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# Teachers' attention to task's potential for encouraging classroom argumentative activity

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*This study investigated secondary school mathematics teachers' attention to task's potential for argumentative activity in the classroom. Analysis of the teachers' choices of tasks and their explanations revealed categories that fall into two dimensions: (1) Attention to the mathematics in which the argumentative activity is embedded, focusing on three aspects: the mathematics inherent in the task; the mathematics related to the teaching sequence that the task is a part of; and meta-level principles of mathematics. (2) Attention to socio-cultural aspects related to the argumentative activity. Four attention-profiles of teachers were identified: Teachers who attended to both dimensions; teachers who were attentive only to the mathematics aspects inherent in the task; teachers who were attentive only to the socio-cultural dimension; and teachers who were attentive to neither of these dimensions.*

**Keywords:** Argumentative activity, teachers, attention, task.

## THEORETICAL BACKGROUND AND GOALS

The approach taken in this study is socio-cultural; therefore, we considered approaches to argumentation as a social process in which two or more individuals engage in a dialogue, and where arguments are constructed and critiqued (e.g., Ayalon & Even, 2015; Conner, Singletary, Smith, Wagner, & Francisco, 2014; Krummheuer, 1995). In recent years, there has been a growing appreciation of the importance of incorporating argumentation into mathematics classrooms: first and foremost because mathematics is intrinsically connected to argumentation. The principal facets of argumentative activity – justifying claims, generating conjectures and their justifications, as well as evaluating arguments – are all essential components of doing and communicating mathematics. In addition, accumulating research suggests that participation

in argumentative activities – which encourage students to explore, confront, and evaluate alternative positions, voice support or objections, and justify different ideas and hypotheses – promotes meaningful understanding and deep thinking (Schwarz, Hershkowitz, & Phrusak, 2010). Moreover, involvement in argumentation in the classroom may provide students with a feeling of sharing responsibility with the teacher for the learning that occurs in the lesson (Forman, Larreamendy-Joerns, Stein, & Brown, 1998).

Research suggests that the teacher plays a crucial role in establishing norms and standards for mathematical argumentation in the classroom (e.g., Forman et al., 1998; Yackel, 2002). These roles include prompting students to establish claims and justifications, encouraging students to critically consider different arguments, explicating the argumentative basis of students' claims, and supplying argumentative support that was either omitted or left implicit, presenting to students what constitutes acceptable mathematical arguments, and modelling specific ways of constructing and confronting arguments. These various roles express the rich and complicated dimensions of establishing learning environment in which mathematics and argumentation are integrated (Yackel, 2002).

Research also suggests that argumentative activities are not often implemented in class, and that teachers need adequate training and scaffolding if they are to improve classroom argumentation (Conner et al., 2014; Hiebert et al., 2003). A possible first step towards offering such assistance may be to investigate teachers' approaches to and sensitivity towards argumentation as a practice in the mathematics classroom. Our study goes to this direction: It investigates teachers' attention to the argumentative potential of mathematical tasks.

The concept of attention is currently used in studies concerning mathematics teaching (e.g.,

Paparistodemou, Potari, & Pitta-Pantazi, 2014; Sherin & van Es, 2005). Common to these studies is the assumption that teachers' ability to notice is a key feature of teaching expertise, and that such an ability can be improved. This study shares the above assumption, and we focus on teachers' attention to task's potential of utilizing argumentative activity, i.e., what teachers take into consideration when they choose tasks from their textbook with the aim of promoting argumentative activity in their mathematics classroom. The specific research question was: What do teachers attend to when they choose tasks that, in their view, encourage argumentative activity in their mathematics classroom? Unlike other studies, our focus on teacher attention was not part of teachers' training; rather we intended to learn on teacher's attention towards argumentation in teaching, within a situation close to their day-to-day individual work of planning their teaching.

## METHODOLOGY

### Participants

A group of 17 secondary school mathematics teachers in Israel, at the beginning of a year-long in-service-course, participated in the study. Their teaching experience varied from 1 to 30 years; each holding a B.Ed. in teaching mathematics or B.Sc. in mathematics. The teachers were not explicitly exposed to the issue of argumentative activity before the research was conducted. All the teachers used the same 7<sup>th</sup> grade mathematics textbook in their classrooms.

### The research tool

We developed a questionnaire that focused on the teachers' practice of selecting mathematical tasks. The teachers were individually asked: (1) to choose three tasks from a unit of the 7<sup>th</sup> grade textbook (Ozrusso-Hagiag, Bouhadana, Friedlander, Robinson, & Taizi, 2012) that they view as encouraging argumentative activity, and (2) to explain and justify their choices. All the teachers taught this unit in their classes while the data were being collected.

### The textbook and the unit

This is the fourth unit of the 7<sup>th</sup> grade textbook, a part of the *Integrated Mathematics* junior-high school curriculum. This curriculum is specifically designed as a series of trajectories of tasks, one of the main characteristics of which is to engage students in conjecturing and justifying. The preceding three units include


some work on algebraic generalization of patterns and some drilling on the properties of real numbers. The unit comprises five lessons; each includes tasks for direct teaching used as part of the whole class discussion and exercise tasks (total of 72 tasks). The main mathematical emphases are: using algebra as a tool for generalization; acquaintance with equivalent algebraic expressions; and purposeful use of simplification and substitution.

### Data analysis

The 17 teachers produced 52 responses (16 teachers chose three tasks as they were asked and one teacher chose four tasks); each response consisted of the choice of a task that will encourage argumentative activity in the classroom and an explanation for their choice. Overall, 21 different tasks from the unit were chosen by the 17 teachers. As a first step, we identified the tasks' distribution within the unit, including the lesson the task was taken from (1–5), its function in the lesson (direct teaching, exercise), and the number of teachers' responses per task. We then took two directions when we analyzed the data: Top-down direction and bottom-up direction. The reason why we used top-down analysis was to learn how researchers, as presenters of the scientific educational authority, conceptualize the chosen tasks as encouraging argumentation. Our intention was to use this conceptualization as a backdrop against which to highlight and characterize the teachers' attentiveness, which emerged from the bottom-up analysis.

*Top-down analysis* of the 21 chosen tasks' affordances for argumentation, by adapting a widely used analytical framework suggested by Stylianides (2009), and was adapted by Bieda, Ji, Drwencke, and Picard (2014). This framework is commonly used to examine the opportunities provided in mathematics textbooks to engage in what Stylianides called reasoning-and-proving (RP), which means "the overarching activity that encompasses... identifying patterns, making conjectures, providing non-proof arguments, and providing proofs" (Stylianides, 2009, p. 259). Following this framework, we coded each chosen task according to the (1) Purpose of the RP Problem, e.g., making claims, making justifications; (2) Intended Outcome of the RP Problem, i.e., the type of justification expected: proof-type argument (demonstration or generic example), or non-proof-type argument (empirical or rational). For example, the task presented in Figure 1 was taken from the direct teaching part of lesson 1. Question a

Below are the first three elements in a sequence of points:



Students suggested algebraic expressions to the number of points in the  $n^{\text{th}}$  element of the sequence ( $n$  is natural).

Shira:  $n + 5 + n$                       Efrat:  $2 \cdot (n + 2) + 1$

Noga:  $n + 4 + n + 1$                   Rachel:  $5 + 2 \cdot n$

Discuss the various suggestions and explain how each student "counted" the points.

- Do all the expressions suggested by the students describe the sequence? Show that the expressions are equivalent by using the properties.
  - Maya said:  $5 + 2 \cdot n = 7 \cdot n$ . Is Maya right? Explain.
  - Write three equivalent algebraic expressions to the expression  $7 \cdot n$ .

Figure 1: A task chosen from the direct teaching section of lesson 1

in the task engages students in generalizing visual patterns algebraically and introduces them to equivalent of algebraic expressions. Several teachers chose question b, which addresses a common mistake. Using Stylianides' framework, and assisted by the textbook's teacher guide (Integrated Mathematics, 2012), we coded the Purpose of the RP Problem as making a claim (Maya is not right) and a justification (e.g., multiplication precedes addition). The Intended Outcome of the RP Problem was coded as a proof-type argument of demonstration (e.g., relying on the properties of real numbers). We analysed in the same way all the 72 tasks within the five lessons unit.

*Bottom-up analysis* of the 52 teachers' responses, through which the categories of attention to the task's potential for argumentative activity were generated and consolidated, and then used to identify teachers' attention profiles, as will be elaborated below.

In both phases of analysis, the data were coded independently by each researcher, followed by a comparison of the codes. All disagreements were resolved by discussion and a consensus was reached.

## FINDINGS

### The distribution of the 21 chosen tasks across lessons

The distribution of the 21 chosen tasks was found to be rather homogeneous across the five lessons, and also across their function in the lesson (direct teaching or exercise). Out of 52 teachers' responses, the

number of responses per each of the 21 chosen tasks was between 1 and 7.

### Top-down analysis of the tasks' affordances for argumentation

Using this framework, we analyzed the 21 tasks chosen by the teachers, which revealed that the purpose of 19 of them was to make claims and justifications and the purpose of the other two tasks was to make claims; this is where 32 tasks of the 72 within the whole unit asked for justification, while the other 40 asked for claims only. In 16 out of the 19 tasks the intended outcome was a proof-type argument of demonstration, which is "at the top of the hierarchy" (Stylianides, 2009, p. 280), In the other three tasks, the intended outcome was an empirical argument, where from the 32 tasks within the whole unit, aimed at justification, 24 were of proof-type and 8 of empirical argument. Thus, we can conclude that the teachers chose tasks that by and large match Stylianides' RP spirit.

### Bottom-up analysis: Dimensions of teachers' attention to the task's potential for argumentative activity

Bottom-up analysis of the 52 teachers' responses led to identifying categories that fall into two dimensions of attention:

- (D1) Attention to the mathematics in which the argumentative activity is embedded (23 out of 52 responses).
- (D2) Attention to socio-cultural aspects related to the argumentative activity, such as student-teach-

er interactions and the nature of the class discussion (20 out of 52 responses).

Next, we describe the various profiles in more detail and with examples.

Note that each response may refer to one of the dimensions, both dimensions, or neither dimension.

*Profile 1:* Teachers whose all responses referred to the two dimensions of attention: the mathematics (D1) regarding its three categories (D1a–c) and the socio-cultural aspects (D2) (three teachers).

Upon further analysis, we found that teachers focused on three aspects of dimension D1: (D1a) the mathematics inherent in the chosen task itself (23 out of 23 responses; this means that each response that was attentive to D1 was attentive to D1a); (D1b) the mathematics related to the teaching sequence that the task is a part of (9 out of 23 responses); and (D1c) global/meta-level principles of mathematics that transcend the particular task (8 out of 23 responses).

Example: A profile 1 teacher chose the task presented in Figure 3, which was taken from the exercise section of lesson 2. In the previous lessons students became acquainted with methods for justifying the equivalence of algebraic expressions.

When looking at each teacher's responses separately we found that each teacher was consistent regarding the nature of his/her attentiveness across his/her 3–4 responses – i.e., the responses associated with the 3–4 tasks that each teacher chose fell in the same categorization in terms of D1 a-c and D2.

She explained her choice (our coding appears in brackets after each utterance):

Here different algebraic expressions may be suggested, some correct and perhaps some incorrect. It is possible to write all of them on the board and ask the students to say what they think about each expression (D2). The answers should be based on mathematical justifications: Use of the properties of real numbers to prove that the expressions are equivalent and use of substitution for a counter example or use of the properties to show that the expressions are not equivalent (D1a+D1c). If there is a disagreement about a certain expression, they (the students) will have to convince each other until they reach an agreement (D2). It is possible

Overall, four different profiles of teachers' attention were found, as shown in Figure 2. It is worth to note that there was no correlation between a certain profile and a certain chosen task: different tasks were chosen by teachers of the same profile, and the same task was chosen by teachers from different profiles.

Profile 1: n = 3				Profile 2: n = 4			
Mathematics dimension (D1)			Socio-cultural dimension (D2)	Mathematics dimension (D1)			Socio-cultural dimension (D2)
Within the chosen task (D1a)	Within the teaching sequence (D1b)	Meta-level principles (D1c)		Within the chosen task (D1a)	Within the teaching sequence (D1b)	Meta-level principles (D1c)	
Profile 3: n = 4				Profile 4: n = 6			
Mathematics dimension (D1)			Socio-cultural dimension (D2)	Mathematics dimension (D1)			Socio-cultural dimension (D2)
Within the chosen task (D1a)	Within the teaching sequence (D1b)	Meta-level principles (D1c)		Within the chosen task (D1a)	Within the teaching sequence (D1b)	Meta-level principles (D1c)	

Note: The shaded area denotes an identifiable dimension of attention.

Figure 2: Profiles of teachers' attention



Write five algebraic expressions that are equivalent to the expression  $1 + 6(x + 3)$

**Figure 3:** A task chosen from the exercise section of lesson 2

that first students will suggest numerical examples as a proof for equivalence, but they will have to convince me and their peers that it is correct (D1a, D2). If no objection is raised in class, I might suggest a counter example in order to show them that an example is not necessarily sufficient to prove equivalence (D1c, D2). I want them to move to a general algebraic justification (D1c), which is one of the goals of this study-unit (D1b). If a student will justify equivalence by substituting a number, it is an opportunity to talk about the idea that one example is not enough for proving equivalence. Here is an opportunity to talk about how, in general, it is acceptable to justify in mathematics (D1c).

In her response, the teacher considers the mathematical justifications related to the task itself, the role of the task in the unit's learning trajectory, and the general principles of proof in mathematics, and interweaves them all in the socio-cultural process of shaping collective argumentation in class.

*Profile 2:* Teachers whose all responses referred to the mathematics within the task (D1a) and not to the other elements in D1. In addition, they did not refer to socio-cultural features (four teachers).

Example: A profile 2 teacher chose question b in the task presented in Figure 1, taken from the direct teaching section of lesson 1. This question was aiming at addressing a common mistake. In explaining her choice, the teacher considered only the mathematical claims and justifications related to the task:

The task involves argumentative activity in which the argument is that Maya is not right because multiplication precedes addition, or you substitute a number on both sides of Maya's equation and receive different values, meaning there is no equality here (D1a).

*Profile 3:* Teachers whose all responses referred to the socio-cultural dimension (D2) and not to the D1 aspects (four teachers).

Example: A profile 3 teacher also chose question b in the task presented in Figure 1:

In this activity students are requested to explain whether Maya is correct or not. Each student holds a different point of view. I will ask for more and more arguments. Students will describe their opinion and justify it and will have to convince their friends and me, or change their opinion and together reach the right answer (D2).

In her response, the teacher considers the socio-cultural aspects of organizing collective argumentation including raising arguments, convincing peers, and reaching a consensus. However, the response lacks any mathematics.

*Profile 4:* Teachers whose all responses were not considered to relate to any of the categories of attention (six teachers).

Example: A profile 4 teacher chose the same task and wrote:

Is Maya right? Explain. Engaging in this activity requires students to raise arguments.

## DISCUSSION AND CONCLUDING REMARKS

We consider as a major achievement of this research the possibility of getting close to a group of mathematics teachers and revealing some of their views concerning the argumentative potential of mathematical tasks in a textbook's unit for the teaching-learning processes in their classrooms. Our methodology was naturally integrated into the teachers' work, allowing us to reveal teachers' actual choices of "argumentative tasks" and their genuine attentiveness to different dimensions of what they consider the task's argumentative potential.

In examining the teachers' choices of tasks through the lens of established research "tools" (Bieda et al., 2014; Stylianides, 2009), we found that most of the chosen tasks are of the proof-type argument (RP) of demonstration, which is "at the top of the hierarchy" (Stylianides, 2009, p. 280), whereas more than a half

of the unit-tasks did not ask for argument of any kind. This suggests that teachers were attentive to tasks that afford argumentation. Moreover, viewed collectively, the teachers attended to important rich dimensions of argumentative activity in their explanations. These dimensions – the mathematics in which the argumentative activity is embedded and the socio-cultural aspects related to the argumentative activity – reflect the complex process of establishing argumentation in the mathematics classroom and the simultaneous tasks teachers need to manage in order to facilitate it (e.g., Yackel, 2002). However, the attentiveness of individual teachers in the group was quite diverse; whereas some teachers were attentive to both the mathematics and social dimensions, for other teachers, such attentiveness was partial or nearly absent; they attended only to the mathematics embedded in the task, or only to the social situation, or did not exhibit attentiveness in either of the dimensions.

Our finding of consistency of the dimensions of attention found within each teacher's explanations for their choices and the fact that a same task was chosen by teachers of various profiles, suggest that the effect of the teacher's approach to argumentation was greater than the effect of a particular task. It is worth to note that no correlation was found between a teacher's profile and teaching experience, neither between a teacher's profile and education.

Restricted to the rather small sample, the findings of this study raise several intriguing questions and issues for further research. One is related to possible connections between what teachers attend to when choosing tasks and their actual teaching in the classrooms: Does a teacher who attends to the social aspects of argumentation but does not attend to the mathematics aspects in which the argumentative activity is embedded find it difficult to interweave the pedagogical practices successfully with the mathematical ideas (e.g., in presenting to students what constitutes acceptable mathematical arguments, in supplying argumentative support that was omitted or left implicit)? Does a teacher who focuses on the mathematics aspects but does not attend to the social aspects of argumentation make less room or find it difficult to support collective argumentation in the classroom?

Another question is related to the finding that the teachers who attended to the mathematics aspects

within the task only, did not attend to the social aspects at all, whereas teachers, who attended to the mathematics aspects within the task, within the teaching sequence, and to global principles, attended to social aspects as well. Can this point at certain connections between attention to the mathematics in a broader sense and attention to social aspects? And if yes, what are the implications concerning the education and support planned for teaching focusing on argumentation? Further study will address such questions.

Paparistodemou and colleagues (2014) showed that teachers can learn to enrich their attention by reflecting on their teaching. We plan to devise ways to support teachers' development of attention, taking into consideration two main issues identified in this study: One is the diversity of the teacher population concerning attentiveness to argumentation in the learning-teaching process. The other is that this diversity ranged between attentiveness to mathematical aspects and socio-cultural aspects, and no attentiveness at all (at least not explicit). The fact that some of the participating teachers considered both dimensions as an integral part of enhancing argumentation processes in the classroom is encouraging; apparently teachers are at least partially open to adopting these "habits of teaching".

Still, another issue that emerges from our findings is related to design. We saw that the same task was chosen by teachers of different profiles; i.e., teachers saw differently "through" the task. This raises the question of how we can make a task more transparent in its argumentative potential so as to be "seen" by varied population of teachers.

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