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# An assessment practice that teacher José uses to promote self-assessment of mathematics learning

Paulo Dias<sup>1</sup> and Leonor Santos<sup>2</sup>

1 Secondary School of Moita, Moita, Portugal, [paulo.dias.7@gmail.com](mailto:paulo.dias.7@gmail.com)

2 University of Lisbon, Institute of Education, Lisbon, Portugal, [mjsantos@ie.ul.pt](mailto:mjsantos@ie.ul.pt)

*In this paper, we present some results concerning an assessment practice, composed of oral interaction between a teacher and students that a mathematics teacher, José, in a context of a collaborative work, uses to develop the capacity of self-assessment of his secondary school students. The assessment practice developed by José includes two ways to promote self-assessment: self-assessment of the answer and self-assessment of performance. While the former is ensured by cognitive and motivational strategies, the latter is mostly based on meta-cognitive strategies.*

**Keywords:** Assessment practices, self-assessment, mathematics learning, collaborative work.

## INTRODUCTION

The tasks that simultaneously serve to teach, learn and assess students' performance have an effect on their school results, mainly because they are not associated with the practice of common assessment and aims to promote self-assessment skills for mathematics learning (Hodgen, 2007). Moreover, when teachers analyze students' answers and give them an appropriate feedback, they deepen the knowledge that students need to organize information and reply (Price, Handley, Millar, & O'Donovan, 2010). This practice helps the student to get a better knowledge structure, allowing an adequate self-questioning, contributing therefore to develop the capacity of the students to self-assess. It promotes mathematical efficiency and self-assessment of the mathematical knowledge, skills and capacities (Quinton & Smallbone, 2010).

But in Portugal, as in several other countries, this kind of practice is really still far from reality in

mathematics classes (Santiago, Donaldson, Looney, & Nusche, 2012). We studied, in a collaborative work context, assessment practices done by secondary school's mathematics teachers, whose aim was to promote self-assessment of mathematics learning. In this text, we present only part of the study (Dias, 2013), focusing upon a secondary school mathematics teacher, José, and one assessment practice, the oral interaction between teacher and students (IT-S). In particular, we aim to answer to the following research question: What is the nature and the characteristics of IT-S assessment practice of a secondary school's mathematics teacher, developed in a work context of collaborative nature, which seek to promote self-assessment of learning?

## THEORETICAL OVERVIEW

The study of teaching practice of mathematics teachers is relevant to understand their performance in the classroom (Ponte & Chapman, 2006). The understanding of the meaning given to the mathematics teachers' decisions contributes to deepen the knowledge of how the mathematics teachers work in the classroom. Regarding assessment practices in specific, it is not sufficient to assess whether students have mastered facts and algorithms or developed attitudes, skills and knowledge advocated in mathematics curriculum. It is necessary that assessment practices reflect teaching and learning processes. For this purpose, rich tasks are required (Smith & Smith, 2014) and the assessment questions have to be constructed so that, when analyzing the answers of the students, it is possible to get an idea of how students organize information (Price et al., 2010). To accept that the students have an essential part in the construction of their knowledge implies that the teacher must pay particular attention

to feedback processes and self-assessment of learning (Santos, 2002).

The focus on self-assessment of learning is justified by the importance of student success in mathematics assignments and consequently in mathematics learning. Self-assessment develops the ability to assess a task and to implement the necessary corrections or adjustments. It is a group of actions that the student develops when regulates his or her own work (Zimmerman & Schunk, 2011). Self-assessment is the process in which students develop the strategies needed to achieve the desired objectives, creating conditions for a successful learning. But self-assessment of mathematics learning do not develops spontaneously (De Corte, Mason, Depaepe, & Verschaffel, 2011). It is the teacher who has the responsibility to promote it through several actions. This study emphasizes the oral interaction, conversation between students and teacher while performing a mathematical task (Henning, McKeny, Foley, & Balong, 2012). Because it is an intentional action from the teacher and occurs in the daily work of the classroom, may be considered an assessment practice for learning (Wiliam, 2007).

Interaction and communication in a mathematics classroom is definitely essential to improve student learning (Santos & Semana, 2012). Nevertheless, some aspects have to be respected. On one hand, teacher has to use relevant information about students reasoning and ways of learning in order to deal properly with the process of teaching and learning (Pinto & Santos, 2006). On other hand, some conditions have to be respected concerning the act of questioning, such as not correcting errors but giving clues, not confirming but asking in a way that leads the students to develop a convincing argument about their reasoning.

IT-S is characterized by questions, stimuli and directions given by the teacher during the implementation of a mathematical task. The impact of this practice may be conditioned by the opportunity of the intervention and the increased student confidence in building their mathematical knowledge (Schwarz, Dreyfus, & Hershkowitz, 2009). In this practice, the teacher should avoid correcting errors and adopt an attitude that contributes to students formulate questions independently. Students can be referred to their own productions or to the proposed tasks, or can be suggested to share and discuss, in pairs or in groups,

their interpretations of the answers (Henning et al., 2012).

## METHODOLOGY

Following a qualitative and interpretative methodology approach, through a case study design (Stake, 2009), the assessment practice of a secondary school's mathematics teacher, José, has been observed along two school years. José was chosen because of his recognized experience (31 years of teaching, both in schools of basic and secondary education), and his willingness to develop assessment practices that promote self-assessment.

The collaborative context (Jaworski, 2007), constituted by José, another secondary school's mathematics teacher and the researcher (first author), was created by invitation from the researcher, and has as main objective to develop and implement assessment practices to promote self-assessment. Episodes of classroom reported in almost all the texts read by the group were considered essential to trigger discussions and to define the assessment practices to study. After each lesson, there were moments of reflection between the three members of the group. The selection/creation of mathematics tasks was a great challenge for teachers when seeking to integrate assessment, teaching and learning (James, 2006; Pinto & Santos, 2006) and to promote self-assessment of mathematics learning (Black & Wiliam, 2006). Noteworthy the assessment practices planned by the group features challenging tasks (Smith & Smith, 2014), including the themes Trigonometry, Geometry and Functions.

Data collection was done in a 11th grade class (students aged 16–17) and included observation of 10 mathematics classrooms (A), that includes 5 tasks, respectively with trigonometry, geometry and mathematical functions, and collaborative work sessions (ST) from February 2009 to April 2010, with audio recording, a structured interview to José at the beginning of the study (E), and documental analysis, that collected documents used by the teacher and the mathematical work done by the students.

Data analysis was performed by content analysis considering three domains, planning, implementation and reflection (Clark & Peterson, 1986). Planning, before class was focused on the role of the teacher in interpreting, managing, planning and putting into

practice his curricular choices. The implementation, during class, focused in the actions of the students during the development of the task and the role of the teacher in the oral interaction between teacher and student. Finally, the third moment, reflection, after class, helps teachers to make progress in their professional development and to build their own way of knowing. In this moment, the teacher makes explicit the strengths and weaknesses of his assessment practice.

In each of the distinct phases sought by José's actions showed his intention to promote the success in the

Self-assessment of the answer	Commitment with mathematical tasks
	Stimulus to individual strategies
	Articulate student ideas
Self-assessment of performance	Self-regulate mathematics efficiency
	Self-assessment

Table 1

task, either by teacher's questioning of the analysis, and the analysis of the self-regulation of mathematics learning by the student. Later, elements were organized as seen in Table 1.

### ORAL INTERACTION TEACHER AND STUDENTS (IT-S)

#### Planning

The assessment practice was defined and planned within the collaborative work. Initially, students' difficulties were identified. According to José, the most relevant ones were written and oral mathematics' communication as well as the understanding of mathematical content, although students master some techniques in solving exercises. In this context, the IT-S planning gave attention to the selection of the task and to the support given to students during the task, avoiding correcting the errors and to give too many guidelines for shaping. According to José, questions and stimuli are guidelines that can help students *to increase student confidence in improving their mathematical knowledge*:

I understand the students' difficulties, but how can I help them? I might answer through interaction. Asking, stimulating, and giving clues to each one. [ST, 12<sup>nd</sup>]

Looking for the best approach in the classroom was essentially one of the texts discussed in the collaborative group (Santos, 2002). To avoid correcting errors was considered an important contribution *to help students to think for themselves* while solving tasks. Some of the questions suggested in the text were harnessed by José as an example of how it should be the role of the teacher in the classroom:

"What have you done?", "Why did you take that option?", "Why did you think like that?" "Where did that idea come from?", "In which other situations does this process may be applied?", "If you want to convince someone that this is true, what would you say?" and I would wait for their reaction. [ST, 12<sup>nd</sup>]

In achieving the tasks, José prefers that students work in pairs, considering that this method may help students to learn:

If students work in pairs, they can help each other, explore, understand and look for the solutions. [E, 1<sup>st</sup>]

José recognizes the importance of questioning to help students to accomplish the task. This action neither includes the correction of errors, nor provides many guidelines. He seeks students to explore their work in pairs or in groups, encouraging discussion among students.

#### Implementation

While the pairs of students develop the task, José observes and interacts with them.

In a task of Trigonometry (Figure 1), José pointed out for the importance of the task assignment and called attention for it, trying *to commit the students to the task*:

- José: Read again. You should go on after understanding the picture \ figure and what it is said about it.
- David: But we just have to look at it and we understand.
- José: The information is important. It is essential to the solution. [A, 1<sup>st</sup>]

In this task, students had difficulty in selecting the information and José *stimulus to individual strategies*,

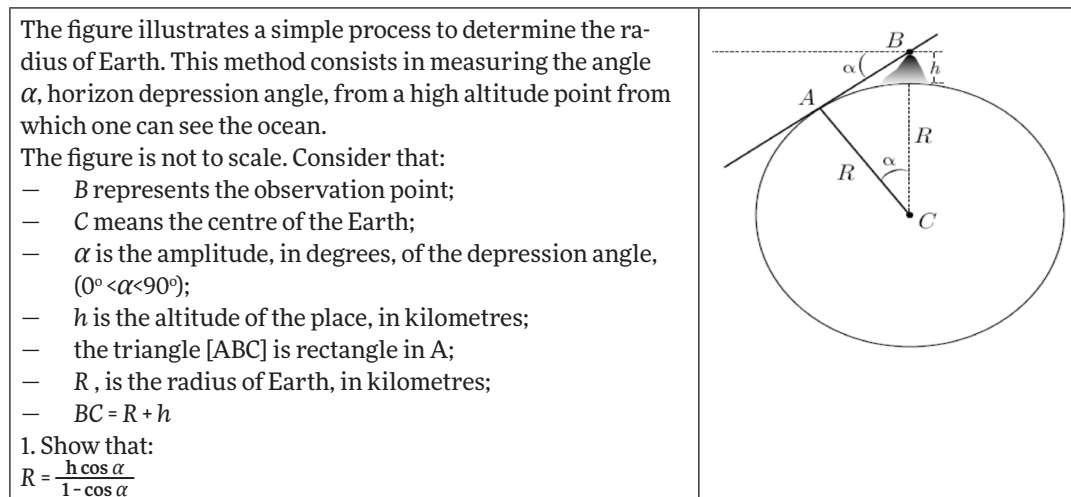


Figure 1

supporting this selection and helping them to find a solution strategy, encouraging them to continue (speaking 9), guiding them to the analysis of the figure and from the strategies designed for each pair of students (speaking 5):

- 1 David: Teacher, we already know, it is the trigonometrically reason: sine, cosine and tangent
- 2 José: Are you going to use the three?
- 3 Alexandre: No, no. we will see. the triangle needs the opposite cathetus AB
- 4 David: No way! AB is useless, what matters is h!
- 5 José: Where is h?
- 6 Alexandre: On the hypotenuse. but we also have R in it
- 7 David: We will do it with the cosine
- 8 Alexandre: We have the adjoining cathetus and the hypotenuse that goes for the cosine and then we find the value of h, can that be?
- 9 José: it can be, there are many ways, that's yours choice. Go on! [A, 1<sup>st</sup>]

In the same task, other item (Determine the value of  $h$ , when  $R = 1000$  meters and  $\alpha = 60^\circ$  and when  $\alpha = 45^\circ$ . Present the results rounded to units. Compare the two previous results), he managed the same with two other students (Magda and Ruth). These students worked  $\cos 60^\circ = \frac{1000}{1000+h}$  and  $\cos 45^\circ = \frac{1000}{1000+h}$  equalities. From these equalities, they said, just watching the figure, that as  $h$  increases, the value of the angle has to increase too. José feels a little hesitant, because he was not expecting this task solution. He articulated students' ideas, not validating the work and asking

students to articulate these ideas so that they may be able to present arguments of the result:

- Rute: Is this true?
- Magda: Adjoining cathetus upon the hypotenuse
- José: Yes, it is right, but you did not use the equality  $R = \frac{h \cos \alpha}{1 - \cos \alpha}$
- Magda: But it is the adjoining cathetus upon the hypotenuse or not?
- Rute: It is the cosine!!!
- José: Yes, but how do you justify the comparison?
- Magda: When  $h$  increases, the fraction value decreases and the obtained angle increases.
- José: But you have to convince me that it is right! [A, 1<sup>st</sup>]

José wanted the students to identify the errors and report it orally. He believed that this explanation would help them to *self-regulate mathematics efficacy* and the ability of self-questioning about some important issues, such as how to work with the calculator:

For instance, a student answered  $R = \frac{2,35 \cos 1,5564^\circ}{1 - \cos 1,5564^\circ}$ , so  $R \approx 0,034$ , and other student answered  $R \approx 1,35$ . I can identify that the first changed in the calculator "Rad" for "Deg". I would accept it as correct and alerted the student for the lapse. But I could not consider correct the second one. After some experiences, I could identify that the second student wrote on the calculator,  $2,35 \times \cos 1,5564 \div 1 - \cos 1,5564$ . In both cases, they are serious mistakes. [ST, 13<sup>nd</sup>]



In this assessment practice, José encourages the identification of mistakes. For example, he gave clues to the students so that they may *self-assess their performance*:

- David: Are these the calculations?  
José: How did you do the equations?  
Alexandre: I think it was very fast, but it's right.  
José: Remember the change of signs...  
David: Let's see but this way leads us to what we wanted [A, 2nd]

In another task (Functions) after identifying a student's difficulty, José questioned him and, in some cases, the error was identified by the student *through self-assessment*:

- David: It is 40 days.  
José: Are you sure?  
David: From 4 to 40 it is 30 plus 10.  
José: But it is 4th January.  
David: January  
José: And the month of January has 30 days?  
David: Okay, so  $t=41$  days [A, 3nd]

When students recognize difficulties to understand the mathematical situation, José do not hesitate to give them a clue to proceed, to pursue the approximation of their work to what the teacher expected them to do (*self-assessment*):

- Alexandre: I am going to need help. I cannot understand.  
José: Write first the formula of the area of the figure that you want to estimate, to calculate.  
Alexandre: The area of the triangle?  
José: Yes and now you can go on. [A, 5nd]

### Reflection

José highlighted the improvements made by the students and the promotion of self-assessment as two aspects to value in his assessment practice. IT-S has raised José knowledge about the students, about the difficulties of writing and oral communication, poor connection between trigonometry and geometry content, the increase mode of understanding of mathematical content, and how to keep them motivated students.

In the first task, he realized through the help he gave the development of the student *capacity to select information*. This support, according to José, was essential

to meet the first items of the task and *to motivate the students* to achieve it:

I had to help them at the beginning, but I think that with that help they improved and accomplished the first questions. It also motivated them, it guided them to go through. [ST, 13<sup>nd</sup>].

José remembered the episode where he was confronted by the unexpected strategy followed by Magda and Rute, which he decided to not validate. He justified his decision by the need to verify the extent to which students arrived to the presented solution and *to understand the knowledge* that students have applied:

They were confident that they were right, but when would they give up? (...) I left it unknown so they could think over it and that really amazed me, they knew what they were doing and rightly. [ST, 13<sup>nd</sup>]

In his opinion, students used strategies that work out things they learned previously. Strategies are not always predictable, but *to use these strategies with confidence* reveals effectiveness:

Magda and Rute answered item solving it in a way I wasn't expecting, but even so it was correct. It is like this, students use methods that I was not expecting, and we must validate them. These students knew what they were doing and were very confident about it. [ST, 13<sup>nd</sup>]

Difficulties concerning the understanding of information given in a short text are obstacles to self-assessment, which, according to José, make it harder to understand the question:

When students read the assignment of a task and are not able to select the information, sometimes they get lost through the text. They do not even read the given information, which makes it much more difficult the approach of self-regulation strategy. They cannot use the knowledge they have, because they do not understand the worksheet. My questions sometimes do not make any sense to the students. [ST, 15<sup>nd</sup>]

One essential characteristic of this assessment practice is the existence of an understanding between teacher and students, so that they can understand

the issues that the teacher poses. It is important that students understand what is asked in the tasks (on paper) or orally (inquiry in the classroom).

This reflection was important to José as it allowed him to identify different behaviors in the students: students participated more, discussed the tasks among themselves and produced work. In addition, José avoided answering directly to the questions of the students, chose to encourage them to overcome the difficulties without help, which resulted in students taking the necessary attitudes to comment on each other's ideas. This outcome has led José to reflect on his practice, making him particularly aware of the positive consequences of a reduction in the number of teacher inputs in the interaction with students.

### FINAL CONSIDERATIONS

After observing and analyzing José's practices, we can infer two ways to promote self-assessment of learning mathematics: answer and performance. Self-assessment of the answer includes the action of the teacher, providing monitoring of the final work of the students, while the performance is concerned to the action to monitor processes and knowledge needed to find a solution. *Self-assessment of the answer* is provided by cognitive strategies and motivation for students who face questioning and oral feedback (Santos, 2002). Since planning, José prepares possible practices such as not correcting errors to help students to be autonomous in their cognitive strategies. He also pays attention to the need of students' confidence in doing mathematics, continuing in encouraging them during the development of the tasks. Once achieved, it leads students to increase the engagement with mathematical tasks, to develop strategies and to promote the integration of students' own ideas about learning mathematics.

*Self-assessment of performance* is assured by meta-cognitive strategies. It occurs to assess the efficiency of the mathematics performance of students and to encourage self-assessment of mathematical strategies to solve the tasks. José demands that their students deem complete answers, and present arguments to support their reasoning. He promotes self-assessment by not confirming the answers and guiding students to read the question again. To promote self-assessment, José also seeks to assess the depth of knowledge of mathematics and to identify errors when needed,

leading students to think about the results (Quinton & Smallbone, 2010).

Although this assessment practice was new to José, he developed it over a whole school year, supported by a collaborative context, which led him to develop his self-confidence and ability in using it, recognizing positive aspects in the capacity of self-assessment of their students and, consequently, in the learning of mathematics. José assumed that his role is to support students' knowledge and learning processes, helping them to learn. Questioning, in IT-S, assumed the characteristics of oral feedback (Hodgen, 2007): to be focus on the task and not on the student; to be challenging, and to require achievable action. The IT-S practice shows progress in how students participate and engage in mathematical tasks, albeit the full understanding of the factors that increase the motivation and commitment of students with the mathematics tasks can and should be further explored.

### REFERENCES

- Black, P., & Wiliam, D. (2006). The reliability of assessments. In J. Gardner (Ed.), *Assessment and Learning* (pp. 119–132). London, UK: Sage.
- Clark, C., & Peterson, P. (1986). Teachers' thought processes. In M. Wittrock (Ed.), *Handbook of research on teaching* (pp. 255–295). New York, NY: McMillan Publishing Company.
- De Corte, E., Mason, L., Depaepe, F., & Verschaffel, L. (2011). Self-assessment of mathematical knowledge and skills. In B. J. Zimmerman & D. H. Schunk (Eds.), *Handbook of self-assessment of learning and performance* (pp. 155–172). New York, NY: Routledge.
- Dias, P. (2013). *Práticas letivas promotoras da regulação da aprendizagem matemática pelos alunos [Promoting practices of regulation of mathematics learning by students]*. PhD thesis, University of Lisbon, Portugal.
- Henning, J., McKeny, T., Foley, G., & Balong, M. (2012). Mathematics discussion: creating opportunities for purposeful participation. *Journal of Mathematics Teacher Education*, 15(6), 453–479.
- Hodgen, J. (2007) Formative assessment. Tools for transforming school mathematics towards a dialogic practice? In D. Pitta-Pantazi & G. Philippou (Eds.), *Proceedings of Fifth Congress of the European Society for Research in Mathematics Education* (pp. 1886–1895). Larnaca, Cyprus: University of Cyprus.
- James, M. (2006). Assessment, teaching and theories of learning. In J. Gardner (Ed.), *Assessment and Learning* (pp. 47–60). London, UK: Sage.

- Jaworski, B. (2007). Developmental research in mathematics teaching and learning. *Canadian Mathematics Education Study Group Proceedings of 2006 Annual Meeting* (pp. 3–16). Calgary, Canada: University of Calgary.
- Pinto, J., & Santos, L. (2006). *Modelos de avaliação das aprendizagens [Models of assessment of learning]*. Lisbon, Portugal: Universidade Aberta.
- Ponte, J. P., & Chapman, O. (2006). Mathematics teachers' knowledge and practices. In A. Gutierrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp. 461–494). Rotterdam, The Netherlands: Sense Publishers.
- Ponte, J. P., Quresma, M., & Branco, N. (2012). Práticas profissionais dos professores de Matemática [Professional practices of Mathematics teachers]. *Avances de Investigación en Educación Matemática, 1*, 65–86.
- Price, M., Handley, K., Millar, J., & O'Donovan, B. (2010). Feedback: All that effort, but what is the effect? *Assessment & Evaluation in Higher Education, 35*(3), 277–289.
- Quinton, S., & Smallbone, T. (2010). Feeding forward: using feedback to promote student reflection and learning – a teaching model. *Innovations in Education and Teaching International, 47*(1), 125–135.
- Santiago, P., Donaldson, G., Looney, A., & Nusche, D. (2012). *OECD Reviews of evaluation and assessment in education: Portugal*. (<http://www.oecd.org/edu/evaluationpolicy>)
- Santos, L. (2002). A auto-avaliação regulada: Porquê? O quê? Como? [Regulated self-assessment: why, what and how?] In P. Abrantes & F. Araújo (Eds.), *Avaliação das aprendizagens [Assessment of learning]* (pp. 77–84). Lisbon, Portugal: Ministério da Educação, DEB.
- Santos, L., & Semana, S. (2012). The teacher's oral communication during whole-class discussions. In Tso, T. Y. (Ed.), *Proceedings of the 36th PME* (vol. 4, pp. 51–58). Taipei, Taiwan: PME.
- Schwarz, B., Dreyfus, T., & Hershkowitz, R. (2009). The nested epistemic actions model for abstraction in context. In B. B. Schwarz, T. Dreyfus, & R. Hershkowitz (Eds.), *Transformation of knowledge through classroom interaction* (pp. 11–41). London, UK: Routledge.
- Smith, J., & Smith, L. (2014). Developing assessment tasks. In C. Wyatt-Smith, V. Klenowski, & P. Colbert (Eds.), *Designing assessment for quality learning* (pp. 123–136). London, UK: Springer.
- Stake, R. (2009). *A arte de investigar com estudos de caso [The art of research with case studies]* (2.<sup>a</sup> ed.). Lisbon, Portugal: Fundação Calouste Gulbenkian.
- William, D. (2007). Keeping learning on track. In F. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 1053–1098). Charlotte, NC: Information Age.
- Zimmerman, B., & Schunk, D. (2011). Self-regulated learning and performance: an introduction and an overview. In B. J. Zimmerman, & D. H. Schunk (Eds.), *Handbook of self-assessment of learning and performance* (pp. 1–14). New York, NY: Routledge.