Development of an algorithm for energy efficient automated train driving

Artem Ozhigin, Pavel Prunev, Victor Sverdlin, Yulia Vikulina

Development of an algorithm for energy efficient automated train driving

Ozhigin A., Prunev P., Sverdlin V., Vikulina Yu.

Siemens LLC, SEN-RU, 191186, Volynskii lane, 3A, Saint-Petersburg, Russia
e-mail: artem.ozhigin@siemens.com, pavel.prunev@siemens.com,
victor.sverdlin@siemens.com, yulia.vikulina@siemens.com

Key words: *energy efficiency, energy management, automatic train operation, optimization problem, genetic algorithm, smart transportation, real-time systems*

1 Abstract

Automated train driving function is greatly demanded in high-speed and commuter trains operated by Russian railways. Siemens Corporate Technology is involved in the development of such real-time function within a "robotised" train control system. The main intention of the system is not only to relieve the human driver from routine control over traction and brakes (allowing him to pay more attention to assurance of safety) but also to increase train efficiency by reducing the amount of consumed energy. The system under development is intended to be integrated into the existing architecture of Siemens high-speed trains as an additional function. Its environment is constituted mainly of control systems of a high safety integrity level. This on the one hand guarantees that the automated train driving system will not have any impact on train safety, but on the other hand it implies additional restrictions on the operation of the system to be fulfilled. This paper presents the experience in implementation and verification of automated train driving algorithms for Velaro RUS and Desiro RUS trains.

2 Introduction

In recent years the question of energy efficiency has become the gravest it has ever been. Railway operation considerably contributes to the overall energy consumption and, therefore, deliberate railway management systems can enhance the global natural resources savings. Significant advances have been made to improve the efficiency of the train motion: minimization of resistant forces, construction improvements, recuperation capabilities [1]. Nevertheless, there still is opportunity for further savings of energy. In particular, taking into account the track profile, optimization of the driving strategies, measuring of the consumed energy and further analysis to some degree can reduce the cost of trips.

Innumerable studies have contributed to the energy efficiency in train control. A detailed review of works on optimal train control can be found in [2]. Diverse methods and approaches can be found in literature concerning optimal train control, optimal schedule construction and railway system management. Beginning with the pioneer works of Milroy [3], Howlett [4], Asnis et al. [5], numerous scientists developed and investigated train control optimization problem, giving birth to great number of specific methods and approaches. Two different models can be highlighted among them: a model with continuous control [3], [4], [5]; and a model with discrete control [6]. The first model considers applied acceleration as a control variable whereas the second one considers switches of traction and brake for this purpose. The second model has received much attention in recent decades, due to the opportunity to model the real train control directly and to obtain as a solution a sequence of a driver's actions for optimal train control.

One of the most effective approaches to energy efficient driving is speed regulation through coasting control, minimization of brakes or a combination of optimal strategies [7]. In [8] driving strategy 'power-speedhold-coast-brake' is considered and the algorithm of optimal switching points finding is proposed. The "speed regulation without braking" strategy is implemented for efficient manual driving on long distance lines in [9].

Among the methods of solution of the train control problem one can point out two main groups widely used by the researchers: exact methods and meta-heuristical.

The first approach was utilized, for instance, by Howlett in [10]. He has elaborated the algorithm to minimize the energy consumption using Pontryagin's principle and Kuhn–Tucker equations. Assuming that the trip shall be completed with a fixed time he finds local optimal points of control switching for each part of the track with a particular slope and obtains a globally optimal strategy.

In the paper [11] Matsuura investigated an algorithm optimizing a train speed profile by the Bellman's Dynamic Programming (DP), taking into account track profile incline, regenerative brake and so on. The parallel computing technique is introduced to deal with the computation complexity issue. Comparison of Dynamic Programming (DP), gradient method and Sequential Quadratic Programming (SQP) is introduced in [12] in application to the complete optimization of multiple trains' speed profiles and energy storage devices with constraints. However, the increase of control input dimension causes explosion of computation time and use of memory space and the introduced methods cannot be applied to real-time control of trains.

As long as the train motion is a multicriterial process, constructing an analytic optimization algorithm, taking into account all the constraints, is still very difficult. Using Pontryagin's principle of maximum or the Bellman equation requires significant simplifications of the problem considered. Furthermore, these methods reduce the problem to the system of differential or even partial differential equations which need to be solved by means of some numerical methods, e.g. Runge — Kutta methods or finite element method respectively. Therefore, the final solution inevitably contains inaccuracies.

The aforesaid is the reason of the ubiquity of the second group of methods involving diverse metaheuristics: genetic algorithm (GA), simulated annealing, tabu search, etc. Although these approaches do not guarantee to find the global optimum point, in most cases it is sufficient for real-time applications to know the quasi-global optimum. During the last decade a lot of papers were published where that kind of algorithm was used for solving the efficient train control problems ([9], [13], [14], [15], [16], [18], [19], [21], [22], [23], etc.).

The genetic algorithm, for example, was utilized in works of Sicre et al. [9], [13], where the problem of constructing an on-line advisory system for efficient manual train driving is solved. The manual driving is modelled by means of fuzzy parameters as long as the exact moment of control switching depends on the reaction speed of the driver.

The GA-based approach is proposed in [14] to search for a control strategy under different operating conditions and the realistic electromechanical simulation model is utilized for energy consumption calculation. GA is also utilized in the works of Besinovic et al. [15] for calibration of the train input parameters using observed data; De Martinis et al. [16] proposed a framework operating on timetable, rolling stock characteristics, signalling system and infrastructure characteristics and an optimization subroutine composed by the genetic algorithm.

In most works the control sequence is represented by a pre-defined array of positions or as a mesh with fixed size. This evidently causes the loss of generality. In this work we propose an approach where the control switching points are computed during the GA routine, allowing to select not only a type of control, but also a particular switch moment minimizing the consumed energy.

Among the works utilizing other meta-heuristical algorithms we should mention the PhD thesis of Kim [17] with an approach to optimize train motion strategies for various track alignments (single or mixed) and maximum operating speed (constant or variable) using simulated annealing. Modified and hybrid meta-heuristic methods applied to optimal train control were considered in works of Wei et al. [18], Bigharaz et al. [19].

Considering the uncertain disturbances arising from the weather, route and locomotive rolling conditions, Li et al. [20] proved the existence of an optimal operation strategy for stochastic train energy-efficient operation.

The main advantages of meta-heuristic methods are relatively simple logic, the capability to expand the problem and an opportunity to take into account any new circumstances and conditions without affecting the optimization algorithm. These methods do not have requirements of differentiability or continuity on the functions involved. The possibility of mutation in the genetic algorithm allows to not be stuck in the local optimum. Moreover, as a result of computations a set of solutions is obtained and if the best one is not suitable for some reason, another candidate can be taken right away. Meanwhile, analytical methods provide only one solution and in order to satisfy new conditions, the necessity of the recalculation of the whole problem arises. In some cases, reconstruction of the method itself may be needed. This can be impossible or difficult to carry out.

Until recently, the question of efficient train control was a responsibility of the driver, who used his own experience, known heuristics and driving patterns as a key to optimality and efficiency. Autopilot (automated train driving) algorithm providing efficient driving will take the responsibility of routine actions and will allow the driver to concentrate on more important trip factors.

Despite the fact that there are a lot of works devoted to energy efficient train control (see [9], [16], etc.), none known to us deals with completely automated control. Apparently, this is a consequence of the local regulations prescribing that the train must be always under human control as a guarantee of safety. An optimization algorithm can be implemented only as an advisory system. On the contrary, in this work we are trying to develop an algorithm which will be able to implement automated real-time train control capable to realize the whole trip without human interference. The genetic algorithm is applied to the problem in order to find an optimal sequence of controls for energy efficient train driving.

Safety is a crucial matter in transportation systems. As mentioned before, the automated train driving function is surrounded by train control systems of a high safety integrity level and it does not control the motion of the train directly. The commands generated by the system under consideration are sent to train control systems and treated by them as recommendations which will only be implemented if safety is not violated. This way of integration isolates the automated train driving system and prevents any impact on train safety. Due to the isolation there are no real safety requirements imposed on the autopilot. On the other hand, in order to be in actual control of motion

and perform designated functions the autopilot shall not violate criteria implied by safety systems for its requests to be implemented. Inappropriate operation of autopilot can affect on passengers' comfort (e.g. frequent switching between traction and braking modes is perceived as not comfortable) and even cause engagement of emergency modes of safety systems (e.g. when autopilot operation causes violation of speed limit). This must be taken in serious consideration in the development of an automated driving algorithm.

The objective of our work is to develop an automatic real-time train control system with a view to reducing of energy consumption. For that purpose we need to find an equilibrium between three antithetic objectives: energy efficiency, fulfilment of schedule constraints and speed restrictions and passengers' comfort.

3 Problem Statement

The aim of this section is to state an optimization problem for energy efficient train control. Firstly, let us consider the mathematical model of the train which is one of the most important parts of a system.

3.1 Train Model Description

A train model is designed to define the velocity and position of a train for each moment of a trip basing on the following parameters: traction intensity (%), brake intensity (%), track profile, adhesion coefficient, weather conditions such as wind velocity. Generalized traction and brake curves were determined from the train documentation.

From the Newton's law we obtain the following dynamic equation of train motion:

$$M\frac{d^2S}{dt^2} = (u_{tr} \quad u_{br}) \begin{pmatrix} F_{tr} \\ F_{br} \end{pmatrix} - F_{fr} - F_h, \tag{1}$$

where M is train mass, S is train coordinate, F_{tr} is traction effort, F_{br} is brake effort, F_{fr} is friction force, F_h is projection of the gravity force, u_{tr} and u_{br} are traction and brake control correspondingly. Traction force values are determined by the train specification. Brake force F_{br} comprises two components: electrodynamic and pneumatic brake effort. Train control system is responsible for actual value of the traction effort and for dynamic distribution of braking efforts between pneumatic and electrodynamic brakes. Control system is trying to fulfil the requests from Autopilot on required speed and acceleration values while simultaneously preventing the violation of safety requirements.

The next term in the eq. (1) is friction force F_{fr} modelled using "Davis equation" [24]:

$$F_{fr} = A + Bv + Cv^2,$$

where $v = \dfrac{dS}{dt}$ is velocity and the coefficients A, B and C are determined by aerodynamic characteristics and rolling resistance. Their values were derived from the train specification. Aerodynamic characteristics include influences of the wind velocity and tunnels. It is possible to take into account other weather conditions, e.g. precipitation, as easy as adding one more term in right-hand side of the eq. (1).

The more sophisticated train models, including non-linear ones, can also be considered and the more realistic electromechanical simulation can be embedded. Complexity of the simulation will not require any changes in the optimization algorithm. Thus, in this work we use the standard simplification in order to pay all the attention to the energy efficiency and finding an optimal speed profile.

Solution of the optimization problem $U(S)$ is represented as a sequence of commands u_i and corresponding switching point s_i.

$$U(S) = [s_i \quad u_i] = \begin{bmatrix} s_1 & u_1 \\ s_2 & u_2 \\ \vdots & \vdots \\ s_n & u_n \end{bmatrix} \tag{2}$$

Control u_i can be equal to $\{0, 1, 2, 3\}$ where 0 designates the brake, 1 – coasting mode, 2 – cruising mode and 3 – traction. Coasting mode is implemented if traction and brake forces are equal to zero and only friction and gravitational forces act on a train. Cruising mode means the keeping by all means of the current speed using traction or brake force depending on the track profile and other factors. Each mode defines the value of control variables u_{tr} and u_{br}: both equal to zero in coasting mode, one of them has some value from an interval $[0, 1]$ in cruising mode, $u_{tr} = 1$ in traction mode, $u_{br} = 1$ in braking mode.

The train model provides information about energy consumption and recuperated energy. Energy consumed from the beginning of a trip up to the current point, is computed according to the relation:

$$E = \int_0^s F_{tr}\, dS,$$

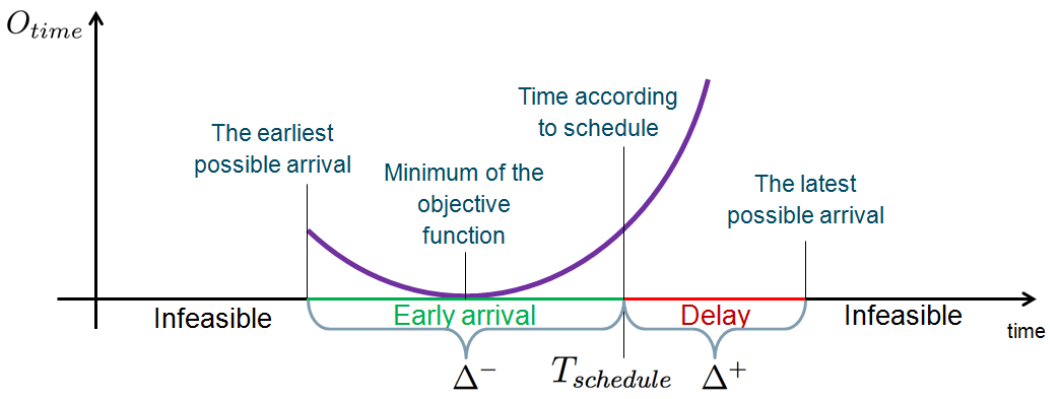

Figure 1: Time component of the objective function O_{time}

where s is current coordinate, E is consumed energy.

Recuperated energy depends only on the electrodynamic brake force which is a component of the whole brake effort as was described above:

$$E_r = \eta \int_0^s F_{edbr}\, dS,$$

where E_r is recuperated energy, η is the maximum percentage of the electrodynamic brake energy which can be returned to the grid, F_{edbr} is electrodynamic brake force.

The regenerative brake opportunity enables to significantly reduce the energy consumption, although, there are some difficulties in efficient utilization of recuperated energy, the first of which is a necessity to absorb the energy by accelerating train or by energy storage device. If the energy is not absorbed, energy consumption is larger because of regenerative failure [12]. However, due to the lack of necessary infrastructure facilities in Russian railways the recuperated energy can not be spent efficiently. Therefore, in this work the regenerative brake energy is not a part of fitness function and is considered as loss.

3.2 Optimization Problem Statement

To complete an optimization problem statement let us define the constraints and restrictions imposed on it. The information defining the train run parameters is described in a digital schedule uploaded before the trip into the automatic train driving system. This schedule contains set of route points with prescribed arrival and departure time, values of speed restriction on different route segments, altitude of the route points. The intention of the system is to make decisions on the driving mode to be implemented at every moment along the route based on current coordinate and schedule data.

Firstly, the optimal train control should satisfy the schedule constraints. We can describe the time condition using the following inequality:

$$T_{schedule} - \Delta^- \geq T \geq T_{schedule} + \Delta^+,$$

where Δ^- and Δ^+ characterise the time tolerance interval, T is trip time for the particular solution, $T_{schedule}$ is time demanded by the schedule.

The boundary conditions define departure and destination points:

$$S(0) = S_{depart}, \quad S(T) = S_{dest},$$

where S_{depart} and S_{dest} are respectively the departure and destination stations coordinates.

Finally, the velocity shall satisfy the speed restrictions:

$$V \leq V_{max}(S),$$

where $V_{max}(S)$ is maximum permitted speed for the coordinate S. Here it is important to note that the length of a train should be taken into account because that constraint should not be violated by any part of a train.

3.3 Objective Function

We understand the term optimal first of all as energy efficient. Nevertheless, the time of the trip should be also included into consideration by an optimization algorithm. Otherwise, the best chosen solution will always

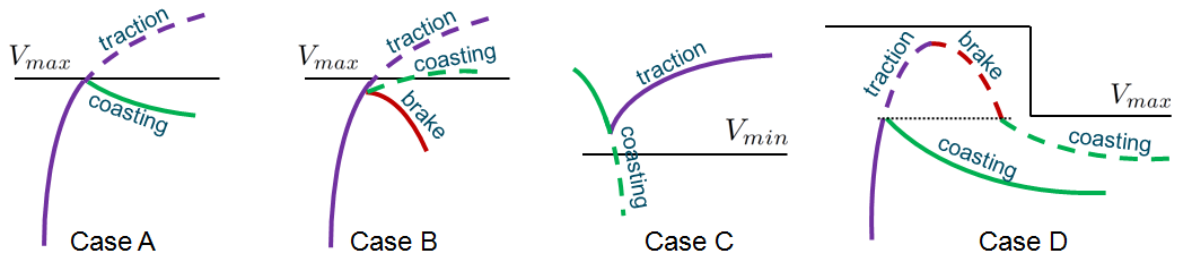

Figure 2: Feasibility check and analytical improvement

implement a trip with longest duration. Therefore, we include in the objective function not only an energy related term, but also a time related one [25]:

$$O = O_{time} + O_{energy} \to \min, \quad O_{time} = \frac{(T_{advise} - T)^2}{(\Delta^- + \Delta^+)^2}, \quad O_{energy} = \frac{E}{E_{flat-out}} + \frac{B}{B_{flat-out}}, \tag{3}$$

where O is objective function, O_{time} and O_{energy} are terms, estimating correspondingly the time and energy optimality.

It is evident that $T = T_{advise}$ provides the minimum for O_{time}, see fig. 1. By default $T_{advise} = T_{schedule} - \frac{\Delta^-}{2}$, but it is to be adjusted by the algorithm of time buffers optimization. This special form of O_{time} gravitates the solution to T_{advise}, not to $T_{schedule} + \Delta^+$ as it would be if only the energy related term in the objective function would be considered.

The energy optimality component consists of two terms. The first is consumed energy E divided by the $E_{flat-out}$ – energy spent during the flat-out trip, i.e. intense mode with minimal trip time. The second one is estimation of brake energy B divided by the one spent during the flat-out mode $B_{flat-out}$. Adding of that component to the objective function allows us to minimize the number and duration of brake modes.

Energy spent in flat-out mode is considered as maximum possible. Although, technically, one can exceed it by using an enormously inefficient solution. Nevertheless, flat-out mode gives us a perfect tool for simple normalization of the energy component of the objective function.

4 Optimization Algorithm

The typical schedule for the trip contains several fixed time points. Some points demand the stop, others – only passing the station within particular time interval defined by tolerances. In order to control the passage time for each point, the optimization process should be separately implemented between all of them instead of considering only points with stops.

For the case when the schedule is unbalanced (too large difference between the route segments' time buffers) we need some preliminary process of estimation and rearranging of the time buffers. For that purpose we move the parameter T_{advise} forward and backward within the tolerance interval for each trip part between two fixed time points.

At the first stage of the optimization process several typical solutions are analysed:

- flat-out solution,
- flat-out solution with cruising modes replaced by coasting modes,
- solution with maximum possible coasting mode duration (maximum time consumption estimation).

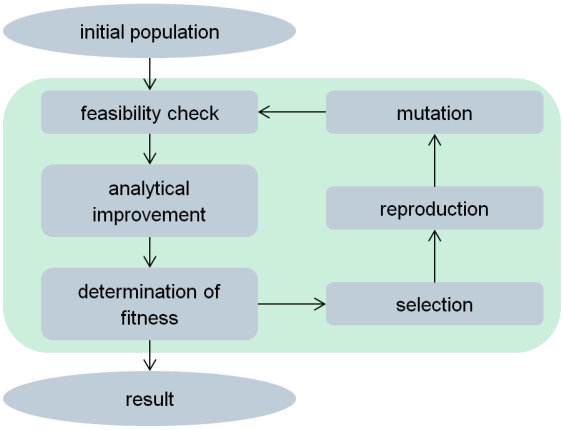

Figure 3: Genetic algorithm chart

If the time buffer after time tolerances optimization is negative, an optimization will not be started and the flat-out mode will be taken as the solution for current trip interval. Otherwise, the time consumption for the next typical solution is checked. If the time buffer is negative, the second typical solution will be taken as a result. If there is a time for improvements, the last check is implemented and if the time buffer for the third typical solution is still positive, then additional brakes will be added to the control sequence during the optimization.

Then, the optimization process based on the genetic algorithm (GA) is started, see its main stages fig. 3. The first step is filling of the **initial population** which consists of the randomly generated solutions. During that process not only controls u_i are generated randomly, but also the moment of switching the command, under assumption that the minimal time between switching the commands equals to 4 sec. This allows to analyse the whole search space of the solutions. Most of the randomly generated solutions are infeasible. But simply discarding the unsuitable solution leads to enormous computation time due to the low value of the relation "feasible solutions / all generated". Therefore a special procedure is necessary to generate feasible random solutions more frequently.

For that purpose, a feasibility check process corrects the solutions if the velocity constraints V_{max} (fig. 2, cases A, B) or V_{min} (fig. 2, case C) are violated. For the V_{max} violation the coasting or brake is added to the control sequence. In case if V_{min} is reached the traction mode is switched on. Secondly, the feasibility check process tracks the time consumed by a randomly generated solution. If the resulting time misses the tolerance interval $[T_{schedule} - \Delta^-; T_{schedule} + \Delta^+]$, then some random controls are added to the control sequence till the solution becomes feasible or the allowed number of attempts is over. During that process the flat-out solution with minimal time consumption allows to have an estimation of the minimal time needed to get to the destination with current velocity constraints.

Finally, an algorithm of analytical improvement attempts to optimize new solutions using several heuristics, see an example in the fig. 2, case D. Starting from the shortest switches, it iteratively changes the control sequence parts with frequent re-switching traction–brake to coasting mode while the solution is feasible and only if the fitness function has not gotten worse due to that improvement.

As a result, the constructed population is a number of feasible and analytically enhanced solutions represented by the sequence of commands for the train. Then, using a train model described in Section 3.1. we estimate the energy and time consumption for each solution in population. The objective function (or fitness function in terms of the genetic algorithm) is computed according to eq. (3).

The next step is **sorting** of the solutions by fitness and **selecting** of the best or elite members in the population to save them till the next iteration while the worst of them are to be deleted. After that the freed place in the population is filled by the solutions obtained by one-point **crossover**, see fig.4. Here a random mutation can happen, which is in fact an addition of a random control: cruise, traction or coasting or replacing one of the previously defined controls, see fig.6. If the time buffer was too big, together with these commands, an additional brake can also be added during the mutation.

New "children" are checked and corrected by the feasibility check algorithm. From this point the algorithm repeats the iteration until the necessary number of steps is reached.

Further, we continue the optimization for the next trip part and repeat these steps.

At the end of a process we have a set of the solutions which provide the minimum for the objective function. In other words, we have energy efficient train control sequence.

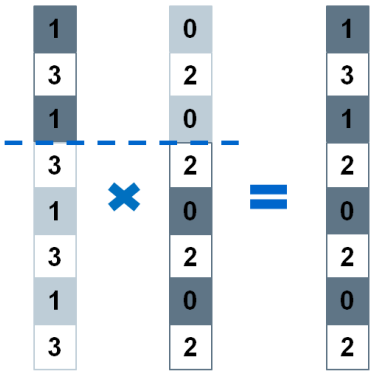

Figure 4: Crossover of two solutions

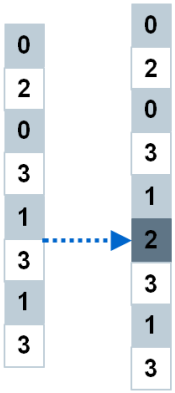

Figure 5: Mutation by adding a control

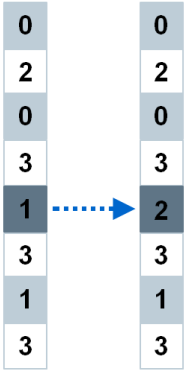

Figure 6: Mutation by replacing a control

5 Results

In order to implement the optimization algorithm described above a C++ program was developed. Estimated time of computations for a 320 km trip is about 40 seconds with Intel Core i5 @ 2.77 GHz, 8 Gb RAM.

An example of the solution for the first 15 minutes of a trip with a real schedule is depicted in the Fig. 7, 8. Despite the fact that the schedule

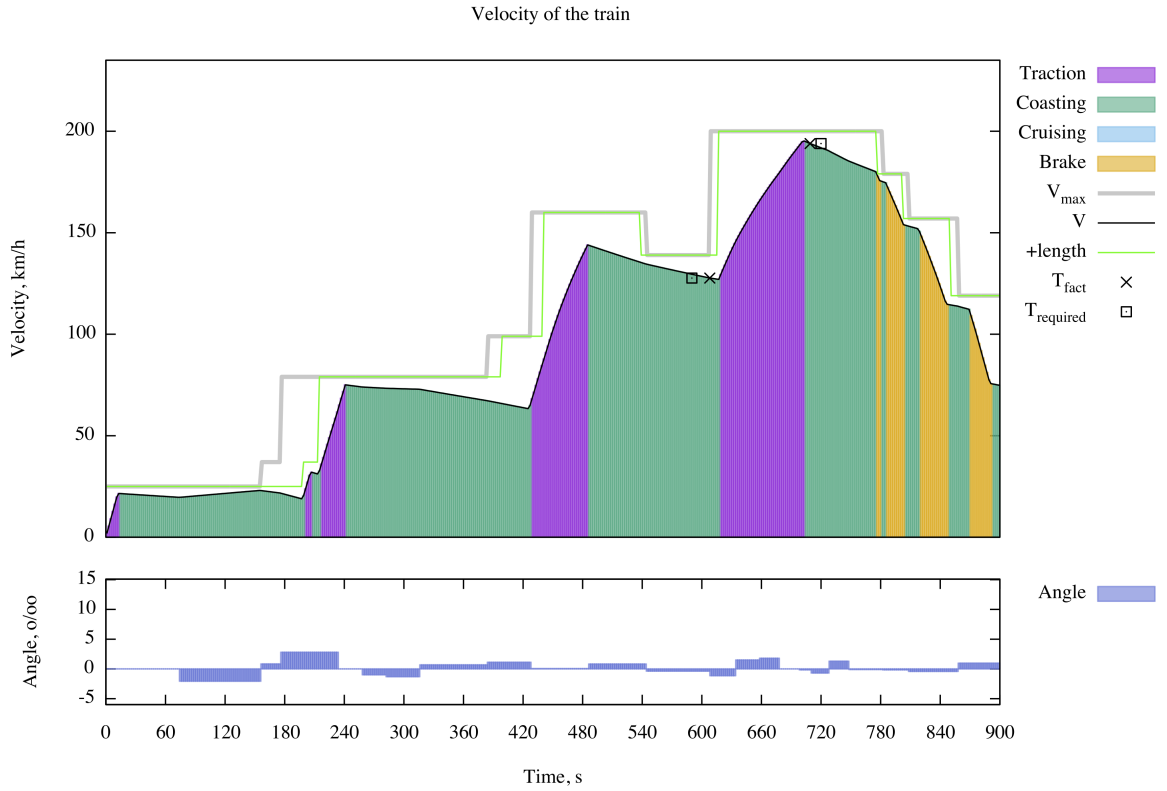

Figure 7: Velocity profile and control modes

is tough, this solution uses the sequences "traction mode → long coasting mode" one after another. Brakes are used only for satisfying the velocity constraints, which is very reasonable. The train is late at the first station (the cross is to the right from the box), nevertheless, it arrives early at the next one (the cross is to the left from the box) without use of the flat-out mode. Cruise mode is not used here at all. Track profile advantages are utilized to hold the speed during the coasting mode: see the interval between 60 and 210 seconds. Here the algorithm does not choose to use brakes or traction, despite the significant change of the speed. The time interval between 420 and 480 seconds also worth mentioning. An algorithm avoids use of a short traction modes here, replacing that with one traction turned on at the end of the 100 km/h constraint and turned off at the middle of the next constraint – 160 km/h. During this speed up the train perfectly fits all the requirements, taking into account its own length (see the green line).

In the optimization process 20 iterations were used. The greater number of iterations would be inefficient because the subsequent steps will lead to diversity reducing in the population whereas no significant improvement of the objective function will be earned. The population size was chosen as a compromise between the computation time and accuracy of the results, i.e. it shall provide the same final solution after multiple experiments. The 200 members population satisfies both requirements.

6 Conclusion

The developed algorithm of automated train control produces an intelligent control sequence which provides an energy efficient trip. The track profile inclines are taken into account, consumed and recuperated energy is computed. There is a capability of consideration of weather conditions and any other additional constraints and circumstances.

We still consider only two possible positions of the traction and brake lever – 0 and 100% intensity. This constraint seems to be reasonable according to the hypothesis that the energy efficient driving means that traction is used for short periods of time but with 100% intensity, where it is possible. However, consideration of several intermediate levels is one of the directions of improvement.

The design of the autopilot with a feature of energy optimality is under discussion and consideration for now. Rough theoretical estimations have shown savings of consumed energy up to 14.5% as opposed to manual driving. Actual values will be computed after the first field trials which are planned for the near future. The authors are looking forward to any reviews and comments.

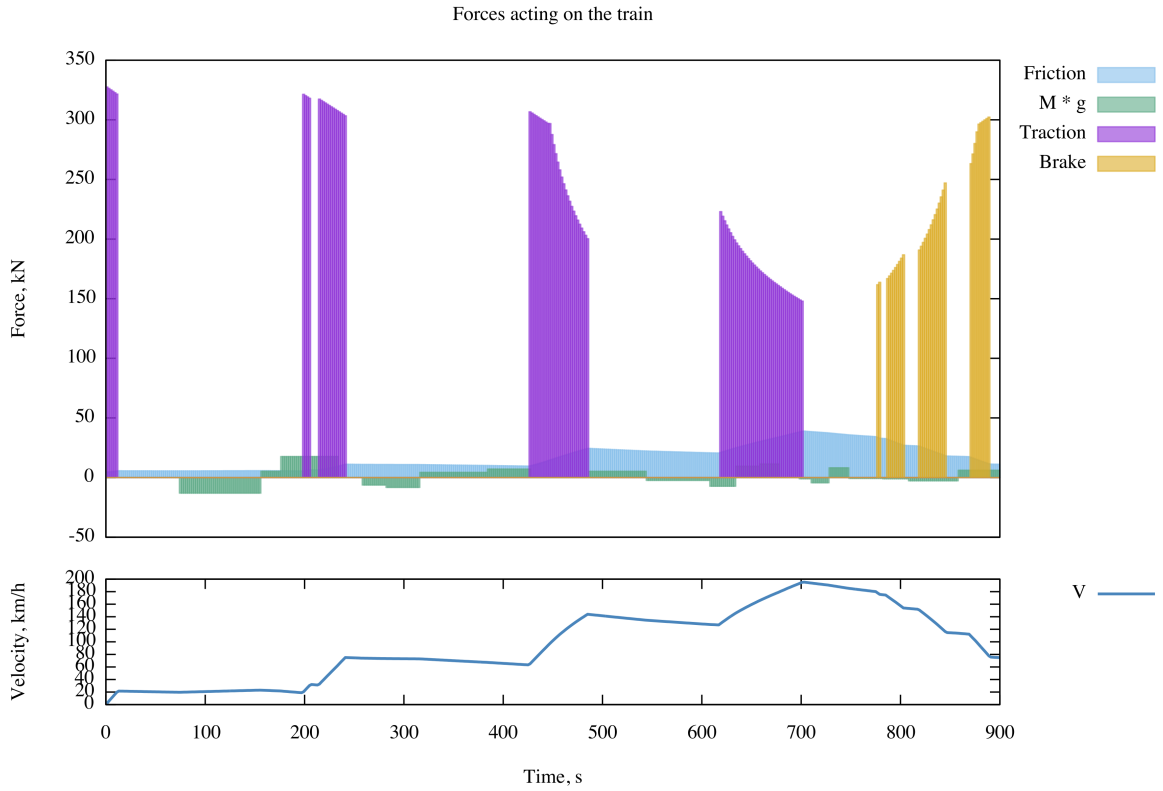

Figure 8: Forces acting on a train

References

[1] Kondo K., 2010. Recent energy saving technologies on railway traction systems. IEEJ Transactions on Electrical and Electronic Engineering 5, 298–303.

[2] Yang J., Jia L., Wei X., 2014. A review on intelligent control for energy-efficient train operation. Proc. of the 11th World Congress on Intelligent Control and Automation, China, 5160–5169.

[3] Milroy, I. P., 1980. Aspects of automatic train control. PhD thesis, Loughborough University, UK.

[4] Howlett P. G., 1984. The Optimal Control of a Train. University of South Australia, Study Leave Report.

[5] Asnis I.A., Dmitruk A.V., Osmolovskii N.P., 1985. Solution of the problem of the energetically optimal control of the motion of a train by the maximum principle. USSR Coput. Maths Math. Phys. 25(6), 37–44.

[6] Benjamin B.R., Milroy I. P., Pudney P.J., 1989. Energy-efficient operation of long-haul trains. In Proc. 4th Int. Heavy Haul Railway Conf., Institute of Engineers of Australia, 369–372.

[7] Howlett P.G., Milroy I.P., Pudney P.J., 1994. Energy-efficient train control. Control Enginneering Practice 2, 193–200.

[8] Albrecht A.R., Howlett P.G., Pudney P.J., Xuan Vu, 2013. Energy-efficient train control: from local convexity to global optimization and uniqueness. Automatica 49(10), 3072–3078.

[9] Sicre C., Cucala A.P., Fernandez A., Lukaszewicz P., 2012. Modelling and optimising energy efficient manual driving on high speed lines. IEEJ Transactions on Electrical and Electronic Engineering 7, 633–640.

[10] Howlett P. G., 2000. The Optimal Control of a Train. Annals of Operations Research 98 (1), 65–87.

[11] Matsuura G., 2014. Optimal train speed profiles by dynamic programming with parallel computing and the fine-tuning of mesh. Computers in railways XIV 135, 767–778.

[12] Miyatake M., Ko H., 2010. Optimization of Train Speed Profile for Minimum Energy Consumption. IEEJ Transactions on Electrical and Electronic Engineering 5(3), 263–269.

[13] Sicre C., Cucala A.P., Fernandez-Cardador A., 2014. Real time regulation of efficient driving of high speed trains based on a genetic algorithm and a fuzzy model of manual driving. Engineering Applications of Artificial Intelligence 29, 79–92.

[14] Boschetti G., Mariscotti A., 2014. Optimizing the Energy Efficiency of Electric Transportation Systems Operation Using a Genetic Algorithm. International Reiew of Electrical Engineering 9(4), 783–791.

[15] Besinovic N., Quaglietta E., Goverde R. M. P., 2013. A simulation-based optimization approach for the calibration of dynamic train speed profiles. Journal of Rail Transport Planning & Management 3, 126–136.

[16] De Martinis V., Weidmann U. A., 2014. The evaluation of energy efficient solutions in train operation: a simulation-based approach. 14th Swiss Transport Research Conference.

[17] Kim K., 2010. Optimal train control on various track alignments considering speed and schedule adherence constraints. PhD thesis, New Jersey Institute of Technology.

[18] Wei L., Qunzhan L., Bing T., 2009. Energy Saving Train Control for Urban Railway Train with Multi-population Genetic Algorithm. Information Technology and Applications 2, 58–62.

[19] Bigharaz M. H., Afshar A., Suratgar A. A., Safaei F., 2014. Simultaneous Optimization of Energy Consumption and Train Performances in Electric Railway Systems. 19th World Congress of the International Federation of Automatic Control (IFAC-2014), Cape Town, South Africa.

[20] Li X., Gao Z., Sun W., 2013. Existence of an optimal strategy for stochastic train energy-efficient operation problem. Soft Computing 17(4), 651–657.

[21] Tuyttens D., Fei H., Mezmaz M., Jalwan J., 2013. Simulation-Based Genetic Algorithm towards an Energy-Efficient Railway Traffic Control. Mathematical Problems in Engineering.

[22] Cucala A.P. , Fernandez A., Sicre C., Dominguez M., 2012. Fuzzy optimal schedule of high speed train operation to minimize energy consumption with uncertain delays and driver's behavioural response. Engineering Applications of Artificial Intelligence 25, 1548–1557.

[23] Carvajal-Carreno W., Cucala A.P., Fernandez-Cardador A., Soder L., 2015. Efficient driving algorithms for non-distributed and distributed trains with the CBTC signalling system. Models and Technologies for Intelligent Transportation Systems (MT-ITS), 418–424.

[24] Rochard B.P., Schmid F., 2000. A review of methods to measure and calculate train resistances. Proc. of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit 214, 185–199.

[25] Bocharnikov Y.V., Tobias A.M., Roberts C., Hillmansen S., Goodman C.J., 2007. Optimal driving strategy for traction energy saving on DC suburban railways. IET Electric Power Applications 1, 675–682.