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The epistemological dimension revisited

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Epistemology and networking was discussed in the last CERME working group on theory. This paper aims to continue the discussion. I reflect on epistemological analysis and the cultural dimension of knowing and present examples which demonstrate how the changes in the cultural context influence the epistemological analysis. Then, I reconsider the epistemological dimension and the networking of theories. In some cases, the epistemological dimension permits the networking. In other cases, we notice how by means of networking, strong epistemological concerns in one theory might be integrated in another theory in a way that reinforces the underlying assumptions of this other theory. I end the paper with an example of networking that demonstrates how the social dimension might influence the epistemological analysis.

Keywords: Cultural dimension, epistemological analysis, networking theories, social dimension.

EPISTEMOLOGY AND NETWORKING THEORIES IN THE PREVIOUS CERME WORKING GROUPS ON THEORIES

The present paper aims to continue the work done at the previous CERMEs in relation to the epistemological dimension in theories. At CERME8, the focus on networking and epistemology was stronger than in the previous working groups on theory. For example, the role of epistemology in the networking of theories was an explicit focus in the paper by Ruiz-Munzón, Bosch, and Gascón (2013). The idea of a “reference epistemological model” (REM) was introduced for networking Chevallard’s Anthropological Theory of the Didactic (ATD) and Radford’s Theory of Knowledge Objectification (TKO). The authors analyzed how each approach addresses the nature of algebraic thinking. The point of view of the ATD was presented with its own REM about elementary algebra as well as the kind of questions addressed by this approach, in relation to the TKO.

In their paper, presented at CERME8, Godino and colleagues (2013) analyzed two approaches to research in mathematics education: “Design-based research” (DBR) and “Didactic engineering” (DE), in order to study their possible networking. DE (closely linked to Brousseau’s theory of didactical situations) focuses on epistemological questions; DBR does not adopt a specific theoretical framework, nor does it explicitly raise epistemological questions. In the working group (Kidron et al., 2013) interesting questions arose like the following one: “is the epistemological focus only a question of ‘cultural and intellectual context’ or is an epistemological reference necessary for each theoretical approach used in design based research in math education?”

Artigue (2002) wrote that the anthropological approach shares with the socio-cultural approaches the view that mathematical objects are not absolute objects, but are entities which arise from the practices of given institutions. These practices are described in terms of tasks in which the mathematical object is embedded, in terms of techniques used to solve these tasks and in terms of discourse which both explains and justifies the techniques. It is interesting to note that the nature of mathematical objects was a theme that appears at CERME4 in the context of the need to be aware of the underlying assumptions of each theory and that underlying assumptions also concern ontological or epistemological questions such as the nature of mathematical objects. This theme reappears in the next CERMEs especially at CERME7 while networking was needed in order to analyze the emergence and nature of mathematical objects. This was well demonstrated, for example, in the paper presented by Font and colleagues (2011). The authors asked “What is the nature of the mathematical objects?” They explored this question by the use of a synthesis between the onto-semiotic approach (OSA), APOS theory (with its four components, Action, Process, Object, and Schema) and the cognitive science of mathematics (CSM) as regards their use of the

concept of “mathematical object”. APOS theory and CSM highlight partial aspects of the complex process through which, according to OSA, mathematical objects emerge. OSA extends APOS theory by addressing the role of semiotic representations; it improves the genetic decomposition by incorporating ideas of semiotic complexity, networks of semiotic functions and semiotic conflicts; it offers a refined analysis due to the way in which it considers the nature of such objects and their emergence out of mathematical practices. Considering mathematical objects not as absolute objects, but as entities which arise from the practices of given institutions, leads us to analyze the role of both, the epistemological dimension and the socio cultural dimension, in theories.

EPISTEMOLOGICAL ANALYSIS AND SOCIO CULTURAL DIMENSION

The following question was asked by Luis Radford at the colloquium at Paris in honour of Artigue (2012): “How can epistemological analysis take into account the social and cultural dimension of knowing?” In the last decades the increasing influence of sociocultural approaches towards learning processes is well recognized. Therefore, the question is essentially *how* the social and cultural dimensions are taken into account in the epistemological analysis. In this section, I will consider this question in relation to the cultural dimension of knowing. I analyze the changes in the cultural context and their influences on the epistemological analysis. In the section about epistemological dimension and networking theories, I will reconsider Radford’s question in relation to the social dimension of knowing.

Changes in the cultural context and their influences on the epistemological analysis

In the last decades we face the changes of our cultural environment as well as the changes of the context in which our theory emerged. I will give an example from my own research on students’ conceptual understanding of central notions in calculus like the notion of limit in the definition of the derivative. In my previous research, using essentially theories that privilege epistemological and cognitive dimensions, I was aware of the cognitive difficulties relating to the understanding of the definition of the derivative as the “limit of the quotient $\Delta y/\Delta x$ as Δx approaches 0”. In my epistemological analysis, my first thinking was that these cognitive difficulties are inherent to the

epistemological nature of the mathematics domain. I realized that students viewed the limit concept as a potential infinite process and I understood that this was a possible source of difficulties. Moreover, previous researches (Tall, 1992) expressed students’ belief that any property common to all terms of a sequence also holds of the limit. I therefore realized that this natural way in which the limit concept is viewed might be an obstacle to the conceptual understanding of the limit notion in the definition of the derivative function $f'(x)$ as $\lim_{\Delta x \rightarrow 0} \Delta y/\Delta x$. In particular, the derivative might be viewed as a potentially infinite process of $\Delta y/\Delta x$ approaching $f'(x)$ for decreasing Δx . As a result of the belief that any property common to all terms of a sequence also holds of the limit, the limit might be viewed as an element of the potentially infinite process. In other words, $\lim_{\Delta x \rightarrow 0} \Delta y/\Delta x$ might be conceived as $\Delta y/\Delta x$ for a small Δx . I therefore looked for a counterexample that demonstrates that one cannot replace the limit “ $\lim_{\Delta x \rightarrow 0} \Delta y/\Delta x$ ” by $\Delta y/\Delta x$ for Δx very small. “Finding such a counterexample... was crucial to my research focus. Such a counterexample demonstrates that the passage to the limit leads to a new *entity* and that therefore omitting the limit will change significantly the nature of the concept. It demonstrates that the limit could not be viewed as an element of the potentially infinite process” (Kidron, 2008, p. 202). In Kidron (2008), I explain that such counterexample exists in the field of dynamical systems which is considered as a new field in mathematics. In the counterexample (the logistic equation), the analytical solution obtained by means of continuous calculus is totally different from the numerical solution obtained by means of discrete numerical methods. The essential point is that using the analytical solution, the students use the concept of the derivative as a limit $\lim_{\Delta x \rightarrow 0} \Delta y/\Delta x$ but, using the discrete approximation by means of the numerical method, the students omit the limit and use $\Delta y/\Delta x$ for small Δx . Students reactions are analyzed in (Kidron, 2008), in particular how students reach the conclusion that passing to limits may change the nature of a problem significantly. The essential point is that the changes in the cultural context permit the new settings for the learning experience. More precisely, the changes in the cultural context permit modern results in research Mathematics which influenced my own research in mathematics education by means of changes in the didactical designs. The didactical design described in (Kidron, 2008) was possible by means of the epistemic status of the new artifacts used

in the research study. The way the students interacted with the software demonstrates that the artifact used in this study should not be considered only as an aid for the students. It had a deep cognitive role while learners interacted with it. The artifact was conceived as co-extensive of thinking: the students act and think *with* and *through* the artifact as described by Radford (2008). In another study (Kidron & Dreyfus, 2010) we also notice this specific epistemic status of the artifact as co-extensive of thinking while the computer is considered as a dynamic partner. Kidron and Dreyfus consider the influence of a CAS (Computer Algebra System) context on a learner's process of constructing a justification for the bifurcations in a logistic dynamical process. The authors describe how instrumentation led to cognitive constructions and how the roles of the learner and the computer intertwined during the process of constructing the justification.

Another example describing how epistemological analysis takes into account the cultural dimension of knowing is described in Artigue (1995, p. 16) in which the author describes her mathematical research in differential equations and the way she notes the epistemological inadequacy of teaching in this area, for students in their first two years at university. By means of epistemological analysis, Artigue described how historically the differential equations field had developed in three settings: the algebraic, the numerical and the geometric settings. For many years, teaching was focused on the first setting due to epistemological and cognitive constraints. Reflecting on these constraints was a starting point towards building new teaching strategies which better respect the current fields' epistemology. By means of the epistemological analysis, Artigue could see the epistemological evolution of the field towards new approaches, the geometrical and numerical approaches. The essential point in this example is that the epistemological evolution is a consequence of the changes in the mathematical culture and the epistemological analysis highlights the crucial role of the cultural dimension.

THE EPISTEMOLOGICAL DIMENSION AND THE NETWORKING OF THEORIES

Epistemological sensitivity

A new view on the epistemological dimension is offered in Kidron and colleagues (2014) by means of the networking between three theories, TDS, the Theory of Didactic situations (Artigue, Haspekian,

& Corblin-Lenfant, 2014), ATD, the Anthropological Theory of the Didactic (Bosch & Gascón, 2014), and AiC, the theory of Abstraction in Context (Hershkowitz et al., 2001; Schwarz et al., 2009; Dreyfus & Kidron, 2014). The foci of the three theoretical approaches are different. In particular, AiC focuses on the learner and his or her cognitive development, while TDS and ATD focus on didactical systems. The three theoretical approaches are sensitive to issues of context but, due to these differences in focus, context is not theorized and treated in the same way. The authors expected some complexity in the effort of creating a dialogue between the three theories in relation to constructs such as context, milieu, and media-milieus dialectic. However, they observed how the dialogue between the three theories appears as a progressive enlargement of the focus, showing the complementarity of the approaches and the reciprocal enrichment. A new term was introduced in this research study: *epistemological sensitivity*.

The authors explain the meanings of the terms context (for AiC), milieu (for TDS) and media-milieus dialectic (for ATD), each of them being a cornerstone for the theory while all of them try to theorize specific contextual elements. The three theories share the aim to understand the epistemological nature of the episode described in the paper but in each of the three theories different questions were asked. Questions for analyses in AiC stressed the epistemic process itself, whereas researchers in TDS and ATD asked how this process is made possible. Nevertheless, these questions indicated that the researchers were able to build on the other analyses in a complementary way. The dialogue between the different approaches was possible because a point of contact was found. In this case, we may talk about a common *epistemological sensitivity* of AiC, TDS, and ATD, which can be noticed in the a priori analyses provided by each frame. This initial proximity was essential for the dialogue to start and become productive, showing the complementarity of the approaches and the reciprocal enrichment, without losing what is specific to each one. The three concepts, context, milieu and media-milieus dialectic were accessed by different data or different foci on data in a complementary way sharing *epistemological sensitivity*, which facilitated establishing connections and reflecting on them.

Epistemological concerns as a consequence of networking

It is not by chance that the common *epistemological sensitivity* of AiC, TDS, and ATD, was noticed in the a priori analyses provided by each frame: the reason is that the a priori analyses take into account the mathematical epistemology of the given domain. In the last years, the AiC researchers decided to implement the idea of a priori analysis in an explicit way. This happened as a consequence of the networking experience with the TDS researchers. An example of such a networking experience is described in Kidron and colleagues (2008). Three theories were involved in this case of networking: TDS, AiC and IDS, the theory of Interest-Dense Situations (Bikner-Ahsbahs & Halverscheid, 2014). Kidron and colleagues (2008) focus on how each of these frameworks is taking into account social interactions in learning processes. The authors wrote that

In a more general way, the different views the three theoretical approaches have in relation to social interactions force us to reconsider these approaches in all their details. The reason for this is that the social interactions, as seen by the different frameworks, intertwine with the other characteristics of the frameworks. (p. 253)

The authors identified not only connections and contrasts between the frameworks but also additional insights, which each of these frameworks can provide to each of the others. In this paper, we only focus on a specific kind of insights: the epistemological concerns which were highlighted as a consequence of the networking of theories. We first characterize the epistemological dimension in each of the three theories before the networking experience:

- TDS provides a frame for developing and investigating didactical situations in mathematics from an epistemological and systemic perspective. TDS combines epistemological, cognitive, and didactical perspectives. TDS focuses on the epistemological potential of didactical situations;
- IDS, the theory of interest-dense situations, is “a social constructivist theory that cannot say much about cognitive processes of individuals and does not provide tools for epistemological analyses” (Bikner-Ahsbahs & Halverscheid, 2014, p. 102);

- AiC analysis focuses on the students’ reasoning; mathematical meaning resides in the verticality of the knowledge constructing process and the added depth of the resulting constructs. An epistemological stance is underlying this idea of vertical reorganization but AiC analysis is essentially cognitive.

Focusing on epistemological concerns as mentioned earlier, we will only characterize the insights offered by TDS to AiC as described by Kidron and colleagues (2008):

According to Hershkowitz et al. (2001), the genesis of an abstraction originates in the need for a new structure. In order to initiate an abstraction, it is thus necessary (though not sufficient) to cause students’ need for a new structure. We may attain this aim by building situations that reflect in depth the mathematical epistemology of the given domain. This kind of epistemological concern is very strong in the TDS, and the notion of fundamental situation has been introduced for taking it in charge at the theoretical level. It could be helpful for AiC. (p. 254)

This was an invitation for AiC researchers to build an a priori analysis that reflects in depth the mathematical epistemology of the given domain. In the same vein the a priori analysis of TDS offers another perspective to IDS to think about the building of situations reflecting in-depth the mathematical epistemology of a given domain and the consequence of such reflection on the analysis of the social interactions.

The social dimension and its influence on the epistemological analysis

In the following, I analyze a case of networking between AiC and IDS which demonstrates mutual insights in the process of networking. In particular, we will observe how the epistemological analysis carried by the AiC researchers is influenced by the social dimension of knowing which characterizes IDS. This case of networking illustrates how the epistemological analysis might take into account the social dimension of knowing.

Kidron and colleagues (2010) focus on the idea of networking and on two theoretical concepts: the need for a new knowledge construct, and interest. IDS considers social interactions as basis which constitutes

learning mathematics. Interest-dense situations provide motivation for processes of in-depth knowledge construction. AiC is a theoretical tool to investigate such processes. As already mentioned, in the AiC analysis, the first stage of the genesis of an abstraction is the learner's need for a new construct. Such a need might arise when the learner's existing knowledge is insufficient to solve a task or to understand a new concept. This individual need is related to the specific mathematical situation at hand. Analyzing this need is a part of AiC epistemological analysis. For IDS the situation is different: interest constitutes a psychological source to gain more knowledge. This need is nested in the situational interest rather than shaped by the epistemic nature of the topic. The aim of the networking was to relate these two concepts: need and interest. As mentioned earlier, the AiC researchers implemented the idea of a priori analysis. Their analysis was based on an a priori analysis of the knowledge elements intended by the design. The AiC analysis focused on the students' reasoning and mathematical meaning resided in the verticality of the knowledge constructing process. The AiC researchers identified students' constructs of the intended knowledge elements. They expected to identify students' need for the new constructs before or during the process of knowledge construction. However, the researchers found it difficult to identify a need for a specific new construct. Networking the two approaches was helpful: The IDS analysis focuses and reconstructs the whole situation sequentially on the basis of utterances that show intense social interactions, whereas the AiC analysis focuses on segments that appear relevant to the constructing process. In fact, the excerpts ignored at first by the AiC researchers did contribute to the constructing process thanks to the social interaction analysis provided by IDS which allowed the AiC researchers to focus on and incorporate these seeds of construction in their analysis. The networking helps AiC researchers realize that there are situations in which constructing actions can occur on the basis of a general epistemic need rather than on the basis of specific needs for new constructs. The benefit of networking was mutual thanks to the epistemological nature of AiC a priori analysis which makes the researchers sensitive for the mathematics at stake and implicit mathematical ideas were identified very early. This was very helpful towards IDS re-analyzing of the epistemic actions in the research study.

CONCLUDING REMARKS

In the last CERME we discussed cases in which the epistemological dimension permitted the networking. This was done, for example, by means of the idea of "reference epistemological model". In this paper, we notice how by means of networking, strong epistemological concerns in one theory might be integrated in another theory in a way that reinforces the underlying assumptions of this other theory. This was illustrated by the insights offered by means of a priori analysis. We also analyzed examples that demonstrate the influence of the cultural context as well as the influence of the social dimension on the epistemological analysis. The cultural context in which the different theories emerged is changing all the time. As a result of these changes, a new view on the epistemological dimension is offered. This new view should be further discussed.

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