discourse of content and students (57), the mentor teacher is now back in a subject matter discourse (m), but she continues to draw upon the discourse of content and students (n) when saying that many students “often add directly to the answer”.

Fiona acknowledges the mentor teacher's statement with a "Mhm", before the mentor teacher continues:

60. Fiona: Mhm  
 61. MT: So you could have taken it further and shown: "Are both sides equal here?" and perhaps made them conscious about this already now, early on, to begin with (o). Because it is likely something many are going to have misconceptions about (p).

In this utterance (61), the mentor teacher draws upon both a discourse of content and teaching and a discourse of content and students. First, she reflects about how Fiona could have acted differently as a teacher (o)—which relates to a discourse of content and teaching—and then she adds a comment about this being something many students will probably have misconceptions about (p). The latter is part of a discourse of content and students. In this example, like in the other examples above, we clearly see that the mentor teacher does not stay within one of these discourses only, as if they were separate, but she continually switches between them—often within the same sentence. Throughout these examples, it the mentor teacher appears more dynamic in how she shifts between these different discourses—all related to a mathematical discourse for teaching—whereas the pre-service teachers are more caught in one discourse at a time.

## CONCLUDING DISCUSSION

In the literature, researchers discuss supposedly distinct sub-domains of mathematical knowledge for teaching (e.g., Baumert et al., 2010). As a potential threat to the idea of the sub-domains of MKT being distinct, some studies indicate that teachers draw upon different aspects of knowledge when responding to MKT items (e.g., Fauskanger & Mosvold, 2013). When using a participation metaphor, however, it is unproblematic to consider that participants draw upon other discourses from which they have experience in participating. These discourses do not have to be completely separate, although they are different types of communication that include some participants and exclude others (Sfard, 2008).

In his attempt to bring mathematical knowledge for teaching under Sfard's (2008) discursive framework of commognition, Cooper (2014) distinguished between two components of what he referred to as math-

ematical discourse for teaching: a mathematical content discourse and a pedagogical content discourse. In doing this, he switched from an acquisition metaphor of learning and knowledge to a participation metaphor (cf. Sfard, 1998). Whereas Cooper used this as a framework for interpreting data from professional development, I have attempted to use it in the context of mentoring dialogues in initial teacher education. I suggest that the mathematical discourse for teaching is even more complex and compound than suggested by Cooper (2014), and indications of this can be found in the results presented.

Using a combination of Sfard's (2008) commognitive framework and the theory of learning as legitimate peripheral participation by Lave and Wenger (1991), I propose that the development of a mathematical discourse for teaching is related to the ability to dynamically draw upon different discourses for teaching mathematics. When discussing the planned and observed lesson on the equal sign, the mentor teacher uses a discursive move that can be referred to as interdiscursivity when she continually draws upon her experience from different parts of the mathematical discourse for teaching. The pre-service teachers appear to have less ability to use interdiscursivity, and this might be related to less experience from participating in the different parts of the mathematical discourse for teaching—or, put differently, that they are still peripheral participants in this discourse. There is a possibility, however, that this apparent difference can be explained by the different roles of the mentor teacher and the pre-service teachers in the mentoring dialogues. Cooper (2014) found that the learning situation in professional development is more symmetrical than what is often found in children's learning. In this study on pre-service teachers' learning in teaching practice, it can be argued that the situation is more asymmetrical. This issue of power relations could be investigated further in discussions among teachers with different levels of experience (but who do not have a mentor/student relationship). I have focused mainly on interdiscursivity in this paper, but this is only one of several issues that might emerge from analyses where the framework of mathematical discourse for teaching is applied. My discussion of this issue is meant to serve as an example, and I suggest that a redefinition of MKT in terms of participation in discourses should be further investigated.

With a discursive definition, a view of MKT – and knowledge in general – as some kind of object or hidden entity can be avoided. Discourse for teaching is not a latent or hidden trait, but something researchers can investigate and analyze more directly. I acknowledge that the framework of mathematical discourse for teaching is still in development, and further studies could for instance investigate a merging of this framework with Skott's (2013) framework of patterns of participation. Applying such a participatory and discursive approach to investigate MKT could then be seen as part of a larger initiative to develop a more coherent approach to understanding the work of teaching mathematics where acquisitionist terms are avoided.

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