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In the last decade, research on instructional practices has been carried out in mathematics classrooms in Iceland, mostly in the lower secondary school. In this study, the structure of 51 mathematics lessons in all compulsory school grades were analysed. The data came from a study called Teaching and Learning in Icelandic Schools, where 518 lessons from all school subjects were observed. To shed light on the structure of the mathematics lessons, diagrams were made which included the categories: non-mathematical work, teachers' public interaction with the whole class, individual seat work, assessment, group work, and playing of games. The analysis revealed that in most of the mathematics lessons the students were working individually in textbooks. There was little public interaction between the teacher and the class but the teacher went around the class and interacted with the students. There were a few examples of varied instructional practices that emphasized group work and discussions.

Keywords: Instructional practices, mathematics lessons, lesson diagrams, compulsory school.

INTRODUCTION

Findings in research on instructional practices in Icelandic mathematics classrooms have indicated that there is little variation in teaching approaches. In research by Savola (2010), where he compared lessons in Icelandic and Finnish schools, it was noticeable that in Icelandic classrooms the teacher often had no public interaction with the whole class and students were often working on their own pace in textbooks, while the teacher walked between the desks and interacted with students.

Karlsson (2009) also observed lessons in Iceland and Finland. According to Karlsson's results, Icelandic teachers were more likely to stay on the sideline and were not as central in the classroom as the Finnish teachers who also used more versatile teaching methods than the Icelandic ones. A recent study on the teaching of mathematics at lower secondary level in eight schools all around Iceland showed that in 56% of lessons students were working individually on workbooks. In 35% cases, there were some interactions between teachers and students around the topic followed by individual work on problems. In 6% of lessons, students were working in groups on tasks and in 3% they were playing some kind of games. During individual work, students often helped each other and the teacher encouraged them to do so both in public and also when walking between the desks (Þórðardóttir & Hermannsson, 2012). The study by Karlsson (2009) was the only one that contained some data from mathematics teaching at lower levels.

In Iceland, limited research has been done on mathematics teaching and learning and it has mostly been done by researchers who are stakeholders in many respects (teacher educators, authors of curriculum etc.). We therefore find it important to use available data, gathered by others, to add to our knowledge. In this paper, we use data from a recent study called *Teaching and Learning in Icelandic Schools* (Óskarsdóttir et al., 2014). In the study, 51 lessons in mathematics at all grade levels from 1–10 were observed. We gained access to the observation protocols and wanted to explore what they could tell us about what goes on in mathematics classrooms in Iceland, or more precisely: How are mathematics lessons structured? Of the observations, 32 were from grade levels 1–7, which have not been studied much previously. We therefore also wanted to find out if instructional practices at different grade levels were similar or if we could spot any differences.

We think that comparing teaching within a culture allows educators to examine their own teaching prac-

tices from different perspectives by widening known possibilities, in a similar way as comparing teaching between cultures does (Hiebert et al., 2003). It can reveal alternatives and stimulate choices being made within a country. It is also important to know what actual teaching looks like on average so that national discussions can focus on what most students experience. Using data from a general study on instructional practices also allows us to examine whether the practices in mathematics are any different from practices in general.

THEORETICAL BACKGROUND

Research on general instructional practices in Iceland

Various studies on general instructional practices in Icelandic compulsory schools have been carried out in recent years. According to Sigurgeirsson, Björnsdóttir, Óskarsdóttir and Jónsdóttir (2014), these studies all indicate that direct and teacher-centred methods are most widely used and to a much less extent methods where students take a more active role. Curriculum materials guide the teaching in academic subjects to a very large extent and the most common structure of lessons is a short introduction by the teacher followed by individual seatwork. There are though indications that instructional practices in lower grades are more varied than on lower secondary level.

According to a 2013 TALIS (Teaching and Learning International Survey) study (Ólafsson, 2014), Icelandic teachers on average seldom review content recently taught, they also more rarely review homework or try to relate new knowledge to daily life, than teachers in the other TALIS countries. There is a considerable difference here. For instance, 38% of Icelandic teachers state that they often or almost in every lesson review content recently taught, where the TALIS average is 78%.

The study *Teaching and Learning in Icelandic schools* (age levels 6–15) was conducted in 2009–2010 (Óskarsdóttir et al., 2014). The study was done in cooperation with many stakeholders from universities, school authorities, schools, teachers and parent organisations, and partners from an architectural firm and an information technology firm. The study focused on many aspects of teaching and learning, like the learning environment, student learning, teaching strategies and internal structures. A special focus was put on the development towards individualised and cooperative learning advocated by school authorities both on the local and national level. The framework for the research project was a model of school practices developed as an evaluation tool by educational practitioners in Reykjavík School District. Data were gathered by using multiple methods like observations, interviews, focus groups, questionnaires and action research in 20 schools out of 175 schools in the country. Three of the schools were chosen because they had been designed with the aim of changing the instructional practices from traditional to more open and individualized learning. Other schools were randomly chosen. In total, 518 lessons in all school subjects were observed, and 240 interviews were conducted with students, teachers, principals and other staff in schools (Óskarsdóttir et al., 2014).

The results showed that teaching strategies that can be labelled as "direct" are most commonly used and strategies like discussions, group work and project work which are recommended in national curriculum guidelines are rarely used (Sigurgeirsson, Björnsdóttir, Óskarsdóttir, & Jónsdóttir, 2014). It is also noticeable that there is a considerable difference between teaching strategies depending on school level. The teaching of the youngest students seemed to be more varied than the teaching of upper grades. Schools in the study were grouped into three categories, based on whether teachers mostly worked alone with their class, or classes if they were subject teachers (6 schools), were team teaching and responsible for a whole year group together or mixed age groups (9 schools), or a mixture of both (5 schools). The teaching strategies used in schools where team teaching was the norm, were more versatile. The results also indicated that the development towards more individualised learning in light of the frameworks used is not very advanced.

International research on instructional practices in mathematics

In recent years, several studies have been conducted in different parts of the world with the aim of identifying common features of mathematics teaching in countries scoring relatively high in studies like TIMSS or PISA or by teachers who are considered outstanding math teachers in their respective countries (Clarke, Mesiti, Jablonka, & Yoshinori, 2006; Hiebert et al., 2003). Even though it is impossible to generalize or identify a common lesson script on the basis of these

studies, some important characteristics of effective mathematics teaching have been located.

The TIMSS 1999 video study brought to light that no single method of teaching 8th grade mathematics, was observed, in all the relatively high achieving countries taking part in the study (Hiebert et al., 2003). However, all eight-grade classrooms in all seven countries shared some general features. Mathematics was often taught through solving problems and 90% of lessons made use of a textbook or worksheet of some kind. Lessons were organized to include some public whole class work and some private individual or small group work. It was most common for students to work individually rather than in pairs or groups. The lessons included some review of previous content as well as some attention to new content and the teachers usually talked much more than the students. It was also observed that a variety of methods were employed rather than a single shared approach of teaching mathematics. Each country combined and emphasized instructional features in various ways, sometimes different from all other countries and sometimes partially the same.

Boaler's (2006) long-term study of mathematics teaching in three different schools in the US, showed that students from a school called Railside both enjoyed mathematics more and reached higher levels of mathematics than the students in the other two schools. According to Boaler, their success was a result of the unusual approach to mathematics at the school. The classes were heterogeneous, the students worked in groups on group-worthy problems that could be solved and represented in different ways. Moreover, the students spent a lot of time discussing mathematical ideas, learnt to help each other and were made responsible for teaching their peers. The lessons were 90 minutes long and the teachers worked closely together while preparing their teaching and shared the same ideas about teaching and learning.

Hiebert and Grouws (2007) identify, by reviewing research on the impact of classroom teaching on student learning, two main features of classroom mathematics teaching that facilitate students' conceptual development. These features are, firstly, an explicit attention to mathematical concepts and connections between ideas, facts and procedures, and secondly, that the students were given opportunities to engage in and struggle with important mathematics. According to Hiebert and Grouws (2007), these features seem to be general and operate across various contexts and teaching systems.

In their review of research on classroom practice in mathematics, Franke, Kazemi and Battey (2007) focus both on the teachers' role in mathematical work and students' experiences in the social context of the classroom. They point out that the nature of mathematical discourse in classrooms is central if teachers are to gain opportunities to learn from their practice. Students' individual work cannot alone provide such opportunities. In creating opportunities for discourse, teachers also need to attend to the social and socio-mathematical norms in the classrooms and develop relationships with their students where they take into account the students' cultural backgrounds. The IRE discourse pattern (teacher *initiated* question, student *response,* and teacher *evaluation)* is still prevalent in many mathematics classrooms, and this needs to change. The IRE discourse pattern falls within the exercise paradigm (Skovsmose, 2001) where the teacher presents some mathematical ideas and the students work with selected tasks from textbooks. The teacher presentation can vary in length. It can take up to a whole lesson and the students could also be working with exercises for the duration of the lesson. Justifications of the relevance of the exercises are not a part of the lesson and there is usually only one answer to the task.

Even though it is clear that classroom practice is complex and many cultural differences can be found when studying mathematics teaching across cultures (Givvin, Jacobs, Hollingsworth, & Hiebert, 2009; Franke, Kazemi, & Battey, 2007; Hiebert et al., 2003), there seems to be consensus regarding the idea that both teachers and students need to play an active role in the mathematics classroom. It is important that students both actively engage in mathematical discussions, making sense of mathematical concepts, and that teachers are able to learn from their students and develop their practice. Students also need to be engaged in solving challenging problems and given opportunities to share and present their ideas (Givvin, Jacobs, Hollingsworth, & Hiebert, 2009).

DATA AND DATA ANALYSIS

Our data consist of observation protocols from mathematics lessons made by researchers in the research project *Teaching and Learning in Icelandic Schools*

described above (Óskarsdóttir et al., 2014). The researchers came from various disciplines within the University of Iceland and the University of Akureyri, mostly from general pedagogy. None of the researchers had specialized knowledge about mathematics teaching and learning. The observers made detailed notes in an observation protocol during the lessons. The focus of the observations was more on the progress of the lesson and the students' activity during the lesson than on the content and how that was dealt with. In some of the observation protocols, it was clear what the focus of the lesson was but in others there was no mention of the content of the lesson. This has its limitation but nevertheless we feel the observation protocols give us an idea of what is happening in the classroom and how the teaching is organized. We, the authors of this paper, have been actively engaged in teaching math teachers and making curriculum materials for a long time and are therefore well known to most math teachers in Iceland. We felt that by using this data we could gain some information about mathematics teaching in Iceland without collecting the data ourselves and thereby probably influencing the results.

As mentioned previously, Savola (2010) studied mathematics lessons in Iceland and Finland. He videotaped 20 lessons (two from each teacher) in each country. He made lesson diagrams for each lesson based on the coding of his data. His main coding categories were review, introducing new content, practice/applying, and other. The category 'other' included classroom management, mathematics management, homework, interruption, social talk and independent learning (IL). Three different types of IL were noted, between-desk instruction, where the teacher walks around the classroom and helps, and teacher or student presenting at the front, addressing only few students while the others are working individually.

Johansson (2006) studied videotaped lessons from three Swedish teachers, considered competent mathematics teachers in their community, at lower secondary level (4–5 consecutive lessons from each teacher). She tried to identify a common lesson script and in coding her data she used four main coverage codes: classroom interaction, content activity, organization of students and textbook influence. She also directed her attention to the teachers' activity and how often specific events like problem solving, assignment of homework, assessment, goal statements, summary of lessons etc. occurred within a lesson.

In our analysis, we started by reading carefully all the observation protocols. We then formed some categories on basis of the data with categories used by Savola (2010) and Johansson (2006) in mind. Our data is much more limited because it is only based on written notes by the observers and not video recordings and therefore does not allow a fine-grained analysis. Classrooms practices are complex and by analysing lesson structure we try to capture some important elements of both the form and the function of the lesson. However, it has its limitations and researchers should be careful not to draw too many conclusions on the basis of this kind of data, but it can shed some light on important aspects of classroom practices (Savola, 2010; Clarke et al., 2006).

Our main categories were: non-mathematical work (a), teacher's public interaction with the whole class including presentation of new material and checking and assignment of homework (b), individual seatwork (c), assessment (d), group work (e), playing of games (f). We made a diagram of each lesson using these codes and also described in few words what was happening in each part of the lesson, for instance, whether the students used textbooks or not. This made it possible to spot differences and similarities across the sample.

FINDINGS

We summarize our findings according to grade levels.

Grade levels 1–4

For grade levels 1–4, we have 19 observations. Most of the lessons (13) were 40 minutes but six had duration of 60–80 minutes. In almost all the lessons it took at least five minutes before the actual lesson could begin and in ten of them the mathematical work was finished five minutes before the actual lesson ended. In ten of the lessons there was some public interaction between the teacher and the students lasting 5–15 minutes. In most cases the teachers explained algorithms and procedures or discussed and showed students how to work with pages from the textbook using an overhead projector. When explaining algorithms and procedures, the teachers were not using textbooks but supplemented the textbooks with material from other resources. The biggest parts of the lessons students were working individually in textbooks or on

worksheets provided by the teacher. Activities like working with attribute blocks, playing dice games, working with Tangram and unit cubes were inspired by textbooks. During individual seatwork, the teacher circulated and assisted students and in nine lessons an unqualified assistant or another teacher/specialist was present either working with specific students or assisting in general. When working individually in textbooks students sometimes worked with the same chapter or worksheets but in at least six lessons they worked at their own speed but with the same textbook or workbook. In three lessons the students worked in groups. Two of these lessons were organized as workstations where the groups worked on different activities. In one class, all the workstations focused on practicing multiplication but in the other, a first grade class, the activities were unrelated but varied. In the third lesson with group work, all students were working with attribute blocks, first making a picture together in a group and then they got some time for free play with the blocks.

Grade level 5–7

From grades 5–7 we have 13 observations. Most of the lessons were 40 minutes but three were 80 minutes long. In ten of the lessons it took about five minutes before the mathematical work started. Here, in six lessons, there was some public interaction between students and teachers and it centred on guiding the students through the textbook and reviewing homework. In one lesson the teacher was discussing properties of geometric forms with the students and then they were to make their own forms but it was not clear whether this was inspired by the textbook or not. In seven lessons the students were working individually on their textbooks almost the entire time of the lesson while the teacher was circulating and assisting them. In one lesson the students were working on a test for 30 minutes and then they handed in the test and were given an opportunity to work on it again the next day with the aim of looking things up at home and then improving their solutions. In one 80 minutes lesson, the students, after completing a self-assessment, worked in groups planning a table tennis tournament and after that they played the tournament and then came back to class and discussed and shared their solutions of the task.

Grades 8–10

From grades 8–10 (lower secondary level), we have 19 observations and eight of the lessons were 60–80 minutes long. Here it took less time for teachers and students to get to work with the mathematics. In most lessons they had started within 2–3 minutes. Eight of the lessons started with some public interaction between teacher and students, either by reviewing homework or presenting new material or problems. Even though in most lessons these interactions only took around ten minutes, there were examples of them taking from 20 and up to 60 minutes, and usually the teachers tried to engage the students by asking questions and encouraging discussions. In three classes the students were divided into two groups where a part of them worked individually while others were taught by the teacher or took a test. In five lessons the students worked individually on textbooks during the whole lesson, and in six more lessons, the students worked individually after a short introduction by the teacher. In many of the lessons, the students seemed to be working on the same topic even though there were examples of individual students working on material from higher grade levels, even upper secondary level, or students working with different material from the rest of the class because of learning difficulties. In three of the classes the students worked independently according to a plan made for the chapter or a month with a daily quota. In one lesson, students worked on an assessment. In one school two lessons were observed where students worked in mixed age groups on problem solving. Here the lesson started with some introduction of important concepts in relation to the problem at hand and then the students were divided into groups.

DISCUSSION

The findings from this study support previous findings on instructional practices in Iceland and add some new knowledge about mathematics teaching in compulsory schools as a whole. In many lessons there was a strong focus on individual seatwork where students worked mostly on textbooks. The teachers seemed to rely on textbooks and their contents for the whole class, aimed at helping or guiding students through the problems or exercises in the textbooks. Students were active in their work and the teachers moved around the classroom and helped them. There were examples of teachers creating opportunities for whole class discussions about topics or ways of working sometimes in the beginning and sometimes in the middle of lessons in connection with the individual seatwork. However, there were

also many lessons with no public interactions and therefore the teachers were not creating enough opportunities for students to elaborate on and discuss with others mathematical concepts and connections. More whole group discussions should also help the teachers to develop and learn from their practice (Franke, Kazemi, & Battey, 2007). The socio-mathematical norms in the classrooms seemed to be that you learn through working on problems/exercises in textbooks, and the communication between teachers and students was centred on supporting the students in completing the work. This is what is often termed as traditional mathematics teaching or the exercise paradigm (Skovsmose, 2001). We do not know much about the nature of the tasks that the students were working on but from the observations it seemed like the teacher put an emphasis on guiding the students through their work with the problems. In several cases, the teachers in the lowest grades worked through the pages in the textbook by using overheads. With the older students, the teacher often discussed specific problems in textbooks. In six lessons, students worked in groups and there the teaching was more in line with what many researchers (Franke, Kazemi, & Battey, 2007; Hiebert & Grouws, 2007; Boaler, 2006) claim to be important features of effective mathematics teaching.

Our findings reveal that instructional practices in mathematics are similar to the general results on teaching practices in the study *Teaching and Learning in Icelandic Schools* (Sigurgeirsson, Björnsdóttir, Óskarsdóttir, & Jónsdóttir, 2014).

The lesson diagrams gave a good overview of the lessons and their structure and drew attention to similarities and differences both within grade levels and between grade levels. In the lower grades, the teachers used more time on non-mathematical work, which is not surprising. It could also be seen that teachers use considerable more time working with the whole class as the children get older.

The data consisted of observations from only one lesson in each class. Previous research has shown that lesson patterns can vary considerably from lesson to lesson with the same teacher (Clarke, Mesiti, Jablonka, & Yoshinori, 2006; Hiebert et al., 2003). In the data there was little information about the mathematical content of the lessons and what the teachers chose to emphasize in their interactions with the students. As mentioned earlier, most of the observers came from general pedagogy and they did not note what the focus of the lesson was. They sometimes referred to pages or specific problems and from that we could see whether the class was working on the same problems at the same time or not, but it provided limited information about the goals with the lessons.

From this study it is evident that Icelandic teachers use a considerable time walking between the desks interacting with students. Research has shown that there can be great variations in regard to what the teacher is actually doing when he is circulating in the class and that affects the quality of the teaching (O´Keefe, Xu, & Clarke, 2006). Therefore, it would be interesting and valuable to find out what characterizes the teachers interactions with students in Icelandic mathematics classrooms.

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