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Applying the structured problem solving in teacher education in Japan – A case study

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In this paper, we examine the implementation of a Japanese teacher educators' lesson, where he applies and, at the same time, inform the students about "structured problem solving". We describe a specific lesson titled "Quantity and Measurement" for elementary school teacher students and we show how the educator make the students aware of the didactic transposition of the material and how he makes the students experience and learn about applying "structured problem solving" in practice. We also show how the Japanese curriculum influences the scale of the mathematical praxeology to be learned and how the students are given opportunities to develop their insight into the PCK during their education in mathematics.

Keywords: Teacher education, pedagogical content knowledge, didactic transposition, anthropological theory of didactics, content representation.

INTRODUCTION

In recent years, the need to train teachers' and teacher students' skills for teaching mathematics has been strongly emphasized by both politicians and mass media in Sweden. One of main reasons for this is that the performance of Swedish students in international surveys of education, such as the TIMSS and Pisa study in mathematics has radically declined since 1990's. According to Brown and Borko (1992), one of the most important purposes of teacher education is the acquirement of pedagogical content knowledge (PCK). It is recognized, that such knowledge forms the essential bridge between the academic subject matter knowledge (SMK) and the teaching of the subject matter. Furthermore, it includes an understanding of which representations are most appropriate for an idea, which ideas are difficult and which are easy for learners and what conceptions and preconceptions students of different ages hold about an idea.

Specifically, if the preconceptions are erroneous conceptions, teachers need to know about strategies for reorganisation of the learners understanding (Shulman, 1986).

This is a part of a future comparative study project between Japan, Finland and Sweden concerning teacher education that aims to identify and analyse differences of institutional settings in several countries (Artigue & Winsløw, 2010). Our intention is to illuminate teacher educators' perception of the SMK and PCK, by analysing the mathematical and didactical organisations in the countries' primary school teacher education. In this paper, we present our first study from Japan, where the focus is on how the teacher educator applies *the structured problem solving* in his lecture. Structured problem solving has the emphasis on creating learning opportunities for students by using challenging problems and to stimulate students' corrective reflection on their solutions.

Shimizu (1999) explains some pedagogical terms which are used daily by Japanese teachers in mathematics class: *hatsumon*: asking a key question, *kikan-shido*: teachers' instruction at students' desks, *neriage*: whole-class discussion, *matome*: summing up. Having such common didactical terms indicates that Japanese teachers have acquired an institutionalised perception about the teacher's role in the classroom. Many Japanese teacher educators in mathematics apply and instruct on this teaching pattern in classes for their students. We observed and analysed the mathematical and didactical organisation of a lesson concerning "Quantity and Measurement" in a course named "Arithmetic Education" for prospective elementary school teachers in Japan.

THEORETICAL FRAMEWORK

Teacher knowledge

The importance of teacher knowledge both in teaching and in teacher education has been cogitated by researchers in several articles (see, e.g., Shulman, 1986; Kind, 2009). A number of models of teacher knowledge have been generated in this field. Although researchers differ in their definitions of various components in teacher knowledge, three areas of teacher knowledge can be seen as the cornerstones of the emerging work on professional knowledge for teaching: subject matter knowledge (SMK), general pedagogical knowledge and pedagogical content knowledge (PCK) (e.g., Shulman, 1986). PCK is a term to describe ‘the ways of representing and formulating the subject that make it comprehensible to others’ (ibid, p. 9). It has been found that PCK is a useful tool for understanding the professional practices of teachers (Kind, 2009). Investigating PCK in teaching practice is a difficult process, but Kind (2009) points out that using Content Representation (CoRe), developed by Loughran, Mulhall and Berry (2006), might give a unique awareness into the teachers’ PCK and their practices relating to specific topics and subject areas. CoRe focuses on different parts of PCK and offers a way to give an overview of the teaching approaches for a specific subject area and to motivate teaching decisions.

The didactical transposition theory and the anthropological theory of didactics

Chevallard developed the conceptualisation of a didactical transposition: how the knowledge content is

adapted for the purpose to be taught within a given institution. It means a transposition from *scholarly knowledge* (Bosch & Gascón, 2006), which is produced in the community of mathematicians, into the knowledge for teaching at different levels within the teaching system. Bosch and Gascón (2006) illustrated the steps of a didactical transposition process through different institutions as *Scholarly knowledge* → *Knowledge to be taught* → *Taught knowledge* → *Learned, available knowledge* (ibid, p. 56). Chevallard’s attempt to describe the mathematical knowledge in an institutional context extended into “*the anthropological theory of didactics*” (ATD) (ibid). There, mathematics learning holds to be modelled as the construction within social institutions of *praxeologies* (ibid). A praxeology supplies both methods for the solution of a domain of problems (*praxis*) and a framework (*the logos*) for the discourse regarding the methods and their relations to a more general setting. The block of “praxis” is usually described as “know-how” and the “logos” is described as “know-why”. The praxis can be described by the set of *tasks* and *techniques* and the logos is constituted of a *technology* that informs and describe techniques and a *theory*, which is used to motivate and establish the technologies. A praxeology that describes some mathematical knowledge is also called *mathematical organisations* (MO) (Barbé, Bosch, Espinoza, & Gascón, 2005). In the same way, *didactical organisations* (DO) (ibid) is a praxeology that describes the knowledge and know-how used by teachers to teach the subject matter knowledge to their students.

Table 1 A CoRe Template

Year level for which this CoRe is designed: _____	Important Science ideas/concepts					
Content Area: _____	Big Idea A	Big Idea B	Big Idea C	Big Idea D	Big Idea E	Big Idea F
What do you intend the students to learn about this idea?						
Why is it important for students to know this?						
What else do you know about this idea (that you do not intend students to know yet)?						
What are the difficulties/limitations connected with teaching this idea?						
What is your knowledge about students’ thinking that influences your teaching of these ideas?						
Are there any other factors that influence your teaching of these ideas?						
What are your teaching procedures (and particular reasons for using these to engage with this idea)?						
Specific ways of ascertaining students’ understanding or confusion around this idea (include a likely range of responses).						

Figure 1: CoRe Template (Bertram & Loughran, 2012, p. 1029)

METHOD

We applied two methods for data acquisition in this study: Firstly, we used classroom observation with video recordings and, secondly, an interview, using a Content Representation (CoRe) template as a reflection document. CoRe, was developed, by Loughran and colleagues (2006), to help to focus on different parts of the PCK. An illustration of a Core Template is given here below.

CoRe is originally developed for science teachers' practice. It used before a lesson is conducted as a collaborative tool helping teachers to identify important aspects of the content within the specific area. After the lessons, the experiences by the teachers' can be documented in a Pedagogical and Professional-experience Repertoire (PaP-eRs), which are linked to the CoRe, and illuminate the decisions underpinning the teacher's actions intended to help the students better understand the content (Loughran et al., 2006). In our study we have used the CoRe template in a different way: After the lesson, the teacher educator answers to the items on the CoRe template to reflect and to consider on the content of the lesson. Thereby making explicit his different conceptions and decisions about teaching the specific topic. The reason we chose to apply CoRe as an interview format is that it is a convenient way to map how the teacher perceives the association between *scholarly knowledge* and the *knowledge to be taught*. Furthermore, by conducting the interview afterwards, it is possible to analyse the association with *the taught knowledge*. Thus we may illustrate the teacher educators take on the discourse on techniques and technologies – the praxis and the knowledge, in the sense of ATD, into account when he/she designs the lessons.

"Quantity and measurement" in Japanese curriculum

The guideline of the Japanese curriculum "The Course of Study" for primary school describes determination of area and volume in the domain "Quantity and Measurement". It states that (Ministry of education, culture, sports, science and technology (MEXT), 2008, pp. 23–26), pupils need to learn to "compare length, area and volume of different objects" in grade 1, learn about "standard units of length and volume (e.g., meters and liters) and measurement" in grade 2, "the units of area and its measurement" in grade 4 and they are obligated to learn "to determine area of triangles and parallelograms" in grade 5. Thus, area and vol-

ume determination is considered as a part of learning "Quantity and Measurement". Hence, it is not located in the domain "Geometry" in the Japanese Curriculum. The introduction to "Quantity and Measurement" in Japanese elementary schools usually consists of four phases (Miyakawa, 2010): 1. Direct comparison of two objects. 2. Indirect comparison of two objects with a third object, having the same kind of quantity. 3. Comparison of two objects with arbitrary object as a unit (e.g., a pencil). 4. Comparison using standard units (e.g., meters).

RESULTS AND ANALYSIS

The lecture "Quantity and Measurement" in "Arithmetic Education"

A main focus for the course "Arithmetic Education" is the content of primary school mathematics and how to teach such content. The lecturer of this course Mr. Matsui, has himself worked as a mathematics teacher in lower secondary school for 14 years. We observed 55 teacher students in Matsui's class.

Mr. Matsui begins the lecture by instructing the students to read the description of the domain "Quantity and Measurement" in the "Guidelines of the Course of study". He remarks that the comparison of two objects' areas and volume is a new addition from 2008 in the Course of Study for grade one. Furthermore, he refers to the Guidelines of the Course of study and explains the four phases in the process of pupils learning about quantities. Mr Matsui picks up two pens and asks his students "How do you compare the length of these two pens?" He requests that a student show the class how to put the pens together in a way so that the difference in lengths demonstrates clearly and he explains the term *direct comparison* of two quantities. Secondly, Mr. Matsui clarifies an example of the *indirect comparison* of two quantities by comparing the length and the depth of his desk. A student answers that one can use an object such as paper tape and so on to compare the length of the two sides of the desk. Accordingly, Mr. Matsui takes up a pencil and describes how to *use an arbitrary object as a unit* to measure and compare the two sides of the desk. He illuminates the disadvantages of this method using an arbitrary unit, (the length of the pencils varies), and he explains the reason why *standard units*, like meters and centimetres, are finally introduced in order to get exact measurements. Finally, Mr. Matsui writes down these four phases on the blackboard and then shows

the class a digital textbook that shows how the first three phases are handled in the textbook.

In the textbook for grade 4, the author uses a small grid of 1cm^2 squares to determine the area of a rectangle by counting and then to demonstrate that it is obtained by multiplication of the width times the length. Moreover, the textbook describes that the area of a square is determined in same way and defining what it calls a *formula* that determines the area of *any* kind of rectangles and squares. Mr. Matsui explains that the area of triangles, parallelograms, trapezoids and rhombs will be learned in the fifth grade and mentions that most textbooks nowadays consider the determination of the area of parallelograms before that of triangles; it used to be taught in the opposite order.

Here, Mr. Matsui’s actions indicate that he is making the students aware of the process of didactic transposition: That is, the transformation from the *knowledge to be taught* to the *taught knowledge*. The knowledge to be taught, i.e. the curriculum as stated in the “Course of Study”, is designed by Chevallard calls the *noosphere* (1992), which is a non-structured set of experts, who have a big influence within the educational system. In Japan, the contents of the textbooks are controlled by MEXT before publishing. Therefore, the contents of the textbooks can be treated almost as the knowledge to be taught. Mr. Matsui’s students verify how the textbook treats the area of rectangles and how to think of a formula as a generalised computation. The textbook treats the determination of the area of rectangles as an initial task and where some techniques are eventually justified by algebraic reasoning. It is clear that the theory, which justifies this technology, is both algebra and geometry and, in that sense, the

praxeology of the textbook is large since it follows the stipulated praxeology in the Course of Study.

The *taught knowledge* is created by the teachers’ praxis when conducting lessons in their classrooms. By referring to the four phases of learning “Quantity and Measurement” in the curriculum guidelines and relating it to the *taught knowledge* – by referring to textbooks – the students are given an opportunity to reflect deeply upon the SMK (or MO) and PCK (or DO). Thereby, the teacher students can avoid the “*illusion of transparency*” (Chevallard, *ibid*), i.e., that one believes that the mathematical knowledge is fixed and known and that one does not question the form it is presented in the curriculum since one feels that it is already known.

The praxeology of the lecture

The students are now going to find out several different methods for the determination of the area of parallelograms with the intention of teaching pupils of grade five. For this reason Mr. Matsui distributes grid papers with parallelograms and reminds that grade five pupils have learned the how to determine the area of rectangles and squares but not of triangles. He gives his students several minutes to reflect, and starts to walk between the students’ desks (*kikan-shido*) and gives them hints and decides which students’ solutions will be presented later, due to the variation of their methods for the solutions. Mr. Matsui lets four students draw and explain their solutions on the blackboard. Student A has combined some incomplete grid squares with the corresponding incomplete squares on the opposite side of the parallelogram. Thus transforming the parallelogram to a rectangle (see Figure 2 in the middle) without changing the area.

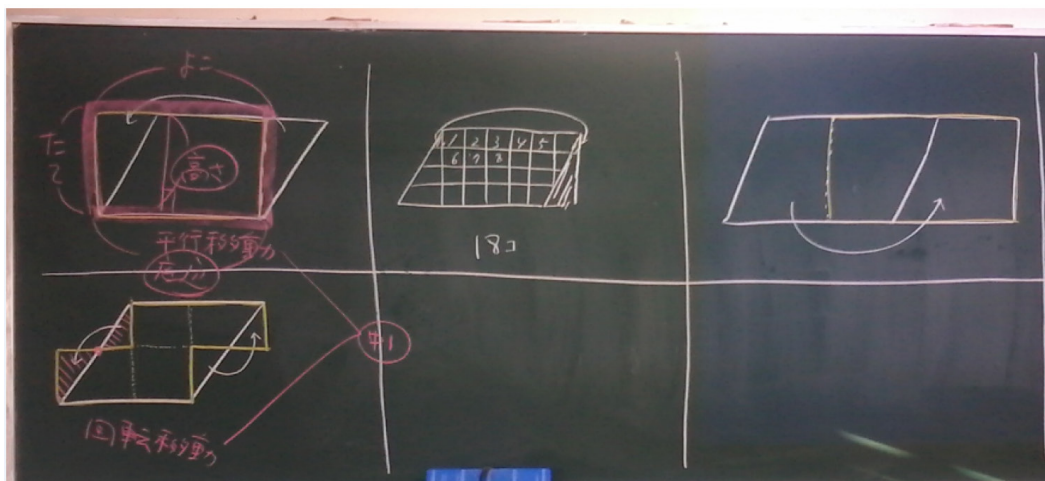


Figure 2: Students’ presented methods for determination of area of parallelogram

Matsui remarks that student A is using an arbitrary unit to determine the area.

Student B has divided the parallelogram into a triangle and a trapezoid to shift the triangle to the other side to make a rectangle (Figure 2 in the top left). Student C has divided the parallelogram into two small rectangles, two trapezoids and two triangles. Then she rotates the triangles in order to construct two rectangles (Figure 2 in the bottom left).

Finally, student D has divided the parallelogram in the middle and shifted one of the two trapezoids to the other side (Figure 2 in the top right). Mr. Matsui points out the different kinds of “shifts” used by student B, D and C. Student B and D used parallel *translation* and student C also used *rotation*.

Mr. Matsui notes that the operations of translation and rotation will be covered in more detail in grade seven. He continues by writing “previous knowledge” on the blackboard. He goes on to explain the formula for the area of the parallelogram as $w \times l$ (width times length), since the geometric transformations shows that the width (or height) and length of the parallelograms exactly corresponds to those of rectangles. He mentions that once pupils have learned how to determine of area of a parallelogram, they are able to consider the area of any triangles.

Thereafter, Mr. Matsui gives as task to find out methods for determining the area of trapezoids, using same didactical approach. He chooses 7 students. Three of them shift parts of the trapezoid in different ways so to transform it to a rectangle. Two of them divide the trapezoid in different ways. One student adds a small triangle on the top of the trapezoid to make a big triangle. Finally, the last student doubles the trapezoid so as to transform it into a big parallelogram. From the last method, Mr. Matsui establishes the formula for the area of trapezoid, which is $(a + b)h/2$.

To finish, Mr. Matsui shows an article, written by a teacher in service, about a case study of teaching the area of trapezoids using the structured problem solving approach. By doing this, he institutionalises the conception of the structured problem solving as a general didactical approach.

Mr. Matsui’s didactical task is to make his students consider how pupils would reason about such tasks

concerning area determination. At the same time, he is letting the students *experience*, how the taught knowledge in domain of “Quantity and Measurement” might *look like*. In this sense, the knowledge disseminated in the lecture has a double focus – one is for the students to construct the didactical praxeology based on structured problem solving and the other is to discuss the viability of different mathematical organisations to be taught in grade five.

The mathematical organisation for the educator/teacher students (MO_1); *Types of tasks*: comparison of lengths and areas of different objects. *Techniques*: measuring with direct comparison, indirect comparison, comparing with arbitrary units and standard units, transformation of shapes, using formulas. *Technology*: comparison, figures, translation, rotation, formulas. *Theory*: Quantity and Units, Euclidean geometry.

The didactical organisation for the educator to be used in teaching teacher students (DO_1); *Tasks*: determine how pupils in various grades would reason during a class with area determination of polygons, by considering the pupils’ previous knowledge. *Technique*: make the student participate in an example lesson using the structured problem solving approach, and follow it up with discussions. *Technology*: statement of previous knowledge, mathematical textbook and curriculum used as reference. *Theory*: structured problem solving.

The mathematical organisation for the teachers/pupils of grade five (MO_2) of this area determination: *Task*: to derive a formula for the area of parallelogram/trapezoid, *Technique*: transformation of shapes, using formulas for rectangles. *Technology*: figures, parallel shift, rotation. *Theory*: Quantity and Units, Euclidean Geometry.

The didactical organisation for teachers of grade five (DO_2); *Task*: making the pupils participate in the lessons and to reason about the determination of area of parallelogram and trapezoids. *Technique*: questioning, giving the task (hatsumon), and using graph paper (grid of 1cm) to draw their ideas on. *Technology*: group discussion, whole-class discussion (neriage). *Theory*: Structured problem solving.

Interview with Core template

As the Big ideas in CoRe template for the theme “determination of the area of a parallelogram and trapezoid”,

Mr. Matsui named: 1. Area of geometrical figures, 2. The concept and properties of geometrical figures, 3. Formula to generalize the calculation of the area. These ideas show how Matsui understands how the knowledge to be taught is derived from the scholarly knowledge. He considers that “determination of area of parallelogram and trapezoid” originates from the domain “Quantity and Units” (according to the big idea 1), “Euclidean Geometry” (according to the big idea 2) and “Algebra” (according to the big idea 3). In order to illustrate his perception of these ideas, the outcomes of the interview, is presented below.

Mr. Matsui’s intention for students learning about this idea is that *Area of parallelogram, triangle, trapezoid and rhombus* can be determined in various ways by using the previously learned knowledge – the area of a square and rectangle by dividing those shapes into standard areas. Matsui mentions also that his students should be able to apply some mathematical terms such as *tosekihenkei* (*same area transformation: transformation of the shape without changing the quantity of the area*), *baisekihenkei* (*double area transformation: transforming the shape of with a duplication of the area*). These terms describe the various ways for determination and helps in understanding the methods. Mr. Matsui intends that the students should learn the process of finding out the formulas for area of various geometrical figures, rather than memorising those formulas.

As a starting point for the planning of teaching, Mr. Matsui takes up the pupils’ previous knowledge. It is an important component in the teacher PCK; the pupils’ previous knowledge has a strong impact when choosing techniques. The sharing of terms like *tosekihenkei* and *baisekihenkei*, (for which we could not find exactly corresponding terms in English), indicates that the technology is institutionalised among teacher educators in Japan. Emphasising the learning process of finding out formulas, shows that Mr. Matsui acknowledges the importance of developing conceptual knowledge, rather than procedural knowledge. He also emphasises the necessity for his students to understand the meaning of the algebraic generalisation.

Matsui names some concepts which the students do not yet need to know within this area: 1. additivity, 2. *bunriryō* (discrete quantity, where the range of possible values are not continuous), 3. *gaienryō* (extension quantity with additivity, e.g., length, time and area), 4.

naihoryō (inclusion quantity, a quantity that does not have additivity, e.g., temperature, velocity and density), 5. other remarks as Cavalieri’s principle.

The mathematical concepts named by Mr. Matsui above, shows his awareness of the technology within the praxeology MO_1 . He links these concepts to the domain “Quantity and Units”. This shows that MO_1 is strongly influenced by the knowledge to be taught, which are denoted by the curriculum.

Concerning teaching procedures listed in the CoRe template, Mr. Matsui names several detailed methods. As preparation, he proposes to use graph papers (grid of 1cm), an article written by a teacher in service and to refer to corresponding pages of in digital textbooks of the subject matter.

He states that the teacher educator should stress the importance of the children’s perspective during the lecture, so that when the students perform their own lessons in the future, they are able to confirm previously learned items and acknowledge various ways of solutions (including wrong answers) during their lessons. According to Mr. Matsui, the teacher students know intimately the flow of working with tasks: reason individually → discuss with neighbors → present the solutions in class → respond to comments from the lecturer.

Mr. Matsui uses different didactical methods in order to transform SMK to PCK, making the subject understandable to the teacher students. For instance, without using graph paper, student A would never come up with the idea for her solution to determine the area of parallelogram. The didactical contract (in this case, both the teacher and the students are aware of the basic flow of working with tasks) is required in order to apply the structured problem solving approach in the classroom. This holds both for the lessons instructing teacher students and classroom lessons for pupils in elementary school.

CONCLUSION

Our aim was to analyse a lesson titled “Quantity and Measurement”. Our impression is that the act of didactical transposition from the scholarly knowledge to the knowledge to be taught is insightfully done by the Japanese noosphere. For example, the area and volume determination is located within the topic of

“Quantity and Measurement” and not in “Geometry”, which is the case in many other countries (e.g., Sweden). Compared to the Swedish curriculum, the Japanese curriculum provides a relatively detailed fundament for the large mathematical (and also didactical) organisations that are also replicated in the textbooks. As a consequence, the transposition from the knowledge to be taught to the taught knowledge becomes more explicit and more uniform. It also makes the progression of the content visible. For instance, the “four phases of comparison” in different grades is clearly institutionalized within the discourse of the teachers in Japan. Furthermore, the curriculum sets the scale of the mathematical praxeologies in the textbooks, which in turn influences the complexity of the mathematical organisations that are discussed in the teacher education classes. The scale is also enhanced by the use of the structured problem solving when reviewing the mathematical organisations in these classes, since it gives a natural setting to discuss and enrich the associated technologies. It is also a way to consolidate the didactical organisations implied by the problem solving approach. By combining an analysis of the interview with the analysis of the didactical transposition and ATD, we have illuminated the teacher educator’s perception of the PCK and the SMK. This case study indicates that prospective teachers in Japan are given good opportunities to develop their insight of the PCK during their teacher education in mathematics.

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