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Secondary mathematics teacher candidates' pedagogical content knowledge and the challenges to measure it

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In this paper, the authors will discuss pedagogical content knowledge of secondary mathematics teacher candidates in Turkey. The discussion is based on comparisons between senior students from secondary mathematics education and mathematics departments in terms of their pedagogical content knowledge measured by Teacher Education and Development Study in Mathematics (TEDS-M) released items. In addition to comparison of two groups, there will be a discussion on the challenges to measure pedagogical content knowledge.

Keywords: Pedagogical content knowledge, TEDS-M, measuring PCK, secondary school, mathematics teacher candidates.

INTRODUCTION

Teacher knowledge and its components have been described and modelled in different ways by different researchers (Shulman, 1986; Ball, Thames, & Phelps, 2008; Franke & Fennema, 1992; Tatto et al., 2008). However, it can be said that many teacher knowledge approaches have been influenced by the Shulman's (1986) model of teacher knowledge. Shulman made an important contribution by categorizing teacher content knowledge as Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK). As Petrou and Goulding (2011) stated, in the Shulman model, the most influential category was the new concept of PCK. Shulman (1986) described PCK as "special amalgam of content and pedagogy that is uniquely the province of teachers, their own special form of professional understanding" (p. 9). According to him as the requirement of PCK, teachers need to know using representations, illustrations, analogies, and demonstrations

and also giving examples and explaining concepts in order to make them understandable.

Shulman's conceptualization of teacher knowledge provided a basis for research field of mathematics education. The knowledge that mathematics teachers need to acquire for teaching was described with the Mathematical Knowledge for Teaching (MKT) model, which is the refinement of Shulman's categorization (Ball et al., 2008). MKT model categorizes SMK and PCK into six subcomponents. Ball (2003) defined the subcomponents of PCK by reconsidering Shulman's categorization. The components are Knowledge of Content and Students, Knowledge of Content and Teaching and Knowledge of Curriculum.

Although the MKT model has been widely used, there is some criticism about it. This model was developed considering elementary and middle school mathematics teachers, but not secondary. Therefore, it is argued that the components of MKT do not meet the mathematical need for secondary mathematics teachers (Zazkis & Leikin, 2010). The claim is that "the higher the level taught, the more the teacher needs to know" (Usiskin, 2001, p. 86), so the nature of mathematics that secondary teachers need to know is at the much higher level than elementary teachers. According to Zazkis and Leikin (2010), Advanced Mathematical Knowledge (AMK), which is defined as knowledge of subject matter acquired during undergraduate studies at universities, is necessary knowledge for teaching mathematics at secondary level. It can then be said that, since generally SMK is prerequisite for PCK (Shulman, 1986) specifically at secondary level, AMK is also necessary for PCK. However, it is not sufficient because PCK includes the knowledge of content and teaching, the knowledge of content and students, and the knowledge of curriculum (Ball

et al., 2008). Therefore, classroom experiences and practices are also important for the development of PCK. Researchers argue that there is an interaction between SMK, PCK, beliefs and practices (Franke & Fennema, 1992; Walshaw, 2012; Türnüklü, 2005). However, PCK has a special importance because it is influenced by all the others: SMK, practice and belief. It can be said that PCK has a multidimensional nature. Wilson (2007) claims that this complex nature makes it difficult to investigate PCK by using efficient measures. Even though developing scalable efficient measures for content knowledge for teaching is difficult (Wilson, 2007), researchers tried to develop rigorous, effective and valid instruments to measure mathematics teachers' knowledge (Hill, Schilling, & Ball, 2004; Krauss, Baumert, & Blum, 2008; Tatto et al., 2008).

One of the instruments to study mathematics teacher knowledge is the Teacher Education and Development Study in Mathematics (TEDS-M) measure. TEDS-M is a cross-national study in which 17 countries participated, even if Turkey was not involved. The characteristics that differentiate TEDS-M measure from others are to consider both primary and secondary levels and to be designed for international usage and national adaptations. Differences in students' achievement level in Trends in International Mathematics and Science Study (TIMSS) encouraged researchers to study on teacher education internationally in order to investigate how mathematics teaching quality differs across countries. Therefore, the TEDS-M measure was developed to examine the mathematical knowledge for teaching of future mathematics teachers, based on TIMSS 2007 framework of content areas and cognitive domains. By considering such characteristics of the measure, in this study, TEDS-M secondary released items were used for the investigation of the mathematical knowledge for teaching of secondary mathematics teacher candidates.

METHODS

Participants

In Turkey, both graduates of secondary mathematics teacher education departments and mathematics departments (after completing teaching certificate program) have chance to be mathematics teachers in secondary schools. Therefore, the participants of the study were senior students from secondary mathematics teacher education departments ($n = 47$) and senior student from mathematics departments ($n = 48$)

of two universities in Istanbul. Totally, 32 females and 15 males senior secondary mathematics education students (the mean age is 24) and 35 females and 13 males students (the mean age is 22) from mathematics departments participated in this study. These two universities were ranked as first and second among the secondary mathematics education departments in the national university entrance exam. In the first ranked university, students enrol mathematics and secondary mathematics education programs by getting similar scores from university entrance exam. In the second university, the minimum score of secondary mathematics education department is a little higher than the one of the mathematics department.

These two programs have different curriculum in undergraduate education programs. The secondary mathematics education program includes 50 % of content knowledge and skills, 30 % of professional teaching knowledge and skills and 20 % of general knowledge courses (YÖK, 2007). However, the undergraduate program in a mathematics department consists of 70 % of content knowledge and 30 % of general knowledge. Moreover, participants of the study were asked to explain whether they had an informal teaching experience like tutoring or teaching in cram school. As they stated, 76 % of secondary mathematics education students and 70 % of mathematics students had informal teaching experiences.

Instrument

The instrument was designed by TEDS-M researchers considering the framework of Trends in International Mathematics and Science Study (TIMSS) 2007 (Tatto et al., 2008). MCK items comprised of four content areas: number, algebra, geometry and data, and three cognitive dimensions: knowing, applying and reasoning. Furthermore, MPCK items consist of two parts: knowledge of curricula planning, and interactive knowledge about how to enact mathematics for teaching and learning. These were aligned with the PCK domains in literature. Table 1 and Table 2 show the distributions of MCK and MPCK items according to their content, cognitive domains and PCK components. (In the appendix, Figures 1 and 2 are examples of MCK items and Figures 3 and 4 are examples of MPCK items.)

These items include 23 mathematics content knowledge (MCK) and 9 mathematics pedagogical content knowledge (MPCK) items with three different item

Cognitive Domain	Content Domain				
	Algebra	Geometry	Number	Data	Total
Knowing	-	2	4	-	6
Applying	5	4	-	1	10
Reasoning	2	1	4	-	7
Total	7	7	8	1	23

Table 1: MCK Secondary Items

	Content Domain				
	Algebra	Geometry	Number	Data	Total
Curriculum and Planning	4	-	-	-	4
Enacting	1	-	3	1	5
Total	5	0	3	1	9

Table 2: MPCK Secondary Items

formats: multiple choice, complex multiple choice and open constructed response.

In order to compare MKT of participants who were studying in different departments, Turkish translated versions of TEDS-M secondary level released items were used. The method which was used while translating the instrument consists of three phases. Firstly, items were translated in Turkish by the researcher who is fluent in English. The translated items were reviewed by a mathematics educator who is an expert in the content area and fluent in English, a three-year experienced mathematics teacher who is fluent in English, and a professional translator. At the second phase, the original tests were administered to a group of pre-service mathematics teachers who are native in Turkish and fluent in English. The same group took the translated versions of the tests three weeks apart. At the last phase, the method of back translation was used to check the quality of translation and to investigate the linguistic or conceptual errors in translation. It was also used to consider particular attention to sensitive translation problems across cultural correspondence of the two versions.

Data collection and analysis

The data was collected from participants in a single point in different times. Instrument administered to senior students during the last two weeks of the spring semester of the 2012–2013 academic year just before they graduate.

After data collection, all items were scored according to the scoring guide of TEDS-M Secondary Items.

Participants' scores acquired from 23 MCK items were calculated and called as MCK scores and scores obtained from 9 MPCK items were calculated and called as MPCK scores. Total scores of participants were also calculated by the summation of MCK and MPCK scores.

The scores of these two groups of participants were compared by using appropriate statistical methods. For total scores and MCK scores comparisons, an independent sample t-test was used since all the assumptions were met. For the comparison of MPCK scores, a non-parametric Mann-Whitney U test was used since the normality assumption was violated.

RESULTS

Participants' scores obtained from the 47 senior students from the mathematics teacher education department and the 48 senior students from the mathematics department were compared. Table 3 shows means and standard deviations of the two groups of participants.

		M	SD
Total	Math Teacher Education	26.83	3.96
	Math	23.63	4.42
MCK	Math Teacher Education	20.45	3.35
	Math	17.50	3.80
MPCK	Math Teacher Education	6.38	1.19
	Math	6.13	1.35

Table 3: Means and Standard Deviations

The results of the t-test indicate that the mean of the total score of mathematics teacher education students is significantly 3.2 points higher than those from mathematics department, $t(93) = 3.72, p < .001$ and Cohen's $d = .76$ with the marginal large effect size (Cohen, 1988). Furthermore, the independent sample t-test results show that students from the mathematics teacher education department have significantly higher MCK scores than those of mathematics departments: $t(93) = 4.00, p < .001$, and Cohen's $d = .82$ with the large effect size. Moreover, according to the non-parametric Mann-Whitney U test, there is no significant difference between them in relation to the MPCK scores: $Z = 1.00, p > .05$.

DISCUSSION AND CONCLUSION

The study aimed at comparing the mathematical knowledge for teaching of students who will graduate from a mathematics teacher education department and others who will graduate from a mathematics department. In Turkey, graduates of both departments have a chance to become mathematics teachers at secondary level, but graduates of the mathematics departments need to take a teaching certificate. However, the knowledge and skills that graduates are able to acquire through these programs are different from each other. For example, the contents of undergraduate education programs of these departments are notably different. The mathematics departments' program does not include any pedagogy or education course, but more advanced mathematics courses than the mathematics education departments' program does. Therefore, the result was unexpected: mathematics students, who were not required to take any teaching related courses, were not significantly different from students of the mathematics teacher education in terms of MPCK scores.

This unexpected result may be explained by discussing the nature of PCK for secondary level mathematics teaching. Even though teacher education programs are the most influential factors that affect PCK of teacher candidates, there are other factors when the nature of PCK is considered. PCK includes knowledge of "the ways of representing and formulating the subject that make it comprehensible to others" (Shulman, 1986, p. 9). It may be conceptualized as not only knowledge of students' thinking and conceptions, but also knowledge of explanations, representations and alternative definitions of mathematical concepts,

and knowledge of multiple solutions to mathematical tasks (Shulman, 1986; Ball et al., 2008; Krauss, Baumert, & Blum, 2008). Therefore, teaching experiences play an important role in the development of teachers' PCK (Ball et al., 2008). Because of this, teacher education programs include many teaching experiences opportunities like field experience and practicum. Moreover, both groups of students who were studying mathematics teacher education in a mathematics department had informal teaching experiences like tutoring and teaching in cram school. Having this kind of teaching experience may explain the result. However, this may not be the only rationale. Measuring and assessing PCK is another issue that should be considered by focusing on its nature in order to explain the results of study.

Achieving the specialized knowledge for teaching mathematics at secondary level requires Advanced Mathematical Knowledge (AMK) which is defined as the knowledge of the subject matter acquired at the university (Zazkis & Leikin, 2010). Mathematics departments' students take many advanced mathematics courses and they develop AMK. It should be noted that AMK is a necessary but not sufficient condition for achieving the specialized knowledge for teaching at secondary level (Zazkis & Leikin, 2010).

Therefore, as it is seen, according to the PCK's multidimensional nature, deep mathematical knowledge plays an important role because it can provide teachers to use effective explanations, representations and alternative definitions. These components may contribute to make an explanation for the unexpected result of the study. For example, when MPCK items were examined according to required knowledge, and skills were needed to provide a correct answer, the need for AMK might be observed. For instance, one of the questions of the instrument (see Figure 3 in appendix) asks to determine what knowledge is needed to prove the quadratic formula. This question measures knowledge of content and teaching, but without knowing how to prove quadratic formula it is not possible to give a correct answer. Therefore, it is not easy to differentiate and measure this kind of knowledge and skills. Difficulty in measuring PCK may explain the unexpected result that there is no difference in MPCK scores between two groups of students.

Moreover, in this study, PCK was tried to be measured by few items (4 questions, 8 items). Therefore,

Questions (Items)	Content Domain	PCK Domain	Intended Ability
1 (b)	Algebra	Enacting	Analyse why one word problem is more difficult than the other.
6 (a, b, c)	Number	Enacting	Determine whether student's response is valid proof.
9 (a, b, c, d)	Algebra	Curriculum and Planning	Determine what knowledge is needed to prove the quadratic formula.
12 (b)	Data	Enacting	Explain student's thinking about histogram.

Table 4: TEDS-M Secondary PCK Items' Characteristics

only some domains of PCK and some abilities were able to be measured with these items. However, as Shulman (1986) and Ball and colleagues (2008) stated, PCK requires different kinds of knowledge, tasks and skills. This instrument can only address some of them. Table 2 shows the distribution of content and PCK domains of items, and Table 4 above shows the intended abilities for each of them.

The reactions of the two groups of participants to these PCK items are different. For example, item 9b (Figure 3 in the appendix) were answered correctly by 97 % of the mathematics department students and by 86 % of the students from the secondary mathematics education department. On the other hand, 72 % of the students from the secondary mathematics education department answered item 1b (Figure 4 in appendix) correctly, while only 52 % of the mathematics department student gave a correct response.

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APPENDIX

Determine whether each of the following is an irrational number always, sometimes or never.

Check one box in each row.

	Always	Sometimes	Never
A. The result of dividing the circumference of a circle by its diameter.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
C. The diagonal of a square with side of length 1.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
D. Result of dividing 22 by 7.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Figure 1: An example of TEDS-M Secondary MCK items (Number, Knowing)

Prove the following statement:

If the graphs of linear functions

$$f(x) = ax + b \text{ and } g(x) = cx + d$$

intersect at a point P on the x -axis, the graph of their sum function

$$(f + g)(x)$$

must also go through P .

Figure 2: An example of TEDS-M Secondary MCK items (Algebra, Reasoning)

A mathematics teacher wants to show some <lower secondary school> students how to prove the quadratic formula.

Determine whether each of the following types of knowledge is needed in order to understand a proof of this result.

Check one box in each row.

	Needed	Not needed
A. How to solve linear equations.	<input type="checkbox"/>	<input type="checkbox"/>
B. How to solve equations of the form $x^2 = k$, where $k > 0$.	<input type="checkbox"/>	<input type="checkbox"/>
C. How to complete the square of a trinomial.	<input type="checkbox"/>	<input type="checkbox"/>
D. How to add and subtract complex numbers.	<input type="checkbox"/>	<input type="checkbox"/>

Figure 3: An example of TEDS-M Secondary MPCK items (Algebra, Planning)

(b) Typically Problem 2 is more difficult than Problem 1 for <lower secondary> students. Give one reason that might account for the difference in difficulty level.



Figure 4: An example of TEDS-M Secondary MPCK items (Algebra, Enacting)