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# Theories to be combined and contrasted: Does the context make a difference? Early intervention programmes as case

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*The constructivist paradigm is the primary underpinning of the Mathematics Recovery Programme (MRP) in Australia, the UK, the USA and Canada. The critical mathematics education paradigm is the primary underpinning of the Early Mathematics Intervention Programme for Marginal Groups in Denmark (TMTM for Tidlig Matematikindsats Til Marginalgrupper). The paper tells the story of how networking strategies have informed the design of TMTM: one strategy is combining elements from the two paradigms; another strategy is contrasting elements from the paradigms. The two constructs 'the six stages-construct' and 'the math holes-construct' have been decisive for the networking. Possible influences from the Danish educational contexts on the networking processes are put forward for further discussion.*

**Keywords:** Networking theories, early intervention in mathematics, math holes, constructivism, critical mathematics education.

## CULTURAL CONTEXT AND OUR CULTURAL CONTEXT

A main issue at CERME9 was cultural contexts in European research in mathematics education. What is meant by cultural contexts and what are the mechanisms by which cultural contexts influence research and practice in mathematics education were displayed in the plenary by Barbara Jaworski, Mariolina Bartolini Bussi, Edyta Nowinska and Susanne Prediger.

This paper primarily focuses at issues in primary school mathematics in a certain cultural context: the Danish educational culture. The development of

school mathematics in Denmark is in no way isolated from Nordic, European or global tendencies and policies. For instance the 2004 OECD review of the Danish primary and lower secondary school emphasized the need to support failing pupils in mathematics in the first school years (Mortimore, David-Evans, Laukkanen, & Valijarvi, 2004). It is evident that the OECD review has influenced Danish policy, and so have Danish teachers' and researchers' initiatives to improve the school support for pupils at risk of falling behind in their mathematics learning.

It seems natural to Danish mathematics teachers and educational researchers to get inspirations from abroad, also when it comes to interventions for pupils at risk. Generally speaking, inspirations from theory and practice in school mathematics in especially the Netherlands have been strong since 1970s, and Realistic Mathematics Education from the Freudenthal Institute is still influential.

Still, some characteristic aspects of the Danish school mathematics seem evident and are visible in national goals and aims despite influences from international politics and tendencies. This paper will look into the tension between national and international trends for educational research and practice, by looking at the development processes behind the Danish early mathematics intervention programme for marginal groups (TMTM for Tidlig Matematikindsats Til Marginalgrupper) as a case. Especially the theoretical networking in the development processes is to be explored. We are ourselves central persons in these development processes, which we aim to document and critically assess.

We generally argue that how theories are being interpreted and networked locally is influenced by local contexts as for instance Danish school mathematics cultures. It is shown that culture does matter for values and practices in mathematics education. As an example, teachers in London and in Beijing hold different views on mathematical learning and teaching. Teachers in London see syllabus and textbooks as less important in determining the content taught than interest and meaningfulness, which is again opposite to teachers in Beijing (Leung, 2006). Similarly, teachers in London see students' ability as more important for their learning than effort, which is the opposite of teachers in Beijing. We claim that it is by now a well-known fact that culture matters for values and practices in mathematics education, but how culture matters in networking theories, is not yet well addressed.

### **CONSTRUCTIVIST THEORY UNDERPINNING MRP**

Among the most common implemented programmes internationally for early intervention in mathematics the Mathematics Recovery Programme (MRP) is to our knowledge the most described and researched one. It is broadly implemented in Australia, the UK, Ireland and the USA (Wright, Martland, & Stafford, 2000) (Wright, Ellemor-Collins, & Tabor, 2011). We have studied programme materials and background papers and we have made personal contacts to leading figures like Robert J. Wright and like Noreen O'Loughlin, University of Limerick.

MRP is an intensive one-to-one tutoring offered to first graders falling behind and at risk of mathematics difficulties. Since 2006 the material *Teaching number – Advancing children's skills and strategies* has been suggested for whole-class teaching, too. Tutor training is a mandatory part of the programme. A learning framework and an instructional framework are described in detail. The tutoring is meant to be diagnostic, so that instruction can be adapted to pupils' reactions.

The mathematical content in MRP focuses on numbers and arithmetic. The MRPM covers knowing and understanding numbers, names and symbols, basic strategies in the four operations (Wright, Martland, Stafford, & Stanger, 2006) and supplemented with

basic understanding of part-whole concept (Wright et al., 2011).

Important for this paper are the theories behind the MRP, which is drawn primarily from Leslie P. Steffe and colleagues. Steffe was engaged as early as 1976 in establishing the research program, Interdisciplinary Research on Number (IRON). As documented in (Steffe, van Glasersfeld, Richards, & Cobb, 1983), (Steffe, Cobb, & van Glasersfeld, 1988) and (Steffe, 1992), a tremendous empirically based work has been going on to explore how children reason and meaningfully grasp processes and concepts related to numbers and arithmetic. The research has led to insights into many diverse aspects of children's mathematical knowledge and how this knowledge is developed.

The constructivist underpinning manifests itself by maintaining that knowing is not passively importing other peoples' knowledge as if you were receiving a birthday gift. It is maintained and underlined that knowing is an active endeavour for the person, who is acquiring/constructing knowledge through genuine problem solving.

### **THE SIX STAGES CONSTRUCTS**

The idea of development through stages runs as a general structuring idea through the work of Steffe et al. Like the construct of the five stages of geometrical reasoning from van Hiele (1985/1959), it seems that an underlying idea of stage has been transformed into several constructs of several and often six stages by Steffe et al. and by their followers.

While the descriptions of the counting stages used in solving addition and subtraction problems is grounded through the work of Steffe et al., the stage constructs are further developed by Wright (1991, 1994) and by Wright and colleagues (2000, 2006, 2011). The stages of children's development of different areas of mathematical knowledge are put forward in the learning framework and in the instruction framework of MRP. The organisation of how tutors' come to observe pupils' development and how tutors adapt instruction to the pupils' present knowledge is facilitated by the lenses of stages.

In Wright and colleagues (2006), the idea of six early arithmetical strategies is used as the main organising principle. The six stages go from stage zero to per-

ceptual, to figurative, to initial number sequence, to implicitly nested number sequence and end with explicitly nested number sequence. Each stage is described. At the last one, the explicitly nested number sequence, the child uses a range of non-count-by-ones strategies. The child also uses a known result, adding to ten and commutativity, and the child knows that subtraction is the inverse of addition (p. 9).

The Wright and colleagues' book from 2006 presents a network of stages in each of the mathematical area covered. Also for *forward number word sequences and number word after* six stages are described. The same for *backward number word sequences and number word before*, while *number identification* has five stages, and *base-ten arithmetical strategies* has three stages.

### CRITICAL MATHEMATICS EDUCATION

The Nordic part of critical mathematics education has primarily been dominated by the Danish scholars Mogens Niss (1984) and Ole Skovsmose (1994, 2001) and the Norwegian scholar Stieg Mellin-Olsen (1987, 1991). Niss underlined that it is an important but difficult task for research and practice in mathematics education to relate to authentic applications of mathematics from a critical stance of view, and to let mathematics education address fundamental social functions of mathematics and school mathematics. For school mathematics this implies mathematical modelling and project work. Skovsmose added the philosophical view of learning mathematics as investigation of mathematical landscapes. For school mathematics this implies a critique towards exercise paradigm and a focus on communication between teachers and pupils. Skovsmose also invented the concept of pupils' foreground as just as important as pupils' background. Mellin-Olsen (1991) added the project work about authentic applications should be for all. It should be for less able and less motivated pupils as well as for all other pupils. He also added the metaphor of travelling as a tool to understand teachers' thinking about instruction. He recognised that teachers talked about their mathematics instruction as being the drivers in the busses full of pupils.

### THE MATH HOLES-CONSTRUCT

Based on Nordic critical mathematics education we have researched special needs education in Denmark.

We found that the ideas from Niss, Skovsmose and Mellin-Olsen of mathematics instruction and learning were all promising. For instance we found the idea of mathematics instruction and learning as a common teacher-pupil investigation of mathematical landscapes promising. Since 2003 Böttger, Kvist-Andersen, Lindenskov and Weng (2004), we have therefore been involved in demonstrating mathematics learning as a journey in landscapes, which evolve with hills and holes as you travel. Many routes can be appropriate for the teachers and students involved, depending of their backgrounds and foregrounds and depending on available authentic (in Freudenthal's sense *realistic*) materials and activities for problem solving and for modelling.

Closely connected to the general idea of investigating or constructing mathematics landscapes, we needed to add a specific idea of pupils at risk of becoming low-level performers. Therefore we invented the math holes-construct.

The math holes-construct metaphorically describes pupils in difficulties as pupils, which for the moment are stuck and are not progressing further into the mathematics landscape. It is as if the pupil has fallen into a trap or – as we finally decided to name it – it is as if the pupil has fallen into a hole.

Sure, it is not our intention with the math holes-construct to see learning mathematics in a negative biased light. Neither is it our intention to avoid mathematical elements, which may cause obstacles, as for instance learning the rules of the positional decimal number system. Our intention is to develop pedagogical constructs to help teachers recognise and support every child's mathematical learning. Therefore it is important to identify when a child is stuck and to provide intervention models, materials and teacher training for how schools and teachers can help getting the pupil moving again.

Within the math holes-construct pupils in difficulties are not pupils lacking behind or pupils with special neurological characteristics as implied by some definitions of dyscalculia. Within the math holes-construct pupils in difficulties are pupils who stopped progressing learning. Gervasoni and Lindenskov (2011) describe the construct with the following:

When mathematics is seen as a landscape it means that whenever students stop learning and feel stuck it is as if they ‘fall into a hole’. There are several ways for a teacher to cope with a student’s ‘fall’. First, a teacher can invite the student to move to another type of landscape, maybe far away from the hole in which the student was stuck; this means that even when students fail to thrive in one area of mathematics there are still many other mathematics landscapes to experience and learn. Second, teachers can help students ‘fill up’ the hole from beneath with mathematical building stones; or third, teachers can ‘lay out boards over the hole’ in order to let the student experience new and smart mathematical approaches. (p. 317)

It is, in our view of the math holes-construct, crucial to draw attention to the individual pupil: Much still needs to be done, following Ginsburg’s (1997) more than 15-years-old call for teaching experiments focusing on pupils with learning difficulties, as children today are exposed to physical and social environments that are rich in mathematical opportunities. In Denmark as in many other countries, children today are exposed for instance to even very big numerals in computer and board games and in family activities. This questions the generality of the existing cognitive models of children’s reasoning and of the theories of instruction. For instance it questions the motivating effect as well as the learning effect of splitting instruction on numbers into first 1–20, and then 1–100, as it is done in MRP.

What children are exposed to rapidly develops, and no one can predict what will happen just a few years from now. This means that the generalisation of identified mathematics learning trajectories in research studies may not be generalizable to all children in the world. On the other hand, the learning trajectories, which are found in research, and upon which MRP have built their stages, do show valuable insight into children’s

learning, which is not visible for teachers even with profound training. We do advocate for teachers to obtain extended knowledge on researched learning trajectories.

### NETWORKING THEORIES TO INFORM DEVELOPMENT OF TMTM

Our design processes towards the Danish model for early intervention for marginal groups TMTM were highly informed by networking theories from constructivism and from critical mathematics education. Tutor material is published in a 191 pages book (Lindenskov & Weng, 2013), but lots of work were done before.

Certainly some authors see the two paradigms as contradictory, and Mellin-Olsen is a prominent example. He disliked the focus on the cognitive development of the individual pupil in the constructivist paradigm.

But as researchers have showed for the last decade networking theories are much more than rejecting or approving specific theories. Bikner-Ahsbahs and Prediger (2010, p. 492) suggested the following illustration of a landscape of strategies for connecting theoretical approaches (Figure 1).

We chose to let the development processes for TMTM be informed by *contrasting* some of the above mentioned theories and constructs and by *combining* others.

The *combining* part includes the deep insight into individual pupils’ problems and development coming from Steffe and colleagues’ constructivistic research *combined* with the math holes-construct.

Also in the Danish programme tutor training is a mandatory part of the programme, and also the tutoring is meant to be diagnostic, so that instruction can be adapted to pupils’ reactions.

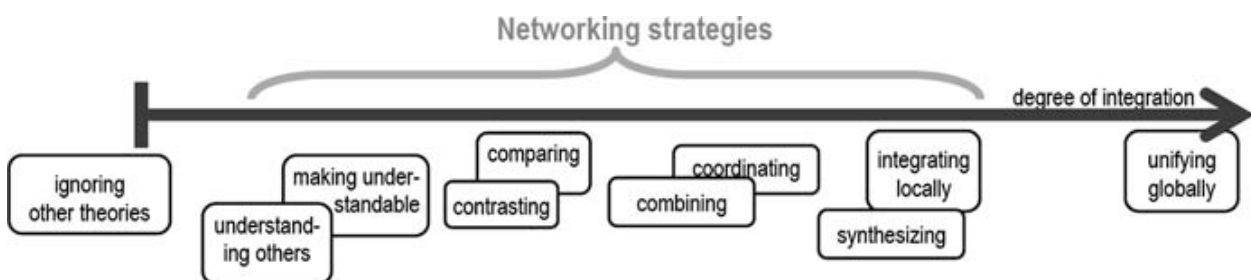


Figure 1

The *contrasting* part includes that Steffe and colleagues' focus solely on numbers and arithmetic *contradicts* the broader goals and aims in critical mathematics instruction. It is not enough for a Danish model to focus on numbers and arithmetic. We find them necessary, but not sufficient.

The *contrasting* part also includes that the stage constructs *contrast* the idea of learning mathematics as a journey where parts of the mathematical landscapes are being constructed as pupils and teachers travel together. The paths are chosen by teachers and pupils collaboratively, and the teachers take into account children's background and foreground in order to motivate the children. The stage construct does not allow that the teachers also take the pupils' foreground into account.

### **CONTEXTUAL INFLUENCE ON THE THEORY NETWORKING**

School cultures in Denmark are influenced by the German/continental tradition in educational philosophy. Danish language has, like German language, two main concepts for 'Education': one is 'Dannelse' (German: Bildung), another is 'Uddannelse' (German: 'Erziehung').

At the level of school mathematics this tradition emerges in the common aim for primary and lower secondary school mathematics by pointing at everyday life, citizen life, creativity, problem solving, and democratic responsibility and impact:

The aim is that students develop mathematical competences and acquire skills and knowledge in order to appropriately engage in math-related situations in their current and future everyday, leisure, education, work and citizen life.

Subsection 2. Students' learning should be based upon that they independently and through dialogue and cooperation with others can experience, that mathematics requires and promotes creative activity, and that mathematics provides tools for problem solving, reasoning and communication.

Subsection 3. Mathematics as a subject should help the students experience and recognize the role of mathematics in a historical, cultural and social context, and that students can reflect and evaluate application of mathematics in order to take

responsibility for and have an impact in a democratic community. (Undervisningsministeriet, 2014, our translation)

On this background it was evident, that the constructivist paradigm had to be combined with the critical mathematics education. The constructivist paradigm gave a too narrow picture of what should be learnt.

According to the choice of subject matter, the school mathematics tradition emerges in broad scope throughout all school grades and for all pupils. Danish pupils are not streamed before Grade 10. This means that skills, conceptual understanding and authentic applications are taught to all pupils. Mathematical competences as well as mathematical conceptual fields like numbers and algebra, geometry and measurement, and statistics are included for all pupils from the very start of primary school.

Besides it is our impression that Danish teachers have a relatively high self-confidence and a relatively strong wish to influence. From the European comparative perspective it seems as if Danish teachers more than teachers in England prioritise the pupils' personal development and see the pupils' mathematical development as a means for personal development (Kelly, Pratt, Dorf, & Hohmann, 2013).

This led us to include a group of teachers, in a decisive way, in design cycles in the development towards a Danish model (Lindenskov & Weng, 2014), and it led us to make the material much open for teachers' adaptation. This also led us to *confront* the stage construct with the math holes-construct. With a stage construct less is up to the tutor to decide, and with a math holes-construct pupils' emotional and motivational background and foreground are easier to take into account.

### **CONCLUSION**

Thus, the Danish developmental and research projects on early intervention are based on an original construct, the math holes-construct. But the whole development processes are very much informed by *combining* and *contrasting* theories.

With Skott (2015), in our view networking theories is very much depending on what we mean by theory, constructs and conceptual framework. Skott points to

decisive elements as a) preliminary understanding of concepts involved, b) theoretical stance on interpretation of these concepts, and c) the overall rationale for engaging in the field of inquiry.

The math holes-construct implies differences compared to MRP concerning all three elements, although similarities between MRP and TMTM exist.

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