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Developing mathematics teachers' pedagogical content knowledge through iterative cycles of lesson study

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This research presents features of knowledge of content and students (KCS) and knowledge of content and teaching (KCT) as empirical evidence of mathematics teachers' pedagogical content knowledge (PCK) utilised and enhanced through their participation in iterative cycles of lesson study. Over the course of one academic year, twelve teachers in two secondary schools engaged in this research as a double case study of teacher learning within a lesson study community. Qualitative data was generated through audio recordings of teacher meetings and through multiple teacher interviews. Dialogue within the lesson study communities was mapped to a framework of PCK as proposed by Ball, Thames and Phelps (2008). Results of this study find empirical evidence of the features of KCS and KCT in teachers' planning and reflection conversations and demonstrate teacher learning over iterative cycles of lesson study.

Keywords: Pedagogical content knowledge, lesson study, reflection, teacher knowledge.

INTRODUCTION

Lesson study is growing in popularity as a form of professional development for Mathematics teachers (Dudley, 2013). Although much research has shown that teacher knowledge is developed through participating in lesson study (Fernandez, Canon, & Chokshi, 2003; Lewis, Perry, & Murata, 2006; Murata, Bofferding, Pothen, Taylor, & Wischnia, 2012) this learning has not yet been explicitly mapped to a framework of knowledge for teaching. Furthermore, lesson study research has mainly focused on primary mathematics teachers and on single cycles of lesson study (e.g., Corcoran, 2011; Lewis, Perry, & Hurd, 2009; Murata et al., 2012).

In this research the development of secondary mathematics teachers' pedagogical content knowledge (PCK) is investigated through their participation in iterative cycles of lesson study. Twelve mathematics teachers in two schools were introduced to this model of professional development and participated in multiple cycles over the course of one academic year. As teachers' participation in lesson study continued, they began to incorporate and develop more elements of PCK in their planning and reflection meetings around research lessons.

Utilising Ball and colleagues' (2008) framework of PCK, qualitative data generated through teachers' conversations and interviews were analysed in terms of knowledge of content and students (KCS) and knowledge of content and teaching (KCT). Features of KCS and KCT emergent from the data are presented here as empirical evidence of PCK in teachers' planning and reflection conversations around research lessons and also as evidence of teacher learning in lesson study.

LESSON STUDY

Lesson study is a systematic inquiry into teaching and learning where teachers collaboratively plan, examine, conduct, observe, and reflect on research lessons (Fernandez et al., 2003; Lewis et al., 2006; Lewis et al., 2009; Murata et al., 2012). The aim of lesson study is not to construct a "perfect" mathematics lesson, nor is it to study lessons in detail, but rather it aims to engage teachers in dialogue around their pedagogical practices. Lesson study also incorporates many of the features of teacher community advocated as a form of sustainable professional development (Grossman, Wineburg, & Woolworth, 2001).

Figure 1: Lesson study cycle – adapted from (Lewis et al., 2006)

A modelled description of the lesson study cycle can be seen in Figure 1 (adapted from Lewis et al., 2006):

1. Study curriculum and formulate goals
2. Plan a research lesson
3. Conduct or observe research lesson
4. Reflect on research lesson and planning process

4a. Option to revise and re-teach the research lesson.

Teacher knowledge is enhanced through participation in lesson study (Lewis et al., 2006; Murata et al., 2012) and this research aimed to consider such learning relative to a particular framework of teacher knowledge. In this paper, detailed analysis focused on fine-graining the features of KCS and KCT evident within teachers' planning and reflection conversations in lesson study as a lesson study community.

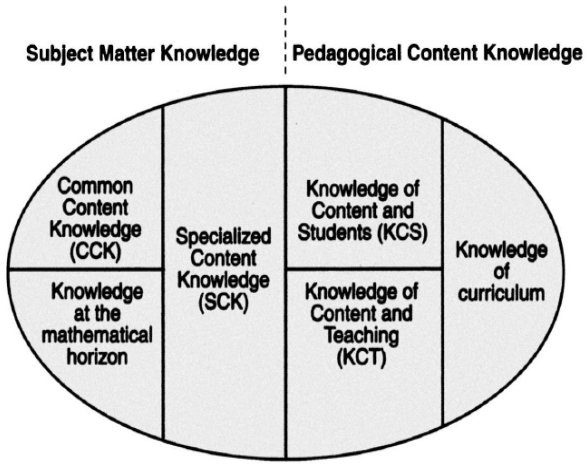

Figure 2: Mathematical Knowledge for Teaching (Ball et al., 2008)

PEDAGOGICAL CONTENT KNOWLEDGE: KCS AND KCT

In attempting to define teacher learning in this research, teacher knowledge was mapped to a particular framework of PCK proposed by Ball and colleagues (2008) (Figure 2).

KCS and KCT are important elements of PCK which combine teachers' knowledge of mathematics with their knowledge of students and of mathematical didactics respectively. KCS and KCT incorporate elements of knowledge specific to mathematics teachers from developing an awareness of students' mathematical thinking (Carpenter, Fennema, Peterson, Chiang, & Loef, 1989; Simon, 1995), to more effectively sequencing learning trajectories (Ball et al., 2008; Simon, 1995). Features of KCS and KCT utilised and developed in teachers' planning and reflection conversations in lesson study were identified as part of this research.

DATA GENERATION

This research took place in two urban secondary schools, Doone and Crannog (all pseudonyms), over the course of the 2012/2013 school year. Teachers were invited to participate in the research with 5 teach-

ers in Doone and 7 teachers in Crannog (varying in teaching experience from 1 to 33 years) agreeing to take part.

Each lesson study community was taken as a case study within which the development of teachers' conversations around the teaching and learning of mathematics could be analysed. Data was generated through audio recordings of each of the lesson study meetings in both schools (3 cycles in Crannog and 4 cycles in Doone) and through individual teacher interviews held at three stages during the research.

Teachers had autonomy over the content they taught, the class group involved, their overall lesson study goal, the construction of their lesson plans, and how they reflected on their students' learning. As participant-observer the researcher was present in each of the meetings and research lessons as an additional member of each lesson study community and participated as lesson study facilitator in the first cycles in both schools. In each of their subsequent lesson study cycles teachers rotated roles of conducting teacher and of facilitator.

In total, 38 hours and 17 minutes of teacher discussions around planning and reflection were recorded (over 18 hours in Crannog and over 20 hours in Doone). Teacher interviews with all participating teachers served as an additional data source in providing teachers with opportunity to self-report on their own learning and on any changes to their classroom practices that may have changed as a result of their participation in lesson study.

DATA ANALYSIS

The data was transcribed and analysed over four phases as a chronological evolution of teachers' planning and reflection conversations over iterative cycles of lesson study. A framework of analysis of KCS and KCT derived from the literature (Ball et al., 2008; Hill et al., 2008) and incorporating codes emergent from the data was utilised in the analysis. Analysis involved reading all of the transcript text, identifying if the text qualified as a legitimate code, and deciding if the text was relevant to the codes within the framework of analysis.

A unit of analysis was defined as any episode of conversation which a) was relevant to the lesson study cycle and b) was relevant to constructing content of a lesson from either the perspective of the student or from a pedagogical perspective. This parsing approach of conversation excerpts, also utilised by Cjakler and colleagues (2013), aimed to encapsulate elements of teachers' conversation where teachers introduced elements of KCS and KCT within their planning and reflection phases. For example, the following conversation excerpt identifies an episode of learning for Lisa who, prior to participating in lesson study, had not recognised how her students might interpret variables within Pythagoras' theorem. This discussion occurred as part of the final cycle in Doone where teachers planned a series of lessons introducing students to Pythagoras' theorem.

Lisa	The thing about learning for the students is that they can learn the theorem but then it is confusion when the diagrams are labelled in any given way... We think it's saying $a^2 + b^2 = c^2$ but it's meaningless to them when you give them a thing and 'a' is the hypotenuse and then you go, $a^2 + b^2 = c^2$ ... They don't actually understand.
Owen	So that's rote learning.
Kate	Yeah, concept rather than formula...
Lisa	*We* know it. We know that this is the formula but we don't look at it from the kids [perspective]. And it's only that you talking about it today – if they label the hypotenuse 'a' – I hadn't actually realised that that is what's causing the problem.

This conversation higlights Lisa's realisation of seeing the mathematical content through the eyes of a student (Fernandez et al., 2003), where she made sense of students' common conceptions – developing her KCS as part of this lesson study community's planning dialogue.

The final two phases of analysis led to a further determining of the categories of KCS and KCT relevant to these elements of lesson study such as: noticing students' mathematical strategies, developing contextualised questions, and reflecting on student talk. These features, linked to existing literature on teacher learning, formed the basis of codes for a final phase of analysis and are presented below as indications of teacher learning through participation in iterative cycles of lesson study.

FINDINGS

Features of KCS and KCT are presented as empirical evidence of PCK in teachers' planning and reflection conversations in lesson study and also as evidencing teacher learning, since the frequency of these features increased as teachers' participation in iterative cycles of lesson study continued. As part of both teacher communities' initial engagement with lesson study, not all of these features were present in the initial cycles of lesson study but began to be incorporated as teachers began to observe research lessons, focus on students' mathematical thinking, and plan subsequent lessons.

Teacher learning through the development of these features of KCS and KCT in iterative cycles of lesson study may be modelled as proposed in Figure 3. As an example of developing KCS, following the observation of students within research lessons (noticing and interpreting students' mathematical responses) teachers became more cognisant of anticipating how their students would mathematically engage in planning subsequent lessons. From an increased focus on anticipating students' thinking, teachers became more aware of the value of engaging students in their own learning and began designing relevant, contextualised questions (KCT) which, in turn, focused teachers on noticing and reflecting on students' mathematical strategies in attempting these activities (KCS).

This model is included as suggested theoretical frame of teacher learning in iterative cycles of lesson study

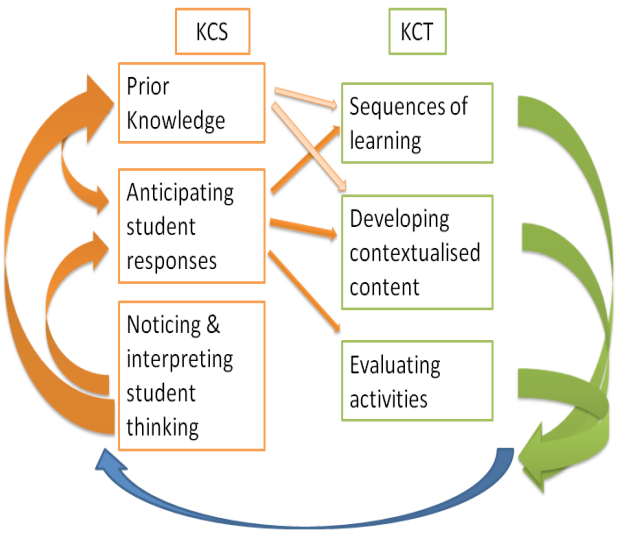

Figure 3: Features of KCS and KCT developed in iterative cycles of lesson study

and further research is required in refining and developing this model.

Findings: Features of KCS

From the analysis of the data three features of KCS were identified as being utilised and enhanced through teachers' participation in iterative cycles of lesson study:

- Identifying students' prior knowledge (Ball et al., 2008)
- Anticipating students' mathematical responses (Ball et al., 2008; Hill et al., 2008)
- Noticing students' mathematical thinking (Carpenter et al., 1989; Jacobs et al., 2010; van Es & Sherin, 2008)

A number of narrative samples of each of these features will now be explored.

Identifying students' prior knowledge

While at the beginning of their engagement in lesson study teachers in Doone did not incorporate students' prior knowledge in their planning, this became more and more important to them as their planning of research lessons continued. From their observation of research lessons, teachers reflected on the need to correctly identify students' prior knowledge in being able to then anticipate students' responses (KCS) and plan a relevant sequence of learning (KCT). In their final research lesson, teachers identified and incorporated the mathematical content students had already met and used this information to build on students' mathematical thinking over a series of lessons. This development of KCS in turn benefitted teachers' KCT in outlining sequences of learning over a number of lessons.

Anticipating students' mathematical responses

From observing particular students within the first research lesson, teachers in both schools began to anticipate how students might engage with and respond to mathematical tasks within subsequent lessons. Within their planning, teachers began to anticipate and identify various strategies which students might employ for particular mathematical activities and also began to articulate how students might think about particular topics. In Crannog's first research lesson, teachers began to anticipate how students

might respond to a planned activity exploring quadratic patterns:

Stephen	Because some would look as if it is 1, 4, 9. Some will look at it as being squared. Some will look at it as being –
Fiona	Add 3 and 5 –
Stephen	And 7, and that is...if you give them a pattern, if you give them a list of numbers of 1 to 9, that is what they do. They will actually see how much it is going up whereas some don't relate it to being anything squared.

This anticipation of students' mathematical responses began to also impact on teachers' practices outside of lesson study as reported by Eileen (a newly qualified teacher) in her mid-point interview:

Eileen	Yeah, I probably would ask myself a bit more how would they react to this or what questions will they have. Pre-empt their questions or pre-empt their confusion. Yeah, I would think about that a little bit more.

Noticing students' mathematical thinking

Through the detailed planning and anticipation of students' mathematical responses within teacher meetings, teachers began to focus on and notice more of students' mathematical thinking during their observation of research lessons. This noticing of students' mathematical thinking was an important element of developing teachers' perspectives on their own pedagogy (Carpenter et al., 1989; Corcoran, 2011; Jacobs et al., 2010; van Es & Sherin, 2008) and as the cycles continued teachers began to explicitly reflect on elements of student strategies and student talk in their post-lesson discussions. Teachers self-reported that this noticing and interpreting of students' mathematical thinking also impacted on their classroom practices outside of lesson study, such as extending the 'wait time' for students to answer questions in.

Findings: Features of KCT

From the analysis of the data three features of KCT were identified as being utilised and enhanced through teachers' participation in iterative cycles of lesson study with their colleagues:

- Sequencing learning trajectories (Ball et al., 2008)
- Designing contextualised questions (Schoenfeld, 2011)
- Evaluating mathematical activities (Ball et al., 2008)

A number of narrative samples of each of these features will now be explored.

Sequencing learning trajectories

It was a surprising result that in both schools, as teachers continued their participation in lesson study they began to plan series of lessons along a learning trajectory within which the research lesson was incorporated. In Crannog's second cycle teachers realised that planning a number of lessons would be more beneficial to students in revising and developing important mathematical concepts.

Fiona	I suppose what we have to do in a pre-runner class, we have to go back with them over the concept of factors: "What are factors?" and then they need to look. Because we kind of gloss over that a bit when we go into factorising usually.
Stephen	That we don't make up two numbers in algebra.
Fiona	Yeah, so go back into factors.
Gerald	Yes and I think the discussion of factors should start with prime numbers because they have only got two factors to talk about then.

In their final research lesson teachers planned a series of 6 lessons which incorporated students' prior knowledge and guided students towards a necessity for differentiation in Calculus.

Similarly in Doone, their final research lesson was planned as part of a series of lessons which teachers felt was far more valuable to both the lesson study community and to students in building their mathematical understanding.

Kate	This is the first time we've actually kind of planned a little scheme.
Nora	For the whole thing. Take you, follow it – follow it through. Because I think you know exactly where you stand or where they [the students] should stand.

Designing contextualised questions

As teachers continued to participate in lesson study it became more and more important to them to design mathematical activities that were relevant and context-based for their students. In Doone's first cycle students' activities were of a traditional textbook format (O'Sullivan, Breen, & O'Shea, 2013) which were not particularly relevant to their group of secondary students. In their subsequent cycles teachers developed activities that were both context and content based but were also of interest to these students, such as a rugby based problem designed for a particular class of 15 year old male students.

Evaluating mathematical activities

As a further feature of KCT, teachers also began to critically analyse and evaluate mathematical activities during planning. This evaluation of activities during planning impacted on how they taught or introduced such activities during the research lesson as exemplified in the following conversation excerpt where teachers modified a question in order to necessitate students multiplying two fractions together:

Lisa	The Ireland rugby squad: 1/5 of these have eye problems. Of this 1/5, ½ wore contact lenses. What fraction of the players wore contact lenses?
Kate	But – they're going to get 30 players.
Lisa	30 multiplied by 1/5 is 6.
Kate	Well, they're going to divide by 5. So unless we said "a squad" instead of saying 1/5 of the "players". Don't give them a number of players because they'll divide by 5 and get 6.
Lisa	I think that's what we're doing wrong. We just want a fraction –
Owen	So it's one whole squad.
Lisa	So it's "a squad". Brilliant!

This evaluation of mathematical activities also encouraged teachers in developing and designing their own activities instead of their traditional reliance on textbook questions (O'Sullivan et al., 2013).

CONCLUSION

Over a number of lesson study cycles, teachers developed in their perspectives of and approaches to teaching and learning mathematics through planning research lessons, observing students' mathematical responses during those lessons, and reflecting on students' interactions and responses. This research maps the learning of these teachers in their lesson study community to a framework of PCK suggested by Ball and colleagues (2008). Furthermore, the research provides empirical evidence of KCS and KCT in the context of teachers' planning and reflection conversations of research lessons in lesson study.

In this paper, three features of both KCS and KCT were identified within the data as part of teachers' planning and reflection conversations in lesson study. While these features were not all present in the data in initial lesson study cycles, they began to be incorporated as teachers' participation in cycles continued where their PCK was developed through structured conversations with their colleagues. While some of these emergent features of PCK were expected from literature on teacher learning and lesson study (such as highlighting students' prior knowledge), others (such as designing contextualised questions and interpreting students' mathematical responses through reflection) were added to this proposed framework of PCK as part of teacher learning in lesson study.

The presentation of these features of KCS and KCT as empirical evidence of PCK in planning and reflection phases of lesson study represents a contribution to the literature in identifying teacher learning through participation in iterative cycles of lesson study.

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