Teacher and textbook: Reflection on the SDT-model

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This study investigates in-depth how two teachers use the textbook in mathematics classrooms. Their use of the textbook was analysed in the light of a socio-didactical tetrahedron (SDT) which proved to be a powerful model to describe many aspects of textbook use that we encountered. Our results showed that the teacher acted as the mediator between the textbook and the students, but the lower part of the SDT helped us to identify the influence of social factors on textbook use which is an aspect that cannot be disregarded.

Keywords: Mediation, socio-didactical tetrahedron textbook, teacher, teaching.

INTRODUCTION

The issue of *how textbooks shape the teaching and learning of mathematics* has been the subject of many studies (Fan, Zhu, & Miao, 2013).Textbooks can be considered as artefacts which are dynamically used in classrooms, influencing the instruction (Johansson, 2006). Their content follows and largely reflects the requirements and intentions of the intended curriculum. Various studies have shown that teachers rely heavily on textbooks for lesson preparation, teaching new subject matter, practicing and giving homework assignments (e.g., Pepin & Haggarty, 2001; Johansson, 2006; Glasnović-Gracin, 2011).This paper investigates in-depth two mathematics teachers' practice in relation to the use of the textbook in the mathematics classroom.

THEORETICAL FRAMEWORK

Textbook-teacher-student

The teacher's role as a *mediator* between the textbook and the students has been considered by various authors (e.g., Luke, de Castell, & Luke, 1989; Love & Pimm, 1996). Pepin and Haggarty (2001) give the research systematization on the use of textbooks through six main domains, one of which relates to the teacher as the mediator of the text. They assert that it is the teacher who decides which textbook to use, when and how it is used, which parts to use and in what order, and when and to what extent the students will work with the text. Traditionally, mediating between the students and textbook has always been the teacher's function (Luke et al., 1989).

According to Remillard (2000), textbooks can alter the teaching strategies of teachers, but conversely it is the teachers who choose what to use and what not to use from the textbook. Sosniak and Stodolsky (1993) found that teachers have autonomy and that they like to mediate between the textbook content and their students. Also, teachers do not feel that the textbooks control their teaching. In addition, the authors believe that in order to understand the use of textbooks in classrooms, the thoughts, actions and working conditions of the particular teacher need to be taken into consideration. This brings us to the socio-didactical tetrahedron.

The socio-didactical tetrahedron

The use of textbooks in the classroom can be considered through the model of the didactical tetrahedron where the vertices are student-teacher-textbook-mathematics (Rezat, 2006). However, this model does not encompass societal and institutional aspects, which are also important factors in mathematics education. Therefore, Rezat and Sträßer (2012) proposed a more comprehensive model of didactical tetrahedron, called the socio-didactical tetrahedron (SDT). We imagine the basic didactical tetrahedron (Figure 1.) where the vertices are artefact-student-teacher-mathematics, as being put in such a position where the vertices student, teacher and mathematics lay on the bottom of the tetrahedron. These three vertices are extended by the social and cultural parameters, forming a compre-

hensive socio-didactical tetrahedron. The new bottom vertices in the SDT are: conventions and norms about being a student and about learning; conventions and norms about being a teacher and about teaching; and the public image of mathematics. These three vertices are connected to each other through the bottom edges. Since these social and cultural parameters lay in a complex relationship, other points on the bottom edges are highlighted, such as institution, noosphere, and peers and family (Rezat & Sträßer, 2012).These social and institutional parameters are often considered to be "hidden" or "less visible" because the persons involved are often not conscious of them. Therefore, we can see the SDT as an iceberg in the water: we often only consider the "visible" didactical tetrahedron involving just the teacher, student, mathematics and artefacts, and we forget (or we are not conscious of) the social and other parameters involved.

With all its highlighted points, the SDT model is a powerful enough tool to provide a structure for textbook use and to show the cultural interplay between educational and social context. For example, Rezat (2013) conducted a qualitative study on students' use of mathematics textbooks in Germany. The findings show that social impacts are important factors which influence students' use of textbooks. Some aspects of textbook use cannot be explained without taking into account the whole SDT.

Previous research and research focus

The study presented in this paper is a follow-up research on the large-scale study reported in Glasnović Gracin (2011). Using a questionnaire with multiple choice items, the previous study investigated the role of mathematics textbooks in lower secondary education in Croatia (grades 5 to 8). The survey involved nearly one thousand mathematics teachers, which is about half of the total number of mathematics teachers in grades 5 to 8 in Croatia. The results showed that teachers use textbooks for lesson preparation to a great extent. Participants said that new material is mainly presented by the teacher at the front of the class followed by students working individually on the textbook exercises. So, the teachers consider the textbook to be an important source of practice exercises for students. The survey also showed that most teachers select a textbook mainly according to the quality of its examples and problems. The coherence between the textbook methodology and the students' age was also an important factor which proved to be relevant for 64% of teachers.

In this new study, we wanted to examine in a more in-depth way how mathematics teachers use the textbook in their teaching practice. Since the previous study encompassed a quantitative method, here we wanted to use a qualitative approach. The aim was to investigate how teachers use textbooks in mathematics classrooms and to identify potential factors that influence such use. The findings are observed in

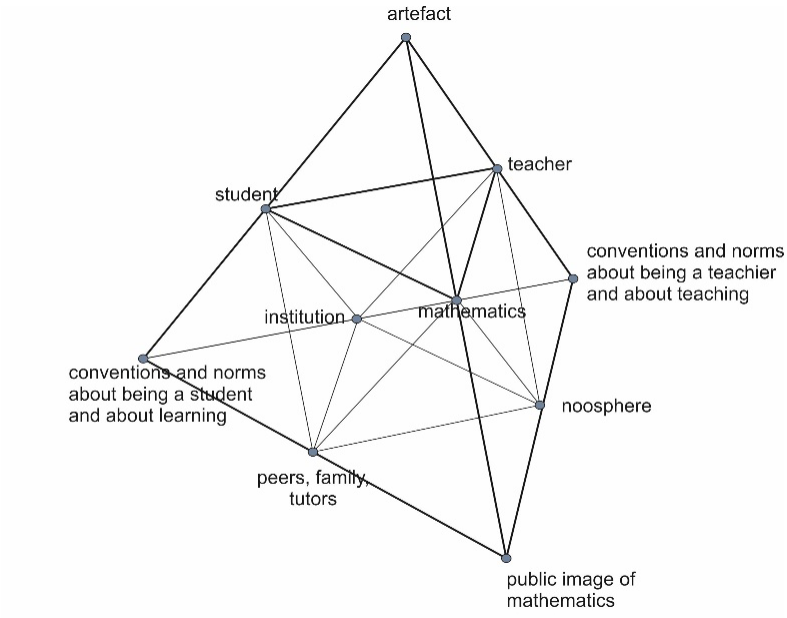

Figure 1: Socio-didactical tetrahedron

the light of the SDT, in order to better understand the teaching and learning of mathematics.

METHODOLOGY

Participants

The study involved two female mathematics teachers, Mrs. S and Mrs. D, from lower secondary education in Croatia (in our education system, this is a primary school, grades five to eight). The teachers chosen for this study are both experienced teachers: Mrs. S has 15 years of teaching experience and Mrs. D 20 years of teaching experience. Both participants have a degree in mathematics education and use the same textbook series. It is the most commonly used textbook series in Croatia, used by about 65% of the whole student population (data retrieved from the Ministry of Education, Science and Sport).The structure of this textbook can be described as the "exposition – examples – exercises" model. It has been found that this kind of model predominates in the structure of mathematics textbooks (Love & Pimm, 1996).

The Croatian education system

In the Croatian education system, primary school is compulsory and lasts for eight years. Secondary school is not compulsory, but the majority continues to go to school after finishing primary school. Secondary school lasts three or four years, depending on the type of school (i.e. vocational or grammar). One lesson unit lasts for 45 minutes in both primary and secondary school. Textbooks are compulsory and all textbooks used in schools are authorized by an official group of experts appointed by the Ministry of Education. As of 2010, teachers jointly select authorized textbooks for their school for the period of 4 years. Textbooks are usually purchased by parents, except for a short period from 2006 to 2008 when the Ministry of Education provided free textbooks for all primary school students in Croatia.

Method

In this study, we used qualitative strategies in the form of observations and interviews. During October 2013, we observed several lessons given by the participants. In that year, Mrs. S was teaching sixth grade, and Mrs. D grades five and eight.

For the purposes of observation, we designed three main observation categories. The categories were:

— *Minutes of textbook use* (How much time do the students and the teacher use the textbook?)

— *Impact of the textbook structure on instruction* (What is the influence of the textbook content and structure (title, language and symbols, order of worked examples, definitions and rules, didactic intentions) on instruction? Does the instruction follow the textbook page by page? What is taken from the textbook?)

— *Use of the textbook* (For what purposes is the textbook used during the lesson? How is new content introduced? Is the textbook used during the teaching of new mathematical content? What is a practice lesson like? Which sources are used for practicing and homework? Is the textbook used for practicing and reviewing? Did the teacher point out any specific figure, frame or picture from the textbook? For what purposes?)

Prior to each observation, we examined the structure of the mathematical content which was to be taught in that lesson. This helped us in making comments and answering the observation questions. After the classroom observations, we conducted a semi-structured interview with each of the participants. The interview questions come under three main categories, but the interviewer was able to ask additional questions based on the observed lessons or expand upon an interesting point arising during the interview:

— *Impact of the textbook structure on instruction* (Describe how you usually prepare for a mathematics lesson. Does the textbook, in your opinion, influence the structure of your instruction, e.g., using the same title as in the textbook for the lesson unit, using the same definitions, symbols, sequence, didactical approach, worked examples, figures? Explain.)

— *Use of the textbook* (Describe a typical lesson where new content is introduced. Describe a typical lesson with emphasis on practicing. Describe a typical revision lesson before a test. Describe how you select homework activities and from which sources. Do you significantly change your teaching style when you change the textbook?)

— *Strong and weak points of the textbook* (What do you find is lacking in textbooks? What do you like and what do you find helpful about textbooks?)

Data analysis was conducted using the constant comparative method (Corbin & Strauss, 1990). Data gathered from the observations and the interviews were coded thematically. This process allowed us to attend to, and report on, those aspects of textbook mediation that were common to both teachers as well as their differences.

RESULTS

Minutes of using the textbook

We observed four lessons of Mrs. S teaching grade six. The textbook was used intensively in all her observed lessons. Students had the textbook open on their desks all the time. In the first lesson, the textbook was used directly for 10 minutes when students worked on their own solving tasks from the textbook, and for the rest of the lesson the students and the teacher used it occasionally. In the other three lessons, the textbook was used for the entire 45 minutes.We observed three lessons given by Mrs. D. She used the textbook directly only in the first observed lesson for 15 minutes. In the other lessons, the textbook was not used by the students or the teacher at all.

Impact of the textbook on instruction

The textbook had significant impact on the lessons observed in Mrs. S's classroom. In the first lesson, the textbook was followed page by page for more than half of the lesson. The influence of the textbook was also visible in other parts of the lesson. The rules written on the blackboard were the same as the rules in the textbook. In the second lesson, to introduce and teach a new topic, the teacher used the motivational example from the textbook, as well as the worked examples and rules. The exercises for practicing were also from the textbook. In the third lesson, the textbook structure was heavily reflected in the lesson, since the textbook was followed page by page. The fourth lesson was preparation for the up-coming exam, where students solved problems from the textbook in the exact order as given in the textbook. In the interview, Mrs. S explained that she always uses the textbook in lesson preparation. The textbook structure influences her lessons and the textbook is the main source for practicing and for homework. Mrs. S explained that in this way she encourages the students to use the textbooks. She wants her students to use textbooks actively for two reasons. The first reason is "so that children can find at home what we did in class, especially if someone did not understand what I said in school". The other reason is so that students become independent and confident in using various resources, like books, encyclopedias or other curriculum materials. In this way she aims to prepare students for the active use of textbooks in upper secondary school, where the subject matter is more demanding than in the lower secondary grades.

Mrs. D did not follow the textbook page by page in any of the lessons observed. In the first lesson, rules were taken from the textbook, as well as some exercises for practicing. In another observed lesson, the textbook was not even open in front of the students; all the tasks were on a worksheet she had prepared, though some tasks were similar to those in the textbook. All of the lesson titles that she wrote on the board were the same as the titles in the textbooks. In the interview, Mrs. D said that the textbook was not her main resource for lesson preparation. Rather, she uses a variety of available materials such as the internet, a range of textbooks (old and new), professional journals, and ideas from colleagues.

The use of textbook

In the first of Mrs. S's observed lessons, the students used the textbook for learning new content. They were required to read the new unit from the textbook (Reciprocal fractions), become familiar with the new content, and work through the examples and exercises from the textbook. The teacher helped those students who had problems with understanding the textbook's content, thus she acted as mediator between the textbook and the student. The second observed lesson involved the division of fractions. The new content was presented through a teacher-led discussion with the class, where the teacher relied on the textbook, using the model of the chocolate bar (quadrilateral shape) as presented in the textbook. The discussion brought up some rules for fraction division, and the teacher also introduced the case $\frac{a}{b} : n$ which was not in the textbook, which some students remarked on. The practice tasks which followed were also from the textbook, as was the homework. The third and fourth lessons had a very similar structure. The teacher gave a long list of textbook exercises to be done, wrote the page number and exercise numbers on the board, and the students opened their textbooks and individually

worked through the tasks. At the end of the lessons, the teacher assigned the homework, which was also from the textbook.

In the interview, Mrs. S said that she uses the textbook for teaching new mathematical content if she finds that the book covers the subject matter well. If it does, she does not devise her own exposition and examples. However, she says that she does not take all the definitions directly from the textbook:

> "I change a definition sometimes, not because my way is better, but so that students can see that the same thing can be said in different ways..."

At the beginning of the first lesson with grade eight, Mrs. D checked and went over the homework from the previous lesson (the homework was from the textbook).The teaching of the new content of powers was not related in any way to the textbook: the teacher used a story from the Rhind Mathematical Papyrus: "There are seven houses. In each house there are seven cats. Each cat kills seven mice. Each mouse has eaten seven grains of barley and each grain would have produced seven hekats. How many grains of barely will be saved?" The teacher pointed out the rules for the powers in the textbook and the exercises which students should do during the lesson. The teacher did not assign homework from the textbook, but gave a link to a web page with numerous tasks for practicing. The second and third observed lessons were in the fifth grade and in both lessons students worked in groups. In the second lesson, the students competed in a quiz. The tasks were prepared on a worksheet and were not the same as the textbook items. The third lesson was a revision of the topic of whole numbers. Here students participated in a game, which consisted of solving various tasks. The solution to each task was a piece of a puzzle that contributed to the solution of the overall puzzle. The tasks were similar to the items from the review section in the textbook. The game, however, did not come from the textbook.

In the interview, Mrs. D explained that she rarely introduces new procedures and concepts according to the textbook; she likes to use her own ideas. She uses the textbook for students to practice, but she uses other resources as well. For reviewing, she uses the textbook items, or composes similar ones herself. She considers that other textbooks do not greatly differ, having similar exercises.

Strong and weak points of the textbook

In the interview, Mrs. S described her attitude towards the official textbooks she was currently using in the classrooms. She was dissatisfied with the introductory sections of the lessons in the textbooks. However, she liked the selection of worked examples, that there are plenty of exercises from simpler to more complex ones, and that the exercises come right after the exposition.

> "I think it's right that when we learn something [a new concept], the exercises should follow on directly. Not be placed at the end of the chapter. This way parents can find them easily... Most of the parents aren't mathematicians and they don't always know what to give their children for practice, they might give them the wrong tasks."

Mrs. D said that she is satisfied with the textbook structure and exercises. But she also finds some textbook definitions and rules to be inappropriate for the students' age, so she rephrases them:

> "Definitions are sometimes unclear to students, so I translate them to be simple and clear".

She says that the main reason for choosing the current textbook series was financially and socially based:

> "Our school uses this textbook because... many of our students have brothers and sisters, we don't want them to have to buy new textbooks so we decided to continue to use this one and not to change it."

DISCUSSION AND CONCLUSIONS

Both participants consider their position as mediators between the textbook and students, in spite of the fact that they have different approaches to teaching mathematics and the use of textbooks. They both highly control what will be learned and practiced, how and when it will be learned. These intentions are related to the norms about being a teacher and about teaching because the teacher is supposed to prepare and shape the lesson actions. This is related to the SDT face *textbook-teacher-student* with the related social components placed on the bottom of the triangle: institution, conventions and norms about being a student and about being a teacher (see Figure 1).

Mrs. S uses the textbook as the central tool for teaching and encourages her students to use the mathematics textbook intensively. All the exercises done in the four observed lessons were taken from the textbook. Although she complained in the interview that she is not satisfied with the introductory sections of the textbook, in the observed lessons she used the material from the textbook. It may be that she relied on the textbook as a guarantee of stable quality (Pehkonnen, 2004) because the approved textbooks offer security and convenience for teaching (Love & Pimm, 1996). On the other hand, she claims that she encourages the use of the textbook because of the students. Learning how to use the textbook will help them when they attend the subsequent secondary school. Here we come to *institution* on the SDT. This also means that for her using textbooks does not mean just "learning mathematics from the textbook", but also "learning how to use a textbook, i.e. learning for life". Mrs. S mentioned another reason why she encourages the use of textbooks by her students – if the student was absent, or did not understand the subject matter in school, he/she is able to look at the textbook at home with the help of peers or parents. Here we come to *peers and family* on the other face of the SDT. The relationship *textbook-student-mathematics-peers and family* corresponds to the explanation given in the interview that the textbook helps parents to help the students at home. This is one of various reasons why Mrs. S follows the textbook so closely. This is also connected with the textbook being clearly organized – so that students and parents can catch up. The importance of the contents of the textbook being set out in a clear and simple way was mentioned by both participants.

The second participant Mrs. D prepares lessons using a variety of materials, the textbook being just one of them. She chose different and innovative ways both of presenting new concepts and practicing them. This is related to the norms and conventions of being a teacher and about teaching (Remillard, 2009). She likes the structure and the amount of exercises in the textbook, but she used it directly in only one observed lesson. The interview and the analysis of the observed lessons showed that she mostly used the same titles, symbols, definitions and rules as found in the textbook. For example, after a creative introduction to the idea of powers, her students opened the textbooks and copied the rule into their notebooks. The copying of rules and definitions into notebooks was also observed in Mrs. S's classes.

In the interviews, both participants said that they sometimes change the wording of the definitions and rules from the textbook to make them more appropriate for the students. This means that the teachers, during the lesson preparation, reflect on the didactics of mathematics in the textbook. This also corresponds to the norms about being a teacher, because the principle of compatibility of mathematical content with the students' age is one of the basic principles in mathematics education (Kurnik, 2009). The textbook content should be appropriate to the students' age, and at the same time it should be mathematically correct. The teacher should reflect on this interplay during the lesson preparation. This finding leads to the tetrahedron face with the vertices *teacher-textbook-mathematics*.

Mrs. D gave a reason of a social nature as to why the textbook is not frequently changed in their school. The same textbooks are used for a number of years, so that the younger generations can use the textbooks of their older siblings. This means that social and financial reasons are important factors in choosing whether to change the textbook or not. Such use of textbooks can be explained with the relationship within the SDT, *textbook-student-parents-public image of mathematics*. In Croatia, parents buy textbooks and this represents a great expense for them at the beginning of each school year. Also, from the public point of view, mathematics is concieved as a static discipline, with a long known set of concepts, principles, and skills (Cooney, 1985), and the learning of mathematics is concieved through practicing various exercises. Thus, from this persepective, there is no need to change textbook if the textbook contains a good amount of exercises.

All these findings lead to the conclusion that the reflection on the actions within the SDT model can help in understanding the teaching and learning of mathematics. The survey results obtained in Glasnović Gracin (2011) were not sufficient in comprehending the deeper reasons related to the teachers' use of mathematics textbooks. The qualitative approach presented in this paper confirmed and extended the survey results: Croatian teachers use textbooks for lesson preparation, exercises and homework. However, the findings of this qualitative study show in-depth how and why Croatian teachers use the textbooks in the classroom. One particular finding is related to the textbook characteristics. Teachers should have good

mathematical knowledge for teaching, i.e. according to Ball and colleagues (2008), knowledge of content, students, curriculum and teaching, and to know the strong and weak points of the textbooks to be able to decide when to be the mediator between the textbook and students, and when to use other instructional resources for teaching beside the textbook. The study implies that teachers' use of the textbook is multilayered. This utilization is connected with the complex interaction between the STD components. Consequently, textbook use should not be examined separately from social and institutional influences.

Using the SDT model offers great potential and ideas for new studies. For example, the triangle *student-mathematics-textbook* is a very important face of the SDT. This aspect needs to be examined in more depth in order to better understand instruction and to improve instructional quality in the classroom.

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