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The epistemological dimension in didactics: Two problematic issues

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This paper presents some theoretical considerations concerning the relationships between epistemology and didactics. We distinguish two big issues that show the mutual enrichment of both fields. On the one hand, considering teaching and learning phenomena as part of the empirical basis of epistemology enables proposing new epistemological models of mathematical bodies of knowledge. On the other hand, these epistemological models provide guidelines for the design and analysis of new teaching proposals, which, in turn, show the constraints coming from the spontaneous epistemologies in school institutions. Some critical open questions derived from these issues draw up the guidelines for a future research programme.

Keywords: Epistemology, didactics, ATD, praxeologies.

PEDAGOGY, EPISTEMOLOGY AND DIDACTICS

What distinguishes didactics of mathematics (or of any other field of knowledge) from general education or pedagogy is the status given to the knowledge or “content” that is taught and learnt. Pedagogy considers the knowledge to be taught as a given, and focuses on the best, conditions or practices to teach and learn it: the knowledge is not problematic, the relationships of the students to it are (Chevallard, 2000). In contraposition, didactics locates the *epistemological problem* at the core of the analysis. A double assumption is meant by this. First, the phenomena underlying teaching and learning processes, at school as well as in other social institutions, are closely dependent on the content that is designed to be taught, actually taught and learnt, and also on how this content is considered by the participants of the teaching and learning process. Second, that the study of these phenomena is also strongly dependent on the way knowledge is

considered and modelled by researchers in didactics. In fact, the main point of the paper is to describe a research programme that seeks to clarify the problem of how to teach (and learn) mathematics and its relationship with the problem of what is considered as mathematics.

The pedagogical dimension of teaching and learning phenomena refers to generic practices, discourses, strategies and regularities that can be described regardless of the content to be taught. The didactic dimension is reached when the concrete mathematical activities organised by the teacher and carried out by the students, as well as any other fact affecting the delimitation, construction, management, evolution and assessment of these activities are considered.

The need to integrate both the pedagogical and the epistemological dimensions was one of the main motivations for Guy Brousseau to promote the construction of a new field of knowledge called didactics of mathematics and which he contributed to with the first formulations of the theory of didactic situations during the decade 1970–1980 (Brousseau, 2002). We are here pointing out two main reasons for this integration that will later on be at the basis of the problematic issues we wish to raise. The first one is the dependence between the dominant *epistemologies of mathematics* (or of any other field of knowledge) at an educational institution and the way teaching is organised in this institution. In other words, the way mathematics and its specific bodies of knowledge are considered in a given institution, usually as implicit assumptions, affects the conditions established for its learning. In this sense, we can say, rephrasing Brousseau (2002), that *teaching organisations* are supported by *spontaneous epistemologies* appearing to the subjects of the school institution as the unques-

tionable and transparent way to conceive the content to be taught.

The second main reason, also put forward by Brousseau, is related to the implementation of research results wishing to improve teaching and learning. Whatever general strategies or conditions we may find at the pedagogical level, teachers will always have to specify them in terms of what ties them to the students: the knowledge-based learning activities. Of course, it is possible to delimit general pedagogical phenomena affecting any content to be taught and to propose general pedagogical actions in order to improve teaching and learning processes. However, eventually, these actions will need to be concretized and converted into didactic facts and strategies, that is, to specific ways of organizing mathematical contents and designing mathematical activities for the students.

Once the necessity to integrate the “epistemological problem” into the “teaching and learning problem” is assumed, there are different levels to take the epistemic dimension of the teaching and learning process into account. In some cases, the focus can be on a given piece of knowledge (“proportionality”, “limits of functions”, “linear equations”), or a whole area (“algebra”, “calculus”, “statistics”), thus considering specific models relying on a more or less explicit conception of what mathematics is and how it can be described. Therefore it can be said that the consideration of the *didactic problem* needs to include, in one way or another, a specific answer to the *epistemological problem*. To interpret the interrelation between epistemological and didactical problems, we now present a historical development of the object of study of each discipline and their respective empirical basis.

THE EVOLUTION OF EPISTEMOLOGICAL AND TEACHING PROBLEMS

In a previous study, Gascón (2001) describes a rational reconstruction of the evolution of the *epistemological problem* and, in parallel, the evolution of the *didactic problem* showing that a certain convergence exists between them. The evolution of the epistemological problem can be interpreted as a successive expansion of what is considered as the object of study and of the consequent empirical basis used to approach it. Briefly speaking, this work shows that the nature of the epistemological problem began as a purely log-

ical problem (EP¹), became a historical problem (EP²) and ended up being considered, at the end of the last century, as an essentially cognitive problem (EP³). Its successive formulations together with its corresponding tentative answers can be outlined as follows:

EP¹: How to stop infinite regress to get a logical justification of mathematical theories?

EPA¹: Euclidean models: logicism (Russell), formalism (Hilbert) and intuitionism (Brouwer).

EP²: What is the logic of the development of mathematical discovery?

EPA²: Quasi-empirical models (Lakatos)

EP³: What are the tools and mechanisms found in history and psychogenesis of the development of mathematical discovery?

EPA³: Constructivist models (Piaget & García, 1982).

This evolution of the epistemological problem can be interpreted as a progressive detachment from logical procedures and an approximation to empirical sciences such as history and psychology. This expansion continues since the 70s, with the inclusion of sociological data. Indeed, sociologists such as Barry Barnes and David Bloor, and later others like Bruno Latour, heavily influenced by the ideas of Thomas Kuhn, tried to highlight the essential social nature of scientific research. Let us notice, however, that, apart from Kuhn’s mention of the “textbooks epistemology” (Kuhn, 1971), none of these approaches seem to consider empirical phenomena related to the teaching, learning and disseminating of mathematics. The division between pedagogy and epistemology appears to be taken for granted in this research domain also.

All of the epistemological models above can be related to general teaching models, ranging from *theoricism* (organizing the teaching of mathematics following the logic construction of concepts) and *technicism* (exercising the main techniques in a given domain without many theoretical tools), to *constructivism*, which aims to enable students to construct knowledge according to certain predetermined stages.

Gascón (1993) shows limitations of the empirical basis used by constructivism to address the epistemological problem. Taking into account personal psychogenesis data, in some sense completed with those provided by the history of science, it does not integrate *didactic facts* and, can thus hardly explain institutional-depending phenomena as the so-called “personal” construction of knowledge. In other words, and according to Chevallard (1991), the study of the *genesis and development* of knowledge (traditional object of epistemology) cannot be separated from the study of the *diffusion, use and transposition* of knowledge (object of study of didactics).

It is at this point where both problems, the epistemological and the didactic one, converge, with the consequently significant expansion of the object of study of both disciplines. Historically, this time corresponds to the first formulations of the theory of didactic situations (TDS) proposed by Guy Brousseau in the early 1970s (Brousseau, 2002). It is no coincidence that at this early stage of didactics of mathematics, Brousseau initially considered to name this new discipline “experimental epistemology”. In particular, didactics of mathematics accepted the responsibility to elaborate and use epistemological models of mathematical bodies of knowledge as a new way to study didactic phenomena, thus turning the *pedagogical problem* into an *epistemological-didactic* one.

New questions arise from this perspective: What new general epistemological theories, based on which empirical data, may serve to support new teaching organizations in order to overcome the limitations of the current ones? To what extent and by what means can the dominant spontaneous epistemologies in a teaching institution be changed in solidarity with the teaching models based on them?

AN ANSWER TO THE EPISTEMOLOGICAL-DIDACTIC PROBLEM

The anthropological theory of the didactic (ATD), following the research programme initiated by the theory of didactic situations, considers a specific model of mathematical knowledge and its evolution formulated in terms of a dynamical sequence of *praxeologies*. Praxeologies are entities formed by the inseparable combination of a *praxis* or know-how made of types of tasks and techniques, and of a *logos* or knowledge consisting of a discourse aiming at describing, ex-

plaining and justifying the *praxis* (Chevallard, 2000). In didactics research, mathematical praxeologies are described using data from the different institutions participating in the didactic transposition process, thus including historical, semiotic and sociological research, assuming the institutionalized and socially articulated nature of praxeologies. Furthermore, a dialogue with the APOS theory shows how data interpreted as the different *levels of development of schemes* by psychogenetic developments, can be reformulated in ATD in terms of the *institutional evolution of praxeologies* (Trigueros, Bosch, & Gascón, 2011).

Reference epistemological models as sequences of praxeologies

To describe and analyse the specific contents that are at the core of teaching and learning processes, the general model in terms of praxeologies is structured in an articulated set of specific models of the different areas of the mathematical activity at stake called *reference epistemological models* (REM) (Barbé, Bosch, Gascón, & Espinoza, 2005; Bosch & Gascón, 2006). The Reference Epistemological Model of a body of knowledge is an alternative description of that body of knowledge elaborated by researchers in order to question and provide answers to didactic facts and problematic aspects taking place in a given institution. This REM prevent researchers to take for granted how this body of knowledge is conceived in the institution considered. For instance, Ruiz-Munzón (2010) and Ruíz-Munzon, Bosch and Gascón (2013) present a REM about elementary algebra which is used to analyse the status and role of this area of school mathematics in relation to arithmetic and functional modelling. The model takes into account the processes of didactic transposition to explain what is currently taught as algebra at school and provides a rationale to this area that does not coincide with the official and more limited one assigned by the educational system. Some of the difficulties in the teaching and learning of elementary algebra can then be referred to these limitations and new teaching proposals can be designed to overcome them (Ruiz-Munzón, 2010; Bosch, 2012).

In this REM, algebra is interpreted as a *tool for modeling* any type of (mathematical and extra-mathematical) systems and the process of *algebraization* is divided into three stages. The first one concerns the passage from the execution of computation programmes (sequences of arithmetic operations on numbers like the ones carried out when solving an arithmetic problem)

to the written or rhetoric description of their structure; the second stage requires the symbolic manipulation of the global structure of written computation programmes (not only simplifying and developing, but also “cancelling”, etc.); at the third stage, the whole manipulation of formulas is reached.

It is important to note that this REM is not a static description of a piece of mathematical knowledge, it also suggests a dynamical process to introduce elementary algebra: starting from the study of arithmetic computation programmes (CP) in order to motivate the entrance into the second stage of algebraization by the limitations of the rhetorical formulation of CPs in the first stage. Encountering problematic questions in this arithmetical work with CP may generate the need to build a written symbolic *formulation* of these CP to globally manipulate their structure, thus promoting the need to establish symbolic codes (hierarchy of operations and bracket rules).

In a similar way, different REM of other specific areas of mathematics have been proposed, all formulated in terms of sequences of related praxeologies: limits of functions (Barbé et al., 2005), proportionality (García, Gascón, Ruiz Higuera, & Bosch, 2006; Hersant, 2001), measure of quantities (Chambris, 2010), real numbers (Bergé, 2008; Rittaud & Vivier, 2013), among others. In general, the organisation of a teaching process based on the REM of a given mathematical content is called *research and study activities*.

From teaching of contents to enquiry processes: Study and research paths

These reference epistemological models correspond to previously established bodies of mathematical knowledge: algebra, limits, proportionality, etc. They provisionally assume the delimitations of mathematical knowledge provided by the school and the scholarly institutions, which are then often redefined. In order to also take into account enquiry processes that start with the consideration of problematic questions to be solved (instead of pre-established contents to be learnt), REM have been enriched with the proposal of the *Herbartian schema* (Chevallard, 2006; Chevallard, 2015). This scheme is a useful tool to observe, analyse and evaluate existing and potential didactic processes that start with the consideration of a generating question and evolve with the search of partially available answers (“contents” to be learnt) and the construction of new answers through the interaction with a milieu.

The study of a specific *question* leads to a rooted-tree of derived questions and provisional answers, which outlines the generating power of the initial question and the possible paths to follow. We thus obtain new reference epistemological models assigned to problematic questions instead of pre-established praxeological contents. Winsløw, Matheron, and Mercier (2013) provide several examples of this kind of rooted-tree REM, such as the dynamics of a population or the trajectory of a three-point shot in basketball. The enquiry process of a particular generating question materializes in an open didactic organisation called a *study and research path* (SRP). During the development of SRP, the need for new knowledge to solve some of the derived questions found in the path usually leads to the activation of study and research *activities*.

Didactic praxeologies emerging from reference epistemological models

The previous section briefly outlined how the design, implementation and analysis of study and research paths and study and research activities call for the activation of specific didactic techniques and creates new types of didactic tasks. For instance, in the case of elementary algebra illustrated above, the didactic technique proposed by Ruiz-Munzon (2010) consists in introducing the study of “mathemagic” games of the sort “Think of a number, apply these calculations [...], you get 73” as generating questions. How do you explain the magician’s trick?” These games generate the need to look for new pieces of answers, in the manipulation of the calculation programmes proposed or in their transformation and generalisation through algebraic symbolism. Questions based on “mathemagic” games allow producing an important number of computation programmes economically. They are presented to the students without much artificiality and their first contact with computation programmes is not problematic. Moreover, the limitations of the rhetorical and numerical formulations of computation programmes inevitably appear and they do so soon enough.

PROBLEMATIC ISSUES

The aim of this paper is to formulate some problematic issues at the crossroads of epistemology and didactics. We will initially explain them within the context of the ATD before extending the questioning to other didactic approaches. If we try to characterize a didactic approach by how “pedagogy” and “mathematics”

are integrated, in the case of the ATD such integration can be formulated in terms of two movements. They appear in the design, management and evaluation of teaching and learning processes and can briefly be described as follows:

- 1) Starting from the analysis of teaching and learning processes at school and considering an empirical basis of study that is large enough to include the processes of didactic transposition, all this empirical work provides tools to design specific REMs for the main mathematical contents or areas that are designed as knowledge to be taught. We can define this movement as “using didactic facts and phenomena to produce epistemological models”.
- 2) Conversely, the principles and criteria that have guided the construction of a REM for a specific area of school mathematical activity and, in particular, the contrast between the rationale assigned by the REM to this area and its official (explicit or tacit) role in school mathematics, all provide some mathematical and didactic tools to design, manage and evaluate teaching and learning processes based on study and research paths sustained by that REM. This movement can be defined as “using the epistemological model as the core of didactic tools”.

This double movement raises different open issues which are at the starting point of the research programme we want to propose in this paper.

New didactic needs

We have seen how previously elaborated REM on mathematical contents or problematic questions (obviously complemented with other methodological design tools) can provide criteria for the design and implementation of teaching and learning processes that are considerably different from the existing ones. In principle, they aim at organising activities that should allow the students to carry out new mathematical tasks and techniques in a more autonomous, functional and justified way. The “mathemagic” games in the case of elementary algebra (Ruiz-Munzón, 2010) or the different enquiry processes described in Winsløw, Matheron and Mercier (2013) are good examples of this enrichment. Obviously, these new didactic organisations should be made available to

the study community and their viability in different school institutions should be tested.

It is important to emphasize that all didactic approaches and theories are also based on general models of mathematical-didactic activities. These general models are a particular way to interpret the mathematical activity and to conceptualize the study process of mathematics (teaching, learning, diffusion and application). Even though these models are not always clearly spelled out, they remain an essential feature of theoretical approaches, as they strongly affect the type of research problems this approach can formulate. Two crucial questions arise:

- 1) In the case of ATD, how to transform the REM into possible didactic organisations that could live in current school institutions? How to take into account the interrelation between the REM and the didactic phenomena appearing in the implementation of these new didactic organisations? How to make this process available to the school institutions, especially to the profession of teachers?
- 2) How is this mutual enrichment between the epistemological and didactic proposals taken into account in other theoretical frameworks?

New epistemological needs

The empirical analysis of the study processes taking place in various institutions (for example, but not exclusively, in schools) clearly shows that the didactic praxeologies are closely related to the epistemological tools available in the institution to describe and manage the mathematical praxeologies. For example, in the institutions where the dominant model is Euclidean, teaching and learning processes are conceived and described in terms of didactic activities around “definitions”, “concepts”, “theorems”, “proofs” and “applications”. In addition, these didactic activities tend to be hierarchically structured according to the logical construction of mathematical concepts (real numbers before limits, limits before derivatives, etc.).

If, instead of analysing traditional teaching processes, we look at those based on didactic research, the situation is very similar: how didactic processes and the dynamics of mathematical praxeologies are designed, described and managed also depends on the tools pro-

vided by the epistemological model which upholds, more or less explicitly, the didactic approach considered. The further this research-based epistemological model is from the dominant epistemological model at schools and scholarly institutions, the more difficult it becomes for teachers to carry out innovative teaching proposals designed within this frame.

In all these cases, the most remarkable feature is the shortage and inadequacy of tools available in the teaching institution to describe, manage, and evaluate the dynamics of mathematical activity. This lack of tools could in the first place be attributed to the scarcity of spontaneous epistemological models and, in particular, to the shortage of the Euclidean epistemological model of mathematics whose supremacy is still present, to a greater or lesser extent, in most institutions.

Which new notions or tools are needed to describe and manage the dynamics of the mathematical activity that will take place in study processes? How to describe these tools depending on the role addressed (didactic researcher, teacher and students)? How to make them available in the teaching institution and to the participants of the didactic process?

The evolution of didactic-epistemological models

In order to establish an alternative and rich REM of a specific mathematical domain or questioning, it is necessary to take into account the didactic phenomena taking place in teaching institutions. This leads to an enrichment of the spontaneous epistemological model during the first design of the REM. However, it is important to keep the process running during the implementation and the evaluation of teaching proposals based on this REM. The consequent evolution of the REM is a clear example of the dynamic and provisional nature of the epistemological models elaborated by didactics, evolving from its initial proposals through the analysis of empirical facts. From a mathematical perspective, these continuous evolutions of the REMs can be seen as the incorporation of new notions and organisations into the field of knowledge. This phenomenon can be related to the transformation of some paramathematical notions into mathematical concepts, as happened with concepts (such as “set”, “function”, “continuity”, “graphs”, etc.), a transformation which takes place as long as researchers deal with new problems. For instance, in

the case of elementary algebra, the notion of “computation programme” is a new and crucial element of the proposed REM. In the experiences described by Ruiz-Munzón (2010), this notion played a very ambiguous role in the management of the teaching and learning processes, given the fact that it did not belong to the official mathematics to be taught and the teacher did not feel at ease with it. A similar phenomenon happened when implementing SRP on population dynamics with notions such as “quantities”, “model”, “system”, “mixed and separated generations”, etc.

Another important and difficult question is the degree of explicitness that should be adopted with the new epistemological models necessary to design, implement and evaluate new teaching and learning processes depending on the participants of the study communities addressed (students, teachers, mathematicians, etc.). Also, what kind of similar experiences can be learnt from other approaches? Did they find similar difficulties? These open questions establish a new research programme where the results of previous investigations carried out within the ATD should be analysed together with analogous research from other perspectives. In all cases, the status given to the epistemological dimension in didactics analysis seems to appear as a crucial question to take into account.

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