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A framework for describing techno-mathematical fluency in beyond-school problem solving

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This study seeks to characterize the mathematical problem solving activity with digital tools that emerges from students' participation in an online mathematics competition. Using a qualitative approach, we elected the case of a 13-year-old participant aiming to understand the ways in which she interweaves her mathematical competence and her technological fluency for solving problems, using GeoGebra. Main results expose the role of the digital tool that permeates every stage of the problem solving process, since the tool is used for solving and for expressing the solution. We further propose a framework for describing the processes that may capture the interplay between mathematical knowledge and technological fluency in solving problems, termed techno-mathematical fluency.

Keywords: Beyond school mathematics, digital literacy, mathematical competitions, problem solving, techno-mathematical fluency.

INTRODUCTION

The constant immersion in a technologically pervaded world is changing the “kind of mathematical abilities that are needed for success beyond schools” (Lesh, 2000, p. 177), especially since the new and powerful tools made available are introducing “new kinds of problem-solving situations in which mathematics is useful, (...) and they radically expand the kinds of mathematical understanding and abilities that contribute to success in these situations” (p. 178). Whilst the kinds of mathematical thinking needed beyond school are shifting, the types of problem solving situations that demand some form of mathematical thinking are also changing. Furthermore, little is still known about the problem solving that occurs beyond the classroom and additional research is needed specially to understand the role of digital technologies in

such activity (English & Sriraman, 2010; Santos-Trigo & Barrera-Mora, 2007).

A glimpse on the context: The mathematical competition SUB14

SUB14^{*} is a web-based mathematical problem solving competition organised by the University of Algarve. Addressing 12-13-year-old students, it is supported by a website where the problems are published. The Qualifying consists of ten problems, each one published every two weeks. Participants may solve the problems using their favourite methods or tools, but they are explicitly required to report on their solving process offering a complete and detailed explanation of their reasoning. At this stage, the rules allow and encourage help seeking from relevant others. Participants who answer correctly to eight or more problems may attend the Final stage, which consists of a one-day tournament at the University of Algarve (see Carreira, 2012).

Our goal is to investigate mathematical problem solving with digital tools in this beyond-school competition, where participants may use their favourite digital tools but, at the same time, are required to use a mathematical stance. We report our progress on analysing how they combine their mathematical knowledge and their technological fluency in solving the competition's problems.

THEORETICAL BACKGROUND

This study is supported by a conception of inseparability between the subject and the digital tool in the development of mathematical thinking. Thus, we consider *humans-with-media* (Borba & Villarreal, 2005) as a central unit in understanding problem solving activity with technology. This metaphor brings forth the

idea that mathematical thinking is an outcome of this symbiosis between people and the digital tools they use.

Mathematical problem solving – the mainstream view

The competition poses non-routine problems, whose context is fully and clearly expressed in the statement, which allow different techniques, procedures or tools. As these problems are not aligned with the mathematics curriculum and a diversity of approaches and tools is encouraged, this problem solving activity can be seen as the development of a productive way of thinking about a challenging situation (Lesh & Zawojewski, 2007). Current frameworks consider a mathematical literate person as someone who is an *active problem solver*, in other words, who has the “capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena” (OECD, 2013, p. 5).

Looking for a way of explaining student’s and expert’s problem solving performance, Schoenfeld (1985) proposed a model comprised of four dimensions: basic resources, heuristics, control, and belief systems. The processes followed by the solvers were organized into five stages: *read* – time spent “ingesting the problems conditions” (p. 297); *analysis* – attempt to fully under-

stand the problem “sticking rather closely to the conditions or goals” (p. 298) that may include a selection of ways of approaching the solution; *exploration* – a “search for relevant information” (p. 298) that moves away from the context of the problem; *planning and implementation* – defining a sequence of actions and carrying them out orderly; *verification* – the solver reviews and assesses the solution. While paper and pencil were the predominant tools used in Schoenfeld’s studies, today’s wide dissemination of powerful technological tools is raising new queries, namely if and to what extent these frameworks still account for the mathematical problem solving proficiency in the presence of digital tools (Santos-Trigo, 2007, Santos-Trigo & Camacho-Machín, 2013).

Bringing together mathematical and technological literacies

In reporting studies that sought to identify the mathematical competencies needed in several workplaces, Hoyles, Noss, Kent and Bakker (2010) highlight an interrelationship of the technology use and the mathematical skills of the workers, proposing the notion of *Techno-mathematical Literacies* as encapsulating both the technological and the mathematical skills needed within those specific work contexts.

Debates concerning the digital skills needed in our daily activities are undergoing. The European project

Process	Problem or Digital Task
Statement	State clearly the problem to be solved or task to be achieved and the actions required.
Identification	Identify the digital resources required to solve a problem or complete a task.
Accession	Locate and obtain the required digital resources.
Evaluation	Assess the objectivity, accuracy, reliability and relevance of digital resources.
Interpretation	Understand the meaning conveyed by a digital resource.
Organisation	Organise and set out digital resources in a way that will enable the solution of the problem or achievement of the task.
Integration	Bring digital resources together in combinations relevant to the problem or task.
Analysis	Examine digital resources using concepts and models which will enable solution of the problem or achievement of the task.
Synthesis	Recombine digital resources in new ways which will enable the solution of the problem or achievement of the task.
Creation	Create new knowledge objects, units of information, media products or other digital outputs which will contribute to the solution of the problem or achievement of the task.
Communication	Interact with relevant others whilst dealing with the problem or task.
Dissemination	Present the solutions or outputs to relevant others.
Reflection	Consider the success of the problem-solving or task-achievement process, and reflect upon one’s own development as a digitally literate person.

Table 1: Processes of digital literacy (Martin & Grudziecki, 2006)

DigEuLit developed a theoretical framework addressing the meaning and operationalization of “digital literacy” by describing the activity of a digital literate person when dealing with a digital task (Martin & Grudziecki, 2006). Those processes (Table 1) can be summarized as actions required before solving the problem (stating, identifying, accessing, evaluating, interpreting, organizing), while producing the solution (integrating, analysing, synthesising, creating, communicating), and actions that occur afterwards (disseminating and reflecting).

To some extent, this list of processes resembles the problem solving stages proposed by Schoenfeld (1985). Due to the rules of the competition, a mathematical problem may be considered as a digital task that requires a number of technological skills, so we conjecture that these two frameworks can provide the necessary level of detail for describing problem-solving-with-technologies. We, therefore, ponder an association of the *stages* and processes: *read* – statement (i.e., appropriation of the situation and the conditions in the problem); *analyse* – identification, accession, evaluation, interpretation (i.e., initial attempt to comprehend what is at stake, namely the mathematics that may be relevant and the tools that may be necessary); *explore* – organization, integration, analysis (i.e., the quest for a combination of mathematical and technological tools within a plausible strategy); *plan and implement* – synthesis, creation, communication (i.e., carrying out the outlined strategy); and *verify* could be complemented by dissemination and reflection since, at this point, there isn’t an overlap between the two frameworks. Thus, the notion of *techno-mathematical fluency* stresses the need to be fluent in a language that entails mathematical and technological knowledge, promoting the skilful use of digital tools, and the efficient interpretation and communication of the mathematical solution produced.

Dynamic Geometry Software in problem solving

Constructing figures and investigating their properties are two of the most commonly known affordances of Dynamic Geometry Software (DGS). In particular, one of the features of DGS is that it “visually make[s] explicit the implicit dynamism of thinking about mathematical geometrical concepts” (Leung, 2008, p. 135) embedded in a challenging task. However, such dynamic environments are not only helpful in visualizing geometric concepts and understanding

rules, but also in producing conjectures and generalizations, and finding connections amongst concepts (Baccaglioni-Frank & Mariotti, 2010; Jones, 2000). When tackling model eliciting problems, students often perceive and make use of the affordances of the software in order to develop a conceptual model of the situation. They undertake a construction, revealing how they are interpreting the problem and depicting the mathematics concealed; they explore and investigate properties (Iranzo & Fortuny, 2011).

Carreira, Jones, Amado, Jacinto and Nobre (to appear) have identified six general affordances of GeoGebra for solving geometrical problems within the competition SUB14: immediate constructions, measurement, referential constructions, setting properties, constructions using parameters or variables, and drag and explore. Claiming that the perception of the affordances of GeoGebra is crucial to this problem solving activity, the researchers observed how several students tackling the same problem with GeoGebra produce different digital solutions that comprise qualitatively different conceptual models. We now argue that such differences may be explained by the solvers’ latent techno-mathematical skills, regardless of where the mathematical and technological knowledge were learnt or developed.

RESEARCH METHODS

Our main goal is to develop a deep understanding of the interplay among mathematical knowledge and technological fluency during the development of the solving process within SUB14. Thus, we developed an interpretative study where the research methods were steered by qualitative techniques for gathering, organizing and analysing empirical data (Quivy & Campenhout, 2008).

We report the case of Jessica (fictitious name), a participant whose productions stood out due to the sophisticated use of technology, namely GeoGebra, for solving these problems (Jacinto & Carreira, 2013). Data include the solutions sent by Jessica in two editions of SUB14; an in-depth interview with Jessica, audio and video recorded, focusing her problem solving activity in the classroom and at SUB14, and asking her to remember and retrace solutions submitted to the competition; and two specific solutions, where she used GeoGebra, were also selected for a deeper analysis. Whilst the data from the interview provide a view of Jessica as a

Consider a sequence of squares of sides 1, 2, 3, 4, ... cm, arranged so that they are connected to each other, as illustrated on the right. Once together, cut up all the squares by a line drawn from the bottom left corner of the smaller square to the upper right corner of the larger square. What is the area above the cut line, if the sequence has 8 squares?

Don't forget to explain your resolution process!

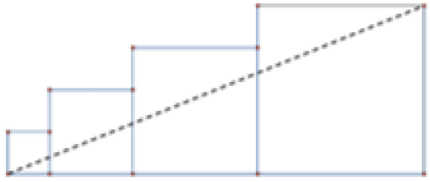


Figure 1: The problem “United and Cropped”

student, a problem solver and a technology user, the GeoGebra’s construction protocols and the written explanations shed light upon the interplay between mathematical knowledge and technological fluency. As the rules of the competition explicitly require to document the solving processes, we assume that the final versions of the constructions performed with GeoGebra account for the interplay between Jessica’s technological and mathematical knowledge to solve the problems and express her reasoning.

The several types of data were organized using NVivo®, where audio and video data were transcribed. The analysis followed an interpretative perspective providing a holistic description of the case, by combining Jessica’s perception of her own problem solving activity (interview), with the analysis of the participant’s productions (GeoGebra file), enlightened by the theoretical ideas discussed above.

THE CASE OF JESSICA

Jessica is a 13 years-old girl who engaged in SUB14 during her 7th and 8th grades. Her answers to the problems are always on time, she describes and properly justifies her processes in a clear language. She developed a particular interest on GeoGebra that stemmed out from her school experience, since her teacher used it often as a way to present geometrical contents. Despite being teacher-centred, it has driven Jessica to download, install and explore GeoGebra at home, independently.

Jessica: As I said, we use technology a lot. We have a board... a white board, and we also have an interactive board. We used GeoGebra very often when we were studying geometry and geometric transformations.

Researcher: When you say “we used”, you mean the teacher?

Jessica: Precisely. And we watched it.

When asked to recall and retrace her solution to the problem “United and Cropped” (Figure 1), she claimed to enjoy solving geometry problems because of the possibility of improving the solutions’ graphical display, afforded by GeoGebra.

Jessica: I think I went straight to GeoGebra. I knew it had something to do with geometry. I realise it was a triangle (...) and that by rearranging it in a simpler manner all I had to do was calculate the whole area and then subtracting the area of this triangle, which is easy: base times height divided by two. And then I thought... Oh, great! Geometry! I’m getting it neat!

Jessica usually resorts to a notepad, coloured pens, a calculator and the computer. Initially, she thinks that GeoGebra only affords “dressing up” the solution that she finds by using paper-and-pencil but, later, she acknowledges that manipulating the construction also led her to a powerful understanding of the problem.

Jessica: Hum... usually I look for the notepad and a pen, then [go to] Word and then I always... well I always use GeoGebra or some other software to add something to the text, for presenting a more complete work.

Researcher: So... you use it [GeoGebra] only after you solved the problem?

Jessica: Yes, but... it depends. If GeoGebra or some other tools would help me understand the problem, then I’d use it firstly and later I’d move to Word.

The image shows a large square divided into 14 smaller squares, coloured in yellow, of different but integer dimensions, and 1 white rectangle, also of integer dimensions. The white rectangle has an area of 30 464 cm². Which is the area of the larger square?

Don't forget to explain your resolution process!

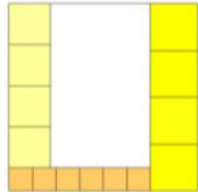


Figure 2: Statement of the problem "A divided square"

Researcher: Ok, so you also use them while you're still looking for the solution...

Jessica: Yes, for instance, in this case [the problem United and Cropped] I started by going to GeoGebra to understand it properly, and then I discovered 'Oh, that is a triangle right there, therefore I have to subtract the area of that triangle'. In that case, I started with GeoGebra for a better understanding.

Her solving activity starts outside the screen but she easily recognizes that digital tools afford powerful approaches to the SUB14's problems. The following section reports Jessica's work with GeoGebra while solving another geometry problem.

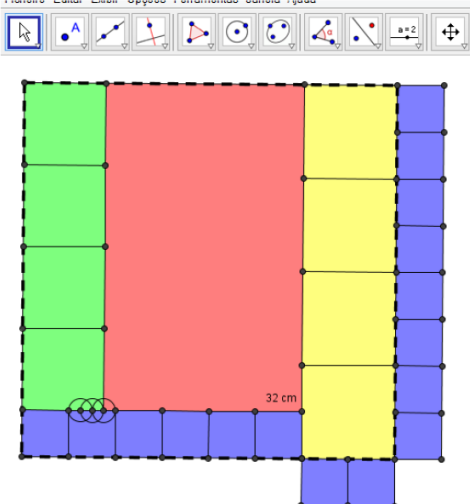
The problem "A divided square"

Jessica's solution (Figure 3) combines the construction of the figure presented in the statement and a written explanation where she presents a 'label' that helps in interpreting the image and her problem solving processes, including finding the area. A closer analysis of the 'construction protocol', which allows tracking steps of the construction, reveals that GeoGebra's role goes beyond 'embellishment'.

Resorting to immediate and referential constructions she represents the larger square that supports the construction: draws two perpendicular lines and a circle centred at their intersection and a radius of length CD. Then, she constructs four squares on the right by finding midpoints, using parallel and perpendicular lines and their intersections. Finally, she builds four squares and colours them in yellow (Figure 4).

As for the lower squares (Figure 5), Jessica marks R as the midpoint of FQ, then uses a central reflection of the point F over R and obtains F'1, which coincides with Q. She marks I' as the central reflection of I over F'1 and, using another central reflection of F over F'1, marks F' and constructs the segment I'F'1. She then marks S as the midpoint of I'F' and proceeds by using circles with given centre and radius, finds intersections and midpoints, and drawing parallel lines to complete the representation of the lower squares. Similarly, she constructs the remaining squares on the left side (Figure 6).

Finally, by setting properties, she colours polygons, adds several squares along the exterior of the initial square and some circumferences whose centres divide the side of a smaller square in four parts (Figure 3). These items emphasise a visual perception of the



The boundaries of the larger square are dashed. I have changed the colours of the smaller squares so I can differentiate them easily. Same colour squares have the same area.

yellow square side = 1/4 larger square side
 blue square side = 1/8 larger square side
 blue square side = 1/2 yellow square side
 green square side = 1,75 the blue square side
 Let us assign x to the side of the blue square.
 Length of the red rectangle = $4,25 x$
 Height of the red rectangle = $7 x$
 Area of the red rectangle = $4,25 x \times 7x = 30\ 464\text{cm}^2$
 $2975 x^2 = 30464$
 $x^2 = 30\ 464 \div 2975 = 1024$
 $x = \sqrt{1024} = 32 = \text{length of the side of the blue square}$
 Length of the red rectangle = $32 \times 7 = 224$
 Side of the larger square = $32 + 224 = 256\text{cm}$
 Area of the larger square = $256\text{cm} \times 256\text{cm} = 65\ 536\text{cm}^2$
 Answer: The area of the smaller square is 65 536 square centimetres.

Figure 3: Solution of the problem "A divided square"

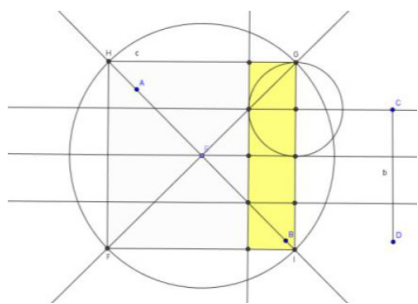


Figure 4: Constructing the initial squares

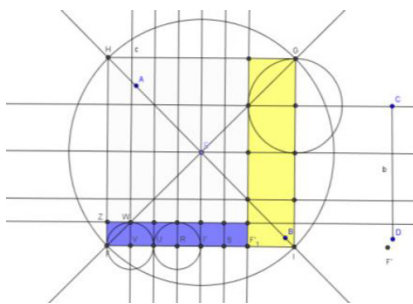


Figure 5: Constructing the lower squares

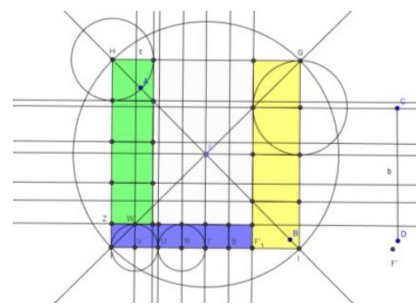


Figure 6: Constructing the left squares

existing relations between several lengths of the geometrical figures. On the right, a label helps interpreting the construction and establishing numerical relations between lengths of the sides of the squares. The unknown is defined as the length of the blue square and, using those relations, she formulates an equation that will provide the measurement that is missing. With this value she determines the length of the side of the larger square and, then, its area.

This case illustrates how GeoGebra is crucial at several stages of the problem solving activity: it is not only affording the construction but, most importantly, the construction activity is allowing to uncover the relations between the geometrical objects, it is transforming what is invisible and concealed inside the proposed figure, into visible and usable ideas for the development of a way of solving this problem and expressing the solution. In fact, the constructions become part of the reasoning, of the process and the solution itself. This can be interpreted as an instance of the problem solving activity of a student-with-GeoGebra (Borba & Villarreal, 2005).

REFRAMING TECHNO-MATHEMATICAL FLUENCY

Taking the problem solving stages proposed by Schoenfeld (1985) and the processes of the digital literacy framework developed by Martin and Grudziecki (2006) as leverage, we have set an analytical tool to account for the ways a participant deals with digital tools whilst solving a mathematical problem and expressing its solution.

Jessica starts by skimming the mathematical topic enclosed in the problem, such as geometrical notions, rules, procedures, and recognizing GeoGebra as a key digital resource (*Read/statement*). The following stage (*Analyse*) is patent through the *identification* of a mathematical repertoire and a technological repertoire

(geometry and GeoGebra's affordances) that are due to her previous knowledge about them and her *access* to them. Moreover, Jessica's choice relies on her *assessment* of the techno-mathematical resources and her *interpretation* of the techno-mathematical outcomes.

She then *Explores* ways of organizing different resources such as notepad, coloured pens, calculator, GeoGebra, text and image editor, e-mail, and several mathematical resources such as properties of parallel and perpendicular lines, circumferences and their representations, areas, algebraic expressions, and combines them in a relevant way to the development of her strategy (*organisation, integration, and analysis*).

Based on the constructions and their manipulation, she *Plans* and *Implements* her strategy recombining the techno-mathematical resources (*synthesis*) in order to produce new knowledge objects: strategies, representations, conceptual models (*creation*). During these processes, she may ask for the assistance of relevant others (like her teacher) to proceed in finding the solution (*communication*). It is important to note that the activity reported by Jessica and the analysis of the construction protocol suggest that the understanding of the problem and the decision on the actions necessary to solve it are not limited to the initial stage but it develops throughout the analysis and exploration stages and it is deepened during the construction and manipulation of the geometrical figures.

The last stage consists of reviewing the process and the solution (*Verify*) but it also includes presenting the solution to others, in this case, the GeoGebra construction and a detailed explanation of the procedure (*dissemination*). As for the personal *reflexion* on the success accomplished during the problem solving activity, there are no other concrete evidences to support it than the fact that Jessica has decided to present this solution to the judges of the competition.

CONCLUDING REMARKS

The two frameworks selected were meant to characterize the problem solving stages and the processes of digital literacy and, as such, their combination seems to offer powerful tools to approach a description of the latent processes underlying the notion of techno-mathematical fluency. However, some improvements must be pondered. Firstly, the processes of digital literacy are a set of actions that occur in a relatively ordered sequence, unlike the stages of problem solving that, as Schoenfeld (1985) disclosed, are flexible enough to describe failed attempts or new appropriations of the problem. So, the descriptors of the techno-mathematical fluency involved in problem solving within SUB14 must comply with this flexibility. There may also be an overstatement of the digital literacy processes, particularly in the level of detail included in the original framework. Our comprehensive knowledge about the competition and the participants allow us to assume that: i) they often choose the tools they are most familiar with, namely everyday digital tools available in their home environment, hence accession, interpretation and evaluation could result in some benefit if they were agglutinated in a broader process, bringing together knowledge and decisions about the digital resources; ii) the communication process, which relates to possible help seeking, permeates other stages of the problem solving activity, namely in understanding the problem or in devising a path; iii) the verification of the solution is not clearly addressed in the digital literacy processes, but it is a very important metacognitive process for assuring the completeness of the solution; iv) the dissemination process, not considered in Schoenfeld's model, is extremely important given the competitive nature of this activity and the unavoidable fact of having to submit a solution to those who are responsible for their acceptance and from whom a return is expected; v) solving and expressing are inter-related activities that are often inseparable (Jacinto, Nobre, Carreira, & Amado, 2014).

In light of the data and the theory, the notion of techno-mathematical fluency that emerges from the 'problem solving with technologies' activity is a useful way of accounting for the intertwining of mathematical knowledge and technological fluency. Future developments will concentrate on the refinement of the framework descriptors based on further analy-

sis of other participants' problem solving activities, observed within the same informal learning context.

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