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# Exploring a framework for classroom culture: A case study of the interaction patterns in mathematical whole-class discussions

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*Research is needed on frameworks that support teachers in the important and challenging work of orchestrating productive problem-solving whole-class discussions. The aim of this paper is to explore a framework for classroom culture with the overarching goal of supporting teachers in conducting class discussions focused on argumentation as well as connection making. Analyses of video-recorded whole-class discussions result in the articulation of some difficulties in clearly distinguishing between certain interaction patterns within different classroom cultures. The overall findings, however, suggest that the framework can be useful for characterizing interaction in terms of an inquiry/argument classroom culture.*

**Keywords:** Interaction pattern, whole-class discussion, classroom culture, inquiry/argument, instructional practice.

## INTRODUCTION

There is great consensus within the mathematics-education field that mathematical instruction needs to provide opportunities for students to participate in instructional practices that develop their mathematical competencies (NCTM, 2000; NRC, 2001). To understand mathematics, reflection and communication are key (Hiebert, Carpenter, & Fennema, 1997). Participating in whole-class discussions of multiple solutions to a challenging problem have great potential to allow students to reflect and communicate. However, interactions in various reform-oriented classrooms differ significantly, and it is important to relate these differences to students' thinking and learning. Supporting teachers in engaging students in interaction that promotes their mathematical thinking is central, and frameworks for teachers' actions can help. More research is needed on such supportive frameworks.

I have previously (Larsson, 2015; Larsson & Ryve, 2011, 2012) investigated ways that teachers can plan and conduct productive whole-class discussions of students' different solutions to challenging mathematical problems and have discussed ways that Stein, Engle, Smith, and Hughes's (2008) model can support teachers in this important and demanding work. In short, the model consists of five practices that build on each other: anticipating, monitoring, selecting, sequencing, and connecting student solutions. However, this model does not explicitly focus on how a teacher could productively interact with students in whole-class discussions.

Among frameworks that focus on interaction (e.g., Boaler & Brodie, 2004; Brodie, 2010), I find Wood, Williams, and McNeal's (2006) proposal especially interesting because they have found that an inquiry/argument classroom culture is closely associated with higher cognitive levels of student thinking. Moreover, Franke, Kazemi, and Battey (2007) see the promise of connecting Wood and colleagues' (2006) interaction patterns to Stein and colleagues' (2008) model, which is central to my research. Wood and colleagues (2006) distinguish between two types of reform-oriented classroom cultures: strategy-reporting and inquiry/argument. In the latter culture, there is a "major shift in participation from an emphasis on the child reporting her/his different strategies to the children as listeners taking over the role of the teacher in questioning, clarifying, and validating mathematical ideas" (p. 235). The role of the listening students is hence crucial for distinguishing between the two types of reform classroom cultures. Wood and colleagues (2006) state that their most important finding is the differences between the two reform classroom cultures.

The overarching goal of my research is to help develop frameworks that support teachers in conducting productive whole-class discussions that focus on argumentation as well as connection making (see Larsson, 2015). In relation to this broad aim, this particular paper explores Wood and colleagues' (2006) framework for interaction patterns. More specifically, it aims to articulate the difficulties, if any, in distinguishing between interaction patterns—in particular, reform interaction patterns. I delineate the conceptual framework and the methodology that I use before presenting my results, illustrated by a fine-grained analysis of one particular whole-class discussion.

### CONCEPTUAL FRAMEWORK

I use Wood and colleagues' (2006) conceptual framework for investigating specific interaction patterns in the whole-class discussion that I analyze. The purpose of their framework is to better understand which opportunities for learning arise in various classroom cultures. Wood and colleagues (2006) divide the interaction patterns into three categories: (i) patterns common to all instruction, (ii) patterns of conventional instruction, and (iii) patterns of reform instruction. The only pattern common to all instruction is *Collect answers*, in which the teacher collects answers to a problem with the purpose of making them public. I have summarized the interaction patterns that characterize conventional instruction and reform instruction in Tables 1 and 2.

### METHODOLOGY

The data source for this paper is a collaboration with a very proficient teacher regarding mathematical problem solving discussions with over 15 years of teaching

experience. I observed this teacher during eight days in one academic year without making interventions, with a particular focus on the teacher's orchestration of whole-class discussions of students' different solutions to challenging mathematical problems. The teacher strives to engage students in inquiry and argumentation in a collaborative spirit, making it interesting to analyze her whole-class discussions with Wood and colleagues' (2006) framework. Data consist of transcribed video-recorded lessons focusing on the teacher during whole-class discussions, audio-recorded pre- and postlesson teacher interviews for every lesson, and collected student solutions. To interpret the videotaped, transcribed whole-class discussions, I performed a fine-grained analysis of four whole-class discussions using Wood and colleagues' (2006) conceptual framework. All lines were coded in segments, each of which was categorized into one interaction pattern according to Wood and colleagues' (2006) descriptions (see my summary in "Conceptual Framework," above). One additional person coded one of the whole-class discussions. The categorizations were then compared, and we discussed them to resolve differences.

### ANALYSIS AND RESULTS

As an illustration of how this particular teacher interacts with her students in a whole-class setting, I use excerpts from a discussion in sixth-grade about students' solutions to the problem "Houses of Cards" (Larsson, 2007). Students allowed for far-reaching generalizations, considering that they were only in sixth grade. This discussion has been chosen because it reflects the typical way this teacher interacts with her students in a whole-class setting.

Interaction pattern	Description	Purpose	Initiator
IRE (Initiate-Respond-Evaluate)	Teacher asks a test question, students' responses are confined to yes/no or right/wrong, and the teacher evaluates.	To check what students know.	Teacher
Give expected information	Similar to IRE, but students' answers can be more open.	To check what students know.	Teacher
Funnel	Teacher leads student(s) to the answer by a number of test questions.	To correct an incorrect student answer without telling the answer.	Teacher
Teacher explain	Teacher gives (often lengthy) explanations of key mathematical ideas/concepts.	To tell students what they are expected to learn and know.	Teacher
Hint to solution	Teacher gives a hint that takes away the challenge of the problem.	To ensure that students reach a correct answer quickly without struggle.	Teacher

**Table 1:** Conventional-instruction interaction patterns (summary of Wood et al., 2006)

Interaction pattern	Description	Purpose	Initiator
Explore methods	Students explain their solution strategy.	To give multiple solution strategies.	Teacher or student(s)
Inquiry	Teacher or student(s) ask questions because they do not understand.	To understand.	Teacher or student(s)
Argument	A student listener challenges an idea because (s)he disagrees, after which students participate, taking turns.	To reach to a resolution.	Student listener
Teacher elaborate	Teacher elaborates on a student's explanation because information is lacking.	To provide more information to the students.	Teacher
Proof by cubes	Teacher uses material either to find the correct answer or to gain understanding.	To get to the correct answer or provide insight.	Teacher
Proof of answer by student explanation	Teacher lets student(s) explain their correct solution.	To ensure that the class hears a correct solution.	Teacher
Focus	Teacher first provides a summary before asking a question that focuses students on what they need to resolve.	To orient students toward key aspects.	Teacher
Build consensus	Teacher tries to have the class agree on a key mathematical idea.	To establish common ground in the class.	Teacher
Check for consensus	Teacher checks with students to see whether they have questions or comments on a student idea.	To open up for questions and comments before moving on.	Teacher
Develop conceptual understanding	Teacher asks a question that addresses a specific idea or concept.	To facilitate students' conceptual understanding.	Teacher
Pupil self-nominate	A student voluntarily offers an idea or insight that goes beyond the topic and explains/justifies the idea.	To have students exercise their autonomy as participants.	Student

**Table 2:** Reform-instruction interaction patterns (summary of Wood et al., 2006)

### Houses of Cards (Larsson, 2007)

Albin and Melvin are building houses of cards as the picture shows.

- 1) How many cards does a house of cards contain that has
  - a) 3 floors?
  - b) 4 floors?
  - c) 5 floors?
  - d) 12 floors?
  - e)  $n$  floors?



**Figure 1**

- 2) A house of cards consists of 408 cards. How many floors does it have?
- 3) Make up a problem of your own and solve it.

Four different student solutions were discussed by the whole class for the general case of the problem. Table 3 displays the solutions in the order they were brought up for whole-class discussion.

Before the whole-class discussion, the students have worked on the problem individually and in pairs. First, Paula and Johanna's unusual strategy is explored. The two students help each other, trying to explain their strategy, whereupon the teacher asks whether anybody understands (checks for consensus) and then asks for the  $n^{\text{th}}$  figure if it is odd. Johanna explains again and inquires: "But I don't really know how to find a formula for it." The teacher asks: "Can someone else find out how they could write it? The  $n^{\text{th}}$  figure is  $n \cdot 3 \cdot \text{something}$  [shows in the table]. If we have 13, it's 7. If we have 11, it's 6. If we have 9, it's 5. If we have 7, it's 4." In several turns, the students and teacher collaborate, which is central to inquiry instruction (Wood et al., 2006), to find that "something" must be  $(n + 1) / 2$ , and Paula concludes, "That works. Then it's  $n \cdot 3$  times  $n + 1$  divided by 2." Then Axel gets to explain his strategy (see excerpt below). The interaction patterns

	<p>1. Paula and Johanna after discussion: <math>(n \cdot 3) \cdot (n + 1) / 2</math></p>		<p>2. Axel after discussion: <math>n \cdot (n + 1) \cdot 3 / 2</math></p>
	<p>3. Lena and Frida <math>((n \cdot n) + n) / 2 \cdot 3</math></p>		<p>4. Andreas <math>(n \cdot (n + 1) / 2) \cdot 3</math></p>

**Table 3:** Student solutions in class discussion of “Houses of Cards,” in sequential order

from Wood and colleagues' (2006) framework that my analysis yielded are included as headings.

**Explore methods**

- 1 Axel: I thought a little like Paula, like she did on this one with—what was it?
- 2 Students: Flowers.
- 3 Axel: With flowers, exactly.
- 4 Students: Aha.
- 5 Axel: Then I thought that you have to add another card house—but sort of upside down, on top of. And then I was doing that for a very long time. Finally, I arrived at that it first gets  $n \cdot 3$  here [points at the tilted pile with four triangles]. Yeah, I have to write that, I think [refers to his expression]. 'Cause this is Figure 4.  $n \cdot 3$  [points at the tilted pile] and  $n \cdot 3$  plus 3 [points at the bottom row]. And then you see here 1, 2, 3, 4, so it's 4—no it's 1, 2, 3, 4, 5—it's five of these stripes here. And then you've got to take times  $n + 1$ .

**Check for consensus**

- 6 Teacher: Does anybody understand what he's saying?
- 7 Students: No. Yes. Hmmm...
- 8 Axel: Should I explain again?

**Build consensus**

- 9 Teacher: We don't understand anything.
- 10 Frida: Or do you mean like this? Wait, wait. Eeh, I think I might know what you mean. First  $n \cdot 3$ . 'Cause that's Figure 4, right?

- 11 Axel: Yes, it's Figure 4.
- 12 Frida: Yes, Figure 4. Yes, then you see there on the edge that it's four triangles going alongside there, and that's the same thing as  $n$ .
- 13 Lena: He counts the cards separately, I think.
- 14 Axel: No, but check this out—
- 15 Frida: First he calculates times, and then he calculates it times 3.
- 16 Lena: Ahaa.
- 17 Frida: Do you understand? So it's  $n \cdot 3 + n \cdot 3 + n \cdot 3 + n \cdot 3$ . And then you divide [.] Yeah, but then it gets [.]
- 18 Johanna: No, it's simply to do  $n \cdot (n + 1)$ . No, I'm just kidding [she laughs]. Yes, but check this out [points at the board]— $n$ , there on the edge, and then  $n + 1$ . And that times, so that's like a quadrangle, but it's nudged. And that divided by 2. So  $n \cdot (n + 1) / 2$ .
- 19 Axel: That you've got to multiply by 3 for each of these [.] It's 3 in each:  
 $n \cdot (n + 1) \cdot 3 / 2$ .
- 20 Johanna: Yeah. So  $n \cdot (n + 1) \cdot 3 / 2$ . [Teacher writes it on the board.]
- 21 Axel: It works.

Lena and Frida's solution and Andreas's solution are then explained and discussed.

**Check for consensus**

- 22 Andreas: Do you understand? [Refers to his explanation of his own solution.]



- 23 Students: Yes, I get it. Yeah.  
 24 Teacher: Do you understand? Yes?

### Argument

- 25 Paula: They're a little different [refers to solutions 2 and 4], or I don't know, because there it's times 3, also divided by 2, but there it isn't times 3 divided by 2. But maybe it doesn't matter.  
 26 Lena: They take times 3 in the end.  
 27 Paula: Exactly. They take times 3 without having divided it.  
 28 Lena: It's the same thing.  
 29 Paula: But there it's times 3 divided by 2, and there it isn't.  
 30 Lena: Yes, here it's—  
 31 Paula: But maybe it doesn't matter.  
 32 Teacher: Exactly. If you think of [writes and talks]  $3 \cdot (8 / 2)$ , what's that, Paula? 12 [writes and talks]. What's  $(3 \cdot 8) / 2$ ?  
 33 Paula: It's also 12.  
 34 Teacher: Yes, that's also 12. So it's the same thing.

### Focus

- 35 Teacher: So now my question is—now we've got 1, 2, 3 different [points at the formulas], and Andreas's here—that's another one—that's four different. My question is: Are all different formulas? Are all different?  
 36 Lena: Actually, they're all the same thing. Everything makes the same thing, the same answer. That means that they're all equal; you just write it in different ways.  
 37 Teacher: Exactly, they're all correct; it's the same answer, and it's the same card house. You just write it in different ways. This means that you can use algebra, the mathematical language, to express the same thing in different ways.

After an initial exploration of Axel's method [1–5], when he tries to explain his strategy and also makes a connection to Paula's solution to a previous problem, the teacher checks for class consensus by asking, "Does anybody understand what he's saying?" [6], with the purpose of opening the floor for questions and comments. Since the students' answers vary [7], Axel asks whether he should explain again [8], whereupon the teacher states, "We don't understand anything" [9]. Now comes the really interesting part [10–21]: in-

stead of leaving Axel alone to try to explain once again, classmates help him out in a collaborative manner that is characteristic of an inquiry/argument culture, according to Wood and colleagues (2006). Together the students try to make sense of the ideas and build a shared understanding; they build consensus.

Note that the teacher does not need to say anything during these turns while the students build consensus. The pupils build upon one another's statements. This must be seen as a result of the social and sociomathematical norms (Yackel & Cobb, 1996) that this teacher had established in her classroom during a long period of time. Since I have observed and interviewed the teacher in connection with many lessons, I know that she strives to foster collaborative discussion in which students help each other and "are listening and participating students at the same time" (Interview). This approach hence relates to the role of students as active listeners who try to understand, question, validate, and build on one another's contributions. In my view, this example amply illustrates the connection between classroom norms and the interaction patterns that develop (Wood et al., 2006). On her own initiative, Frida helps Axel out, saying, "Wait, wait. Eeh, I think I might know what you mean" [10]. She ensures that they are talking about the same figure number and continues to explain [12]. Lena also contributes by describing how she thinks Axel has reasoned [13], which helps her understand [16] after she gets additional input from Frida [15]. Frida actually addresses Lena directly with her question, "Do you understand?" [17]. When Frida continues to explain, she hesitates [17], whereupon Johanna steps into the discussion [18], making clear that "No, it's simply to do  $n \cdot (n + 1)$ ." To Johanna's formula,  $n \cdot (n + 1) / 2$ , Axel adds [19] "That you've got to multiply by 3 for each of these [.]. It's 3 in each:  $n \cdot (n + 1) \cdot 3 / 2$ ." Johanna clearly shows her agreement [20]. Axel seems content in his concluding comment: "It works" [21]. A correct formula for Axel's strategy has now been produced through the class's collaboration. The students act as "listeners taking over the role of the teacher in questioning, clarifying, and validating mathematical ideas" (Wood et al., 2006, p. 235), a salient feature of an inquiry/argument classroom culture.

After Lena and Frida's and Andreas's solutions have been discussed in a whole-class setting, four algebraic formulas are displayed on the board for the number of cards. Andreas checks the class for consensus by

asking, “Do you understand?” [22], and the teacher repeats his check [24], whereupon Paula argues that Axel’s and Andreas’s solutions are different. Paula and Lena take turns [25–31] in an Argument interaction pattern, trying to resolve whether the two formulas are equivalent (the teacher steps in to help [32–34]). The teacher summarizes and focuses the discussion on a key aspect by asking whether all four formulas are different [35], and Lena says that “Actually, they’re all the same thing.” [36]. The teacher concludes the discussion by restating this key mathematical idea [37].

## DISCUSSION

I find Wood and colleagues’ (2006) framework straightforward for distinguishing between conventional and reform interaction patterns. The categories cover the interactions well. However, I see some difficulties in making a clear distinction between certain reform interaction patterns. Wood and colleagues (2006) state regarding the pattern *Check for consensus*, “The teacher participates by checking with the students and listening to find out if they have any questions or comments about an idea, strategy, or concept that a student explained.” So far so good. They continue: “The student explaining may be asked further questions or to re-explain by the listening students. In some cases, listeners give another different strategy for solving the problem or offer further explanation. The outcome is public agreement on the validity of an idea or concept given by the student explaining” (p. 255).

I am concerned about the second part whose interpretation constituted the major difference in the interrater coding. In my interpretation, a *Check for consensus* consists solely of checking with the students to see whether they have questions or comments on a student’s idea. This aligns with Wood and colleagues’ (2006) statement that “Checking for consensus initiated by the teacher appeared to be a final attempt to open the discussion so any child could make comments or ask questions before moving on in the discussion” (p. 235). The teacher’s check can then be followed by, for example, a student asking a question in order to understand (*Inquiry*) or challenging an idea (*Argument*) or by the teacher trying to establish common ground on key ideas (*Build consensus*) in the class (cf. the short *Check for consensus* [6–8] and [22–24], followed by *Build consensus* and *Argument*). However, the interrater coding made clear that another interpretation

could be that the entire interaction is a *Check for consensus* since “the outcome is public agreement” (i.e., [6–21] constitutes one extended *Check for consensus*).

Making clear that a *Check for consensus* is solely a check and is followed by other interaction patterns would render the framework more straightforward. Further, my analysis suggests that in addition to the teacher (as stated by Wood et al., 2006), a student might also initiate such a check (cf. Andreas’s *Check for consensus* [22]). Just as it is difficult to determine when the pattern *Check for consensus* ends, it is not completely clear when the *Inquiry* pattern ends. My interpretation is that the *Inquiry* pattern consists solely of the act of asking and does not include the clarifications that follow. Again, outlining clear criteria not only for when an interaction pattern starts but also for when it ends would make Wood and colleagues’ framework less ambiguous.

Solutions are purposefully selected by the teacher when using Stein and colleagues’ (2008) five-practices model. Therefore, the pattern *Exploring methods* is hard to distinguish from *Proof of answer by student explanation* in terms of correct solutions; only their purposes differ (see Table 2). The purpose of deliberately selecting a correct solution for display can be both to provide multiple solution strategies and to ensure that the class hears a correct solution (cf. solutions 3 and 4 in Table 3).

The difficulties with Wood and colleagues’ (2006) framework relate mainly to distinguishing between certain interaction patterns that are specific to different classroom cultures. Hence, the difficulties in interpretation affect only the relative distribution of the interaction patterns within a specific classroom culture, not whether the culture should be regarded as inquiry/argument or strategy-reporting.

Since Wood and colleagues (2006) have shown that an inquiry/argument classroom culture is closely related to higher cognitive levels of student thinking, I contend that it is desirable for teachers to strive to establish inquiry/argument interaction patterns. I see tremendous potential in using Stein and colleagues’ (2008) model as a tool to guide teachers’ actions and support teachers’ development over time in their orchestration of whole-class discussions. My ongoing efforts intend to take into account argumentation as well as connection making in the Stein and colleagues

(2008) model. The exploration of Wood and colleagues' framework in this paper contributes to those efforts.

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