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Mathematical opportunities: Noticing and acting

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The aim of this study was to investigate how three pre-service teachers (PSTs) listen to students, notice Mathematical Opportunities (MO) and scaffold ideas based on MOs. There were 12 videos of three PSTs' interactions with a pair of 6th grade students respectively while studying fractions. We analysed videotapes and identified different number of MOs for each PST. The findings revealed that with the help of this research and teaching environment, all PSTs listen to the students to understand their mathematical thinking initially (meaning catching MOs) and try to follow-up on them in action in differing levels of sophistication. While most of the investigated MOs resulted in a mathematical solution, PSTs need to further develop appropriate scaffolding practices.

Keywords: Pre-service teacher education, mathematical opportunities, fractions, scaffolding, listening to children.

INTRODUCTION

Reform oriented teaching supported the idea that “all students should have the opportunity and the support necessary to learn significant mathematics with depth and understanding” (NCTM, 2000, p. 50). Reform-oriented teaching and its natural necessity, student-centred education, require teachers to attend to students' thinking both in planning and in action. Many researchers investigated how this mechanism work in terms of how teachers listen to, pay attention to students' mathematics and act accordingly (Franke, Webb, Chan, Ing, Freund, & Battey, 2009; Sherin, 2004). Giving importance to students' mathematics and using this construct in pre-service teacher education programs as component for quality teacher-education have been also argued in different research venues (D'Ambrosio & Campos, 1992; McDonough, Clarke, & Clarke, 2002; Philipp et al., 2007). While the research in pre- and in-service education of teachers helps us illuminate the issue of how to use students' thinking

for practice, there is more to investigate for how this opportunity pre-service teachers become to notice children's mathematical thinking and act on such situations.

In this paper, we will report an intervention study that focused on three pre-service teachers and their interactions with three pairs of 6th grade students. The purpose of the study was to understand how pre-service teachers' (PSTs) notice children's mathematical solutions and steer their interactions using those notices i.e., how they use those constructs for the purpose of “scaffolding activities.” In relation to this concern, we defined *Mathematical Opportunities* (MOs) occurred in the interactions and used those opportunities as basis for our web of analysis. Therefore, we had following research question:

To what extend do the pre-service teachers notice MOs and scaffold students' mathematical thinking during interactions (with the students)?

THEORETICAL FRAMEWORK

Mathematical opportunities

Our definition and use of mathematical opportunities for pre-service teachers might show parallelism with what Leatham et al. noted for teachers. It is that teachers should recognize such opportunities initiated from the students to build on the students' mathematical thinking (Leatham, Peterson, Stockero, & van Zeist, 2015). They called those opportunities as *Mathematically Significant Pedagogical Opportunities to Build on Student Thinking* (MOSTs) and described them as being observable student actions that enable teachers to make inferences about students' mathematical thinking, as being appropriate and important *mathematical point* to be focused on and as having potential to help students to understand the essence of the *mathematical point*. We build further on MOs for the purpose of understanding how they might pro-

vide opportunities for PSTs and what the affordances of those opportunities in action for the students.

Instruction based on students' thinking

A teacher-initiated or student-initiated conversation between the student and the teacher is valuable in terms of having a potential for creating teaching-learning opportunities. Van Es and Sherin (2002) stated that in reform-oriented teaching, teacher's noticing and interpreting students' actions are key components when adapting instruction in the moment. While many experienced teachers can learn how to notice and interpret over the years of practice, it is important that these concepts should be integrated in the teacher education programs as basis for reform-oriented teaching centered on students' thinking. From their research, van Es and Sherin (2002) identified the components of "noticing" as follows: "(a) identifying what is important and noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; and (c) using what one knows about the context to reason about classroom interactions." (p. 573). For the purpose of providing learning opportunities for PSTs about "noticing," instead of the whole classroom interactions, we provided a setting where PSTs observed the two individual students solved mathematical problems. Then PSTs continued to observe when two students discussed about their solution of the particular problem and lastly we let them to interact with the students to understand, and extend students' thinking with their questioning and scaffolding activities. In this sense we think that providing such micro-classroom environment is important for PSTs to focus on "noticing" of a pair of students' mathematical thinking and provide in-action instruction around their observations of students' thinking.

Questioning and scaffolding

With the demands of reform-oriented teaching, the nature of the interactions between the students and the teachers has to change from traditional *show* and *tell* to more advanced interactions. Teacher's use of language, teacher's intentions, the use of representational and instructional tools, how an interaction is started, continued and ended are important for providing learning opportunities for the students. Anghileri (2006) discusses three levels of "scaffolding practices" (for further explanations, see Anghileri, 2006). The first level is basic and it is related to the en-

vironmental and physical materials in the classroom and their effect on orienting students for preparing learning. She defines Level 2 scaffolding as "explaining, reviewing and restructuring." Anghileri (2006) states that usually showing and telling had been an accepted explaining in traditional teacher initiated actions. However, she states that there are alternatives to showing and telling, such as reviewing and restructuring. Probing and prompting questioning types are mainly used in reviewing students' mathematical ideas. While probing questions are "to gain insight into students' thinking, prompting their autonomy and underpinning the mathematical understanding that is generated" (p. 42), prompting questions might "lock the teacher in the center stage" (p. 43) and might put the student into guessing mode of teacher's intentions behind the questions. Level 3 scaffolding is identified as the highest level of scaffolding such that we rarely observe this in classroom discourse, it is the scaffolding that helps students' extend their thinking and "specifically focused on making connections and generating conceptual discourse" (Anghileri, 2006, p. 47). While Anghileri's framework helped us to how to focus on scaffolding actions, we needed to develop a new framework, which helped us to explain PSTs' teaching practices in this study (see the Analysis part).

METHODS

Research setting and participants

Data collected for this analysis occurred in an after school program with the partnership of a university in Istanbul, Turkey and a local low SES middle school 6th grade classroom. Participants included three senior PSTs specializing in teaching middle and high school mathematics (Anna, Betty, Carol; all names are pseudonyms) and three pairs of 6th grade students (Pair A, Pair B, and Pair C; worked with Anna, Betty and Carol, respectively). PSTs and the students participated in this study voluntarily. All of PSTs had some informal teaching experiences such as tutoring mathematics, teaching in an after school programs voluntarily, however their teaching experiences were not homogenized.

Mathematics workshops

Mathematics workshops originally started with the idea to teach 20 local 6th grade students difficult mathematics concepts using manipulative as an after school program. There were two university professors (authors) who did team-teaching in the workshops, five

to seven PSTs who voluntarily came to workshops and helped the 6th graders whenever needed and one research assistant videotaped the whole class interactions. After each session, the professors and PSTs came together and discussed their observations related to what happened in the workshop and their observations related to students' understanding. About 11th week in the workshop program, we realized that PSTs were doing "show and tell" instead of paying attention to students' mathematical thinking and they were very directive in their questions. For the following eight weeks, we recruited three PSTs from the group, assigned them to particular three pairs of students, and focused on one mathematical topic, *fractions*. Sixth grade students were chosen based on the observations and performances they showed during the first 11 weeks. There were two mid-achieving pairs (Pairs A and B) and one-high achieving pair (Pair C).

In this paper, we will focus on smaller part of the data-4 weeks. During this 4-week period, each session was about 45 minutes and organized as follows: The first author introduced the activity and 6th graders individually worked on them. Later, 6th grade students worked in pairs and had chances to discuss their solutions with their partners. During this time peri-

od, PSTs only observed and video recorded students' individual work and their work in pairs. PSTs did not talk to students during this time period. This was on purpose since we wanted to have PSTs focus on students' activities while freeing themselves from the urge of teaching. This enabled them to observe the students' mathematical thinking process, how students communicated and negotiated their thinking to their partners. Later, PSTs were allowed to talk, interact and ask questions for 10–15 minutes. Finally, time permitting we had whole classroom discussions with 6th graders that the first author led.

ANALYSIS

We analyzed four sessions of three PSTs own-recorded videotapes. Data analysis consisted of analytic induction (Bogdan & Biklen, 2003). We reviewed all the videotapes one by one and identified MOs. We define MOs as 6th grade students initiated solutions and these solutions are usually interesting ways of mathematical thinking related to the fractions topic. For example, 6th grade students were asked to draw a number line and locate unit fractions such as 1/2, 1/3, 1/4, 1/5 on it. Betty's students treated the unit fractions as if they were whole numbers and they located them accord-

Identifier	Description	Coding
Opportunity	The mathematical situation that 6 th grade students provide and overall how PSTs approached the situation	Y: Yes N: No
Opportunity in action	The interactions such that what kinds of questions PSTs asked to assess and advance students' thinking, what kinds of mathematical or mathematical pedagogical knowledge they used in their interactions, and how they proceeded and closed the conversations	N/A: If the PST missed the opportunity Level 1: If PST has surface level questioning. Conversation is mostly described as individual questions-answers. There is no big mathematical idea communicated in the conversation. Teacher mostly asks for explanation but does not take it to the further; she does not do anything with the explanation. Level 2: If PST asks probing and prompting questions. There is progressive conversation, which might include students' contribution, but it is mostly teacher-dominated conversation <i>with her mathematical goals in mind</i> . "Reviewing" and "show and tell" could be indicators of this level. In addition, PST might attempt to give examples and use materials to help students achieve her mathematical goal in mind but the students may/may not understand or make the teacher's intended connections. Level 3: If we saw evidences of PST's guiding students. Purpose of questioning is guiding towards a legitimate mathematical idea. The students also positively perceive questioning. Questioning might advance the students thinking. Students might have some ownership of the ideas developed during conversation. PST addresses misconception (if there were any) by providing mathematically valid examples to help the students to understand the teacher's mathematical goal.

Table 1: Coding scheme for analysing MOs

ingly with equal distances. This is an example of a MO that we investigated in detail. We made coding based on a grounded framework we developed ourselves (see Table 1) but it also shows some similarities to the focal points of van Es and Sherin's (2002) "learning to notice" and Anghileri's (2006) "questioning and scaffolding" frameworks.

We will introduce two examples from the common opportunities and discuss how the opportunity was used by two PSTs. In the third opportunity (see Table 2) the students were asked to find a fraction between $\frac{1}{2}$ and $\frac{1}{3}$, make a number line and place that fraction between $\frac{1}{2}$ and $\frac{1}{3}$.

Anna and Pair A

Anna's students' answer to the given problem is given in Figure 1. The students thought $\frac{1}{2}$ and $\frac{1}{3}$ as integers of 2 and 3 respectively. Therefore, a number which was bigger than 2 and smaller than 3 was 2.5 (In Turkey, comma is used to show decimals).

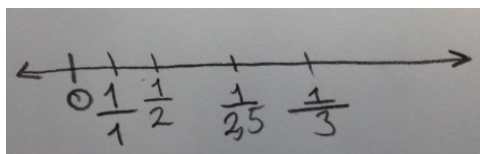


Figure 1: Anna's students' work

From their drawing we inferred that students probably thought that, except "0", the numerators of the fractions would be "1" and denominators would be "in order" starting from "1." Because they knew that 2.5 is between 2 and 3 then $\frac{1}{2.5}$ would be the answer. Anna had the idea that fractions should be represented in a/b form where "a" and "b" are natural numbers but not decimals. In the following vignettes, Anna first wanted to discuss this point:

Vignette 1

- Anna: How did you find two and a half? What kind of number is two and a half?
- Pair A1: It has comma
- Anna: Decimal
- Pair A2: It is with comma, that is, it is not whole.
- Anna: What kind of number system are we interested in? That is, how do we call the numbers [pointed to $\frac{1}{2}$ and $\frac{1}{3}$] in this system?
- Pair A2: Proper fraction [it was supposed to be unit fraction].

- Anna: Proper [Unit] fraction, right? Even it is improper, it is fraction. We show fraction like this [pointed to $\frac{1}{2}$]. So, why did you write a decimal here [pointed to $\frac{1}{2.5}$]?
- Pair A2: We couldn't find anything else.
- Anna: OK. How can you represent two and a half in other way?
- Pair A2: $\frac{2}{4}$
- Pair A2: The half of 5. That is $\frac{1}{2}$ of 5.
- Anna: What do you do with 5, that is, how can you represent this as a fraction?
- Pair A2: $\frac{1}{5}$...
- Anna: Your thinking is good but you are missing something. What did you do here? [Pointed to the previous number line] What did you do to show $\frac{1}{4}$? What did you do 1?
- Pair A2: We divided into 4 pieces.
- Anna: What did you do with 5 and got 2.5?
- Pair A2: 5, again we divided it into two.
- Anna: OK. What kind of expression is that?
- Pair A2: $\frac{1}{2}$.
- Anna: You are saying that 5 but then how did you divide it into 2.5? Where that division operation comes from?
- Pair A2: Division operation...
- Anna: Don't get confused by division. Here [pointed to previous drawing for $\frac{1}{4}$] you divided into 4 because you have $\frac{1}{4}$ at hand and to find it you divide one by four. Then, I am just talking about this 2.5. Because, why you wrote 2.5 here, it's interesting. Very interesting. I cannot think of it.

Then Anna drew a number line placing 1, 2 and 3 and then placing 2.5 between 2 and 3. The conversation continued with her questioning but because of the limited space we will not include it. Anna realized that students' thinking would lead to a correct answer if the problem statement asked a number between 2 and 3; an answer of 2.5 as a decimal would be acceptable. Her questioning shows us that she realizes students are not necessarily thinking in the same way that Anna thinking about conception of fractions. Anna's interaction with the students evolved around many topics. She first questioned students whether their answer was a fraction or decimal, then she wanted to have the students show 2.5 as a fraction (where she differed from the original answer of $\frac{1}{2.5}$).

In our framework, PST noticed the MO (coded as Y for Opportunity). Once she received answer of “one half of 5” from the students, she moved asking questions about how to represent it (students said $1/5$ that Anna did not investigate). She then asked them to relate to earlier example of $1/4$ “where 1 was divided into four pieces” (as students verbalized). Anna then focused on centralizing the conversation on “division operation” and how students might have used “division” to get 2.5 from 5. For a while, the focus of the conversation and leading questions were unclear. Eventually, Anna asked how they thought $1/2 \cdot 5$ as in the same way they would give meaning to $1/4$ (where students said it was one piece out of four pieces). Students did not give a definite answer to the question but they indicated that it seemed “illogical” after talking to Anna. The conversation ended without a satisfying situation neither for the students nor for Anna.

When we analysed the conversation, we observed that Anna controlled the conversation. Nature of her scaffolding and questioning changed throughout the conversation: she used some probing questions (e.g., how can you represent this as a fraction? So, why did you write a decimal here?) But interestingly the conversation did not lead to any productive ways of thinking on the students’ part. The probing questions sometimes did not help the students and the teacher did not know how to use it to steer the conversation to help students gain some understanding. We coded these interactions as Level 2, for Opportunity in Action. We did coding for all three PSTs and on five common MOs (see Table 2 for the results).

Betty and Pair B

Betty’s students’ answer to the given problem is given in Figure 2. They thought a number between $1/2$ and $1/3$ could be found if the numbers are rewritten as $2/2$ and $3/3$. A number in between would be less than $3/3$ and more than $2/2$, so it would be $2/3$.

Betty started the conversation by summarizing what the problem was asking. She wanted to make sure that students were also viewing the situation as she was. Then, students’ answer was not an answer she expected. She asked “how” they found the answer. We coded

whether teacher noticed this Opportunity as Yes. The conversation continued with teacher’s questions focused on understanding how students came up with the answer of $2/3$. Students introduced “wholes” and used number line as a conveying representation of their ideas. When Betty did not agree with students’ answer of $2/3$, she did not say this directly but asked them to locate $1/2$ and $1/3$ on the same number line. This was a good move in terms of reorienting students’ thinking to what was asked in the original question situation. But she moved to using fraction strips, i.e., a tangible linear manipulative with colored parts and unit fraction symbols written on them. Betty first asked a general question, such as how to use the fraction strips to transfer that knowledge into the number line. Then the focus of the conversation moved to comparing $1/2$ and $1/3$ with the colored fraction strips. Even though with questioning, students were able to say, “ $1/2$ ” is bigger than “ $1/3$ ” they had hard time to understand what fraction might be in between those two numbers when fraction strips were used. Eventually, since the students were so immersed in the context, i.e., fraction strips, they were not able to look at all the other possibilities that included proper fractions such as $5/12$ or $2/5$. They were thinking that it should be a unit fraction that was overly written on the fraction strips. Betty indicated that she was surprised: “Now you say there is no fraction. But you were saying there was $2/3$ before...” Betty’s purpose of questioning evolved depending on what kinds of answers she received from the students. We coded Opportunity in Action as Level 2 since PST asked probing and prompting questions. There was a progressive conversation, which might include students’ contribution, but it was mostly teacher-dominated conversation with her mathematical goals in mind.

FINDINGS

In the initial round of coding, two researchers (authors) checked all the videotapes that three PSTs recorded and identified different number of opportunities for each PST. We identified 9 opportunities for Anna and pair A (mid-level achieving), 10 opportunities for Betty and Pair B (mid-level achieving), and 15 opportunities for Carol and Pair C (high-level achiev-

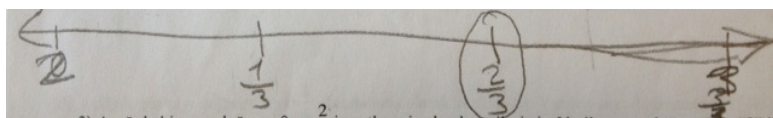


Figure 2: Betty’s students’ work

Pre-service Teacher	Description of the Opportunities	Opportunity	Opportunity in Action
Anna	Opportunity 1: Showing one fourth of an equilateral triangle Opportunity 2: Ordering unit fractions on a number line Opportunity 3: Placing a fraction in between two unit fractions (e.g., 1/2 and 1/3) Opportunity 4: Sharing an unknown amount, fraction multiplication and comparison Opportunity 5: Fraction division with manipulative and transferring to paper	Y: 5	Level 2: 5
Betty		Y: 5	Level 2: 4 Level 3: 1
Carol		Y: 3 N: 2	N/A: 1 Level 1: 2 Level 2: 2
Total		Y: 13 N: 2	N/A: 1 Level 1: 2 Level 2: 11 Level 3: 1

Table 2: Summary of the frequencies of the coding items

ing). This different number depended on the nature of the interactions PSTs and the pair of 6th graders had about the specific fraction problem. In the second round of coding, we overviewed all of the occasions and we identified 5 opportunities common for all the three PSTs (See Table 2). Based on the coding scheme we coded the PSTs interactions with students separately and achieved 0.88 consistency initially. Then we discussed the different coding and reached full consistency in coding.

With this experience, none of the three PSTs did “show and tell”. They developed better questioning skills (Sleep & Boerst, 2012). Although, this was the case, as stated in other studies (e.g., Morris, Hiebert, & Spitzer, 2009; van Dooren, Verschaffel, & Onghena, 2002), one PST’s lack of content and pedagogical content knowledge hindered to catch all MOs and address them effectively (see Carol’s case, in Table 2). For Anna, while her interactions in defined MOs indicated as Level 2, her communication skills in Turkish deterred her from finding ways to use her content knowledge. This situation was directly related to her developing pedagogical knowledge for teaching fractions. Even though, PSTs listen to the children to figure out students’ mathematical thinking, they need to further develop appropriate scaffolding activities. For instance, working with high-level achiever students who can argue their thinking more firmly might benefit PSTs growth more (see Carol’s case). In addition, PSTs’ own preparation, thinking ahead the necessary mathematical connections and planning towards some mathematical goal can improve their scaffolding actions (see Betty’s case). Eventually, this

might result in advancement of the conversations and 6th grade students’ mathematical gains. In the working group, we will provide further evidences related to these claims and seek contributions from the group members.

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