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# Learning trajectory for conceptualizing the fundamental theorem of calculus using dynamic and multiple linked representations tools

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*This study was designed to evaluate and elaborate a hypothetical trajectory for learning the fundamental theorem of calculus (FTC) by high school students using dynamic and multiple linked representations tools. The study focuses on 13 pairs of 17-year-old students. Data analysis identified the learning foci involved in conceptualizing the FTC, based on which I suggest a trajectory for learning the FTC.*

**Keywords:** Learning trajectory, Fundamental theorem of calculus, technological tools.

## INTRODUCTION

How to promote students' development of new mathematical topics, especially topics whose development is unsure, is one of the significant problems facing mathematics education. One of these topics is the fundamental theorem of calculus (FTC). Many students miss out on the conceptual learning of the FTC (Thompson & Silverman, 2008; Bressoud, 2011). I suggest a hypothetical learning trajectory (HLT) for conceptualizing the FTC using multiple linked representations and dynamic tools that can help students construct an understanding of mathematical concepts. The proposed learning trajectory can make the mathematical learning process more profitable (Steffe, 2004). My aim is to evaluate and elaborate the HLT for learning FTC by high school students learning with a dynamic and multiple linked representations tool.

## THE HYPOTHETICAL LEARNING TRAJECTORY

I construct the HLT for FTC based on the functional approach for learning calculus notions. In this approach, the derivative is considered to be a rate of change function and integration is considered to be

an accumulation function. At the first level of the HLT, students are asked to conceptualize the Riemann accumulation function as a sum of products. Next, they are asked to conceptualize the accumulation function as a convergence of Riemann accumulation functions. At the third level, they are asked to conceptualize the accumulation function as a rate of change function. Finally, they are asked to conceptualize the evaluation part of the FTC.

## THEORETICAL FRAMEWORK

Simon (1995) offered the HLT as a way to explicate an important aspect of pedagogical thinking involved in teaching mathematical understanding. In particular, he described how mathematics educators can conceive the design and use of mathematical tasks to promote mathematical conceptual learning. An HLT consists of three components: (a) the students' learning goal, (b) a hypothesis about the process of the students' learning of mathematical concept, and (c) tasks to be used to promote the students' learning (Simon, 1995). The HLT perspective provides no framework for thinking about the learning process. Therefore, I used the mechanism of reflection on activity-effect relationships to explain the relationship between conceptual learning and mathematical tasks (Simon & Tzur, 2004). This mechanism provides a description of how the learner's goal-directed activity can lead to the generation of new and more sophisticated conceptions for the learner.

## METHOD

The present study explores approximately 55 hours of learning by 13 pairs of 17-year-old students. The students volunteered to participate in four after-school meetings. To study the progression processes of the

FTC, we asked students to explain and explore the possible connection between two given function graphs and the table of values. We designed the tasks to be learned by the students based on the mathematical structure of the FTC. The data for the present paper were collected from all the students who participate in the study. All the pairs of students in each session were video-recorded as they engaged in solving the task assigned to them.

## PRIMARY FINDINGS

The data analyses suggest that the HLT constructed based on the mathematical structure of the FTC helps students conceptualize the FTC. Based on the data analysis process, I detected the learning foci used by the students to find the possible connections between the graphs, and the mathematical relationships involved in the FTC. These foci can be used to elaborate the HLT for conceptualizing the FTC. In this poster, I present the elaborated learning trajectory for FTC and discuss the role of the learning focus in conceptualizing the components of the FTC. I also present the ways that students used to connect between the differentiation and integration.

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