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Digital interactive assessment in mathematics: The case of construction e-tasks

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Our research focuses on the e-assessment of challenging calculus construction e-tasks designed to function as a dynamic interactive environment of multiple linked representations (MLR) that provide feedback to the learner. A construction e-task requires students to use technological affordances to construct examples that satisfy specific conditions. The e-task is checked automatically and intermediate actions and submitted answers are reported. We present here an example of a construction e-task and report on a pilot experiment designed to elucidate the role of the dynamic MLR environment in solving and assessing construction e-tasks. Specifically, we examine the student's submitted solutions and analyse whether it helps reflect the reasoning behind the answer.

Keywords: Calculus, assessment, representations, technology, examples.

THEORETICAL BACKGROUND

The Joint Information Systems Committee (JISC)¹ defines e-assessment as the end-to-end electronic assessment process that uses information and communications technology (ICT) to present the assessment activity and to record the responses. By the term e-assessment we refer to forms of assessment that are created to be delivered, answered, managed, and marked mostly automatically, using ICT. We are especially interested in e-assessment of conceptual understanding of the content of calculus by high school students. In the present study we define an e-task as a technology-aided mathematical activity that engages students in seeing and doing mathematics. Scalise and Gifford (2006) introduced a categorization of innovative item types that may be useful in e-assessment. These types are based on categories of ordering involving successively decreasing response constraints, from fully selected responses (as in conventional multiple-choice questions) to fully constructed one (as in

the traditional essay). The latter can be a challenge for computers to analyze meaningfully, even using sophisticated tools. According to the authors' review, technology makes a limited contribution to tasks that attempt to assess higher-order mathematical skills. Our research focuses on designing and studying challenging e-tasks in a dynamic MLR environment that provides reflecting feedback through e-assessment. We are particularly interested in high-school level e-tasks, which until recently have been limited mostly to closed multiple-choice questions.

The dynamic linkage of MLRs has been used as a design strategy that attracts attention to the relation between different representations. Yerushalmy (1997) and Yerushalmy and Schwartz (1999) have incorporated these strategies in the design of e-tasks in the areas of functions and calculus. The ability of digital tools to translate instantaneously across representations enable students to exhibit and evaluate actions in more than one representation system. Cognitive and pedagogical research suggests that appreciating the manner in which multiple representations are related is not automatic (e.g., Tall, 1991). Using multiple representations supports and requires tasks that involve decision-making and other problem-solving skills, such as estimation, selecting a representation, and mapping the changes across representations. E-tasks involving MLRs provide feedback to students that reflects the process of inquiry during the examination (reflecting feedback).

In computer-based instruction, feedback is any message or display that follows the learner's action or response. Vasilyeva and colleagues (2007) provided an overview of feedback studies and classified feedback into several types. For more than two decades, Yerushalmy and colleagues have explored the main differences between reflecting and judgmental feedback. Reflecting feedback provides immediate feedback to students about

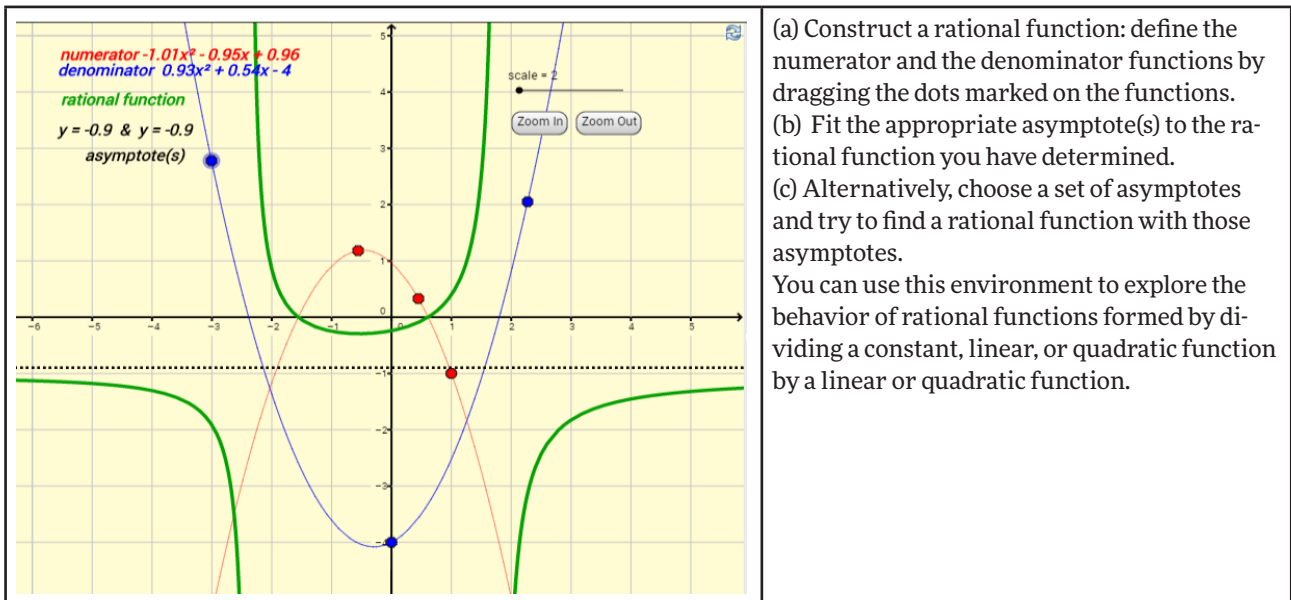


Figure 1: Dynamic MLR learning object: rational functions and asymptotic behavior, <http://tube.geogebra.org/student/m97472>

their actions in multiple linked representations, and offers students the opportunity to judge and reflect upon the action taken. By contrast, judgmental feedback provides a short right/wrong answer, without other representations or explanations. Yerushalmy (1991) compared three groups of students performing symbol manipulations in algebra: one group received a reflecting graph feedback, the second judgmental feedback without a graph, and the third group used a symbolic manipulation aid. Yerushalmy found that judgmental feedback had a positive effect on the process, but that the students still lacked the motivation or ability to complete the tasks correctly. The effect of feedback was more significant on the first and third groups than on the second one (the manipulator aid motivated the students to obtain a correct product, and the graph feedback motivated them to make algebraic investigations).

Examples may be used for assessment in several ways. One obvious use is in refuting conjectures, either by citing standard counter-examples or by constructing new ones. Higher-level skills are needed when constructing instances of mathematical objects that

satisfy certain properties, because typically there are many correct solutions but no general method by which such a solution can be constructed (Sangwin, 2003). When students are asked to create their own examples, they experience the discovery, construction, or assembly of objects and of their relationships (Liz, Dreyfus, Mason, Tsamir, Watson, & Zaslavsky, 2006). Occasionally, students create an example based on a ready-made example, which helps them reach the correct answer. A major affordance of technology is that it allows the easy creation of many different examples.

Figure 1 shows the GeoGebra applet, a challenging dynamic MLR learning object (<http://tube.geogebra.org/student/m97472>). To support inquiry learning of rational functions and of their asymptotic behavior, this object provides graphic and symbolic representations of the numerator and denominator (linear and quadratic functions of a single variable) and of their quotient. Users create instances of the rational function (Figures 1 and 2), explore and discover which functions have specific types of asymptotes, how many, and what are the reasons for it. During

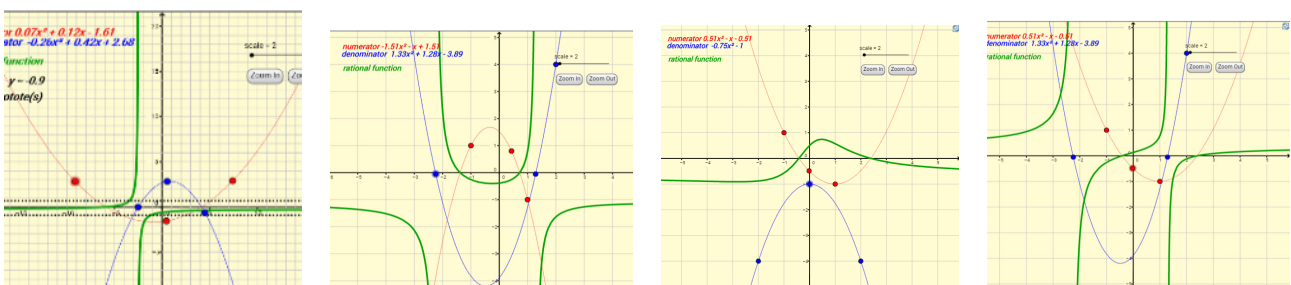


Figure 2: Examples/instances of rational functions of two quadratic functions

the exploration, the GeoGebra applet provides many instances of the same object (Figure 2), and students can evaluate their actions, reflect upon the feedback they receive, and change their conjectures accordingly (Santos-Trigo & Camacho-Machín, 2013).

GOALS AND FRAMEWORK

The current research is part of the Digital Interactive Assessment project at the University of Haifa (<http://assess.gigaclass.com>). A main challenge for the developers is to design e-tasks that on one hand invite opportunities for active personal learning and on the other set limits that pertain either to pedagogy or content (Watson & Mason, 2006). Our research arises from the challenge to design e-tasks that faithfully assess future learning and teaching. We are also seeking to create e-tasks that check automatically not only the correctness of the answer but also its justification, without the need for human check of written explanations. Because examples play an important role in justifying answers (e.g., Buchbinder & Zaslavsky, 2013) and can be checked automatically, we decided to include them in the present study. In this paper, we focus on construction tasks requiring students to construct examples that satisfy specific conditions. These e-tasks can be used to generate examples and provide tools for exploration. The tools are interactive MLR artefacts designed for mathematical experimentation, and may be part of the GeoGebra applet (the words tools and artefacts are used without any theoretical connotation; the tools in the e-task shown in Figure 3 are the value table, the coordinate system, and the

symbolic input line). Each designed e-task appears as multimodal text. The designed tools are suitable for experimentation, and the answers submitted are checked automatically, almost without human intervention. The answers appear as live screenshots of the relevant representations within the tools, for example symbolic expressions, tables of values, and graphs. Therefore, they can be checked automatically. Figures 3 and 4 contain an example of a construction e-task (<http://tube.geogebra.org/student/m440111>) created by the authors. The e-task, which was inspired by the applet shown in Figure 1, also deals with vertical and horizontal asymptotes and has three representations: numeric (the value table), graphic (coordinate system), and symbolic (input line for the algebraic expression of the function). Students are asked to construct the requested function by typing an appropriate symbolic expression. Before the exercise, students receive instruction about using the numeric table (by typing x and deltaX as shown in Figures 3, 4), dragging points to create different instances of functions, using sliders (Figures 1, 2) using the zoom in and zoom out buttons (Figures 1, 3, 4), and about other technical issues. At this initial stage of the study, we intend to follow the students' reasoning mainly by asking them to submit their answers and to highlight the relevant component that justifies the answer. To make this possible, the task must provide multiple tools or operations for sending the answer. Moreover, students must be able to choose an appropriate numeric interval or a point in the table, or indicate on the graph the relevant segment. The value table in the e-task shown in Figure 3 is initially empty. Students

In each of the above parts you are required to submit a screenshot that supports your answer. You may write further explanations, if necessary. The diagram allows construction of the function in the form: $h(x) = \frac{f(x)}{g(x)}$. Define the numerator function $f(x)$, the denominator function $g(x)$, and the asymptotes by entering their expressions. You may use the value table and change x, deltaX.

Construct a function $h(x) = \frac{f(x)}{g(x)}$.

- 1) With one vertical asymptote $x = 4$ and one horizontal asymptote $y = 2$
- 2) With two vertical asymptotes $x = 4, x = 2$
- 3) With two vertical asymptotes $x = 4, x = 2$ and one horizontal asymptote $y = 2$
- 4) With two horizontal asymptotes $y = 2, y = -2$.

x	h(x)	y_1(x)	y_2(x)
-14	0	0	0
-4	0	0	0
6	0	0	0
16	0	0	0
26	0	0	0

Figure 3: Example of construction e-tasks

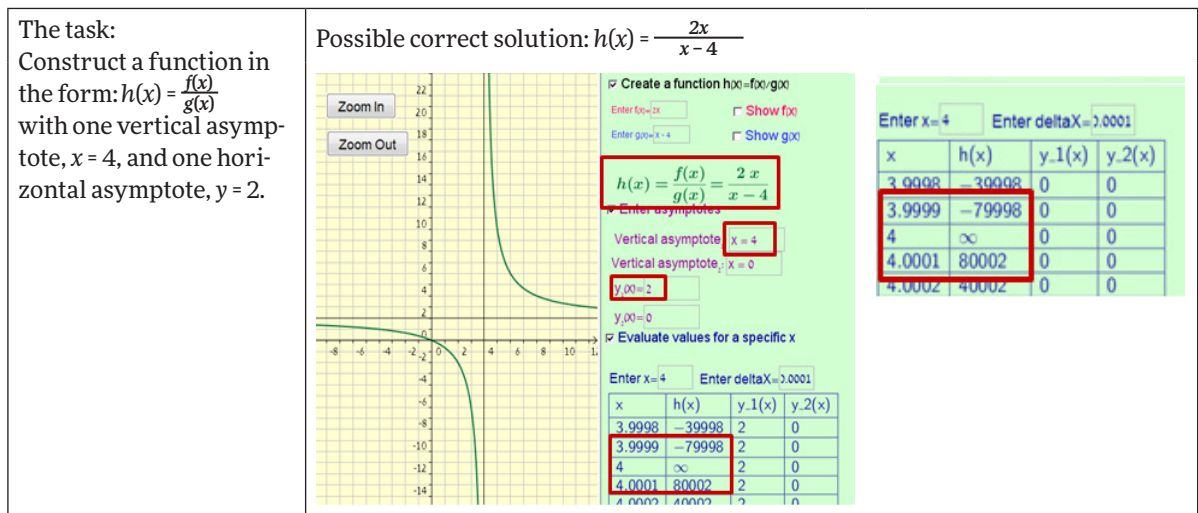


Figure 4: A possible correct submission for a construction e-task

may enter an infinite number of values of x and its surroundings (Δx). To support their construction, students must choose the appropriate values of x and its surroundings. Figure 4 shows a possible correct solution (the relevant components are surrounded by rectangles): the student constructs the function $h(x) = \frac{2x}{x-4}$ and builds the value table around $x = 4$, with $\Delta x = 10000$ to support the argument that $\lim_{x \rightarrow \pm\infty} h(x) = 2$. The second value table around $x = 4$, with $\Delta x = 0.0001$ supports the arguments that $\lim_{x \rightarrow 4^+} h(x) = \infty$, $\lim_{x \rightarrow 4^-} h(x) = -\infty$. Both tables support the student's construction and demonstrate that $y = 2$ is a horizontal asymptote and $x = 4$ is a vertical asymptote of the function $h(x) = \frac{2x}{x-4}$.

The automated checking system is under construction, therefore we can report only on a pilot experiment. The submission and checking of the answers were conducted manually rather than automatically. Our research question is: What is the role of the dynamic MLR environment in completing construction e-tasks. Specifically, we explore how students use the environment to solve the task and what is the added value of marking certain parts on the screenshot they submit as the solution in reflecting reasoning behind the answer. To obtain initial answers to the above questions, we conducted a pilot experiment.

PILOT STUDY

We report a few excerpts from the experiment we conducted with two pairs of 11th grade high school students: Iddo and Ilay, and Shira and Ayala. Each pair studied with the same teacher but in different schools. They studied the standard curriculum of functions and calculus in a regular classroom, without special

emphasis on technology, and they successfully passed the Israeli matriculation exams.³ Each pair worked on eight calculus e-tasks created with the GeoGebra software (each e-task had up to four parts). Most of the e-tasks in the pilot experiment were construction e-tasks related to calculus, not necessarily asymptotes. In each e-task students were asked to mark certain parts on the submitted screenshot that reflect additional reasoning on their part regarding the answer. One of these e-tasks is shown in Figures 3 and 4. At the beginning of the experiment, the first author demonstrated all the functions of the applet. She was present during the experiment and answered technical questions (for example, how to enter the square root function into the input bar). The experiment was videotaped to capture the complete sound track and everything that happened on the computer screen. All subject matters included in the e-tasks are from the standard curriculum, and the participants were already tested on these topics at their matriculation exams. The students were asked to say out loud whatever they were looking at, thinking about, doing, and feeling as they went through their task. This enabled us to see the process of task completion as it was taking place, rather than only its final product, and to listen in on the problem-solving process. Figures 5 and 6 present different trials of the two pairs, including their conversation, as they were working on the e-task that appears in Figure 3. Both pairs were asked to construct a function in the form $h(x) = \frac{f(x)}{g(x)}$ with two horizontal asymptotes, $y = 2$, $y = -2$.

First pair: Ilay and Iddo

Ilay: It is difficult... It (the result) has to be square of something.
(Writing in their papers for several minutes.)

Iddo: Aaah! When you have $\sqrt{x^2}$, you have plus and minus. Try to do $4x^2$ divided by... no... it'll be reduced.

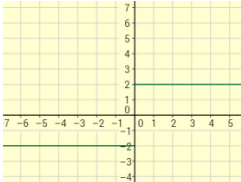
One minute later:

Iddo: Let's try $\frac{2x}{\sqrt{x^2}}$, then think that you divide the numerator in x and it equals 2, and then you divide the denominator in x and you put it into the $\sqrt{x^2}$ so that it equals 1. But there is another option of -1.

(Iddo refers to: $h(x) = \frac{2x}{\sqrt{x^2}} = \frac{2x}{\sqrt{x^2}} \cdot \frac{x}{x} = \frac{2}{\sqrt{x^2}} \Rightarrow \pm\sqrt{\frac{x^2}{x^2}} = \frac{2}{\pm 1} = \pm 2$)

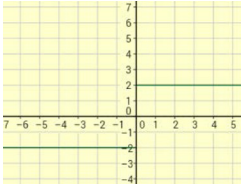
(First try on the screen.)

Iddo: Yes, right,
Ilay: It's all from our yellow book...
Iddo: Let's see what's wrong here.



On the screen:
 $h(x) = \frac{2x}{\sqrt{x^2}}$
 $y = 2, y = -2$

Iddo: These are not the asymptotes.

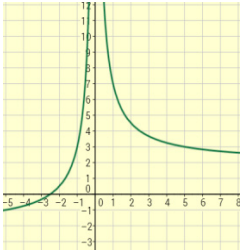


On the screen: $h(x) = \frac{2x}{\sqrt{x^2}}$
They remove the horizontal lines $y = 2, y = -2$

Two minutes later:

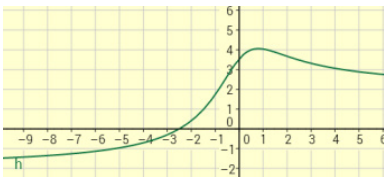
Iddo: Add 5 to the numerator. No, no... actually yes, it has to work, I think.

Iddo: Oh, add 2 to the denominator, because then if you have a negative value for x , then $x^2 + 2$ is positive.



On the screen:
 $h(x) = \frac{2x+5}{\sqrt{x^2}}$

Ilay: Great!



$h(x) = \frac{2x+5}{\sqrt{x^2+2}}$

Figure 5: First pair's conversation and attempts at construction

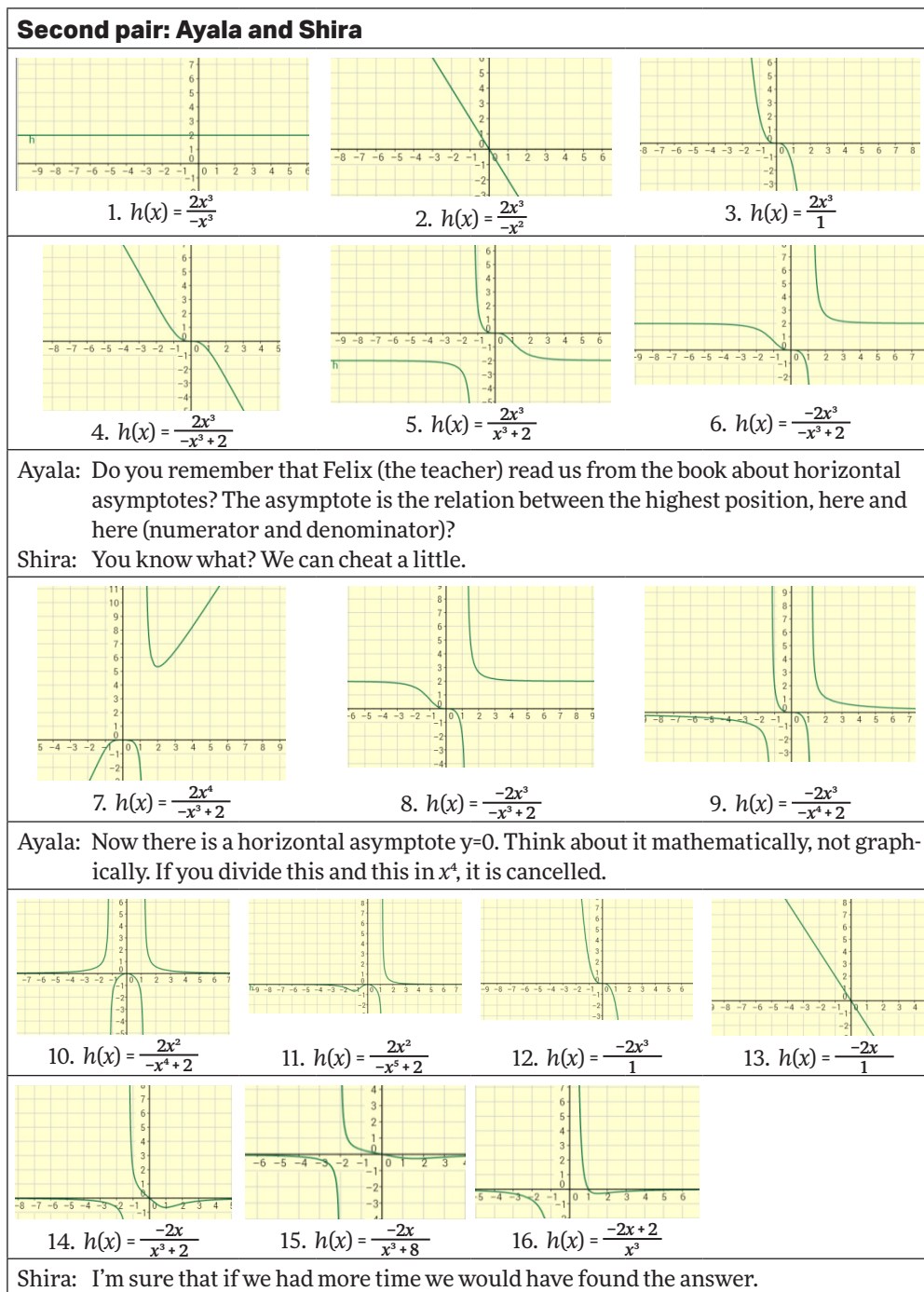


Figure 6: Second pair's conversation and attempts at construction

DISCUSSION AND CONCLUSIONS

Construction e-tasks have a great potential for e-assessment of higher-order mathematical skills. The e-tasks may have an infinite number of possible solutions, which all can be checked automatically. The solution stages (Figure 5) attest to the fact that submitting the correct answer is not accidental. Below we discuss the functionality of the dynamic MLR environment in solving and assessing construction e-tasks.

The first pair had no difficulties constructing the required examples (Figure 5). In each of the three trials they obtained a function with the required horizontal asymptotes, $y = 2$, $y = -2$. They did not submit the first trial because they were not familiar with the Heaviside step function, and did not submit the second trial because it had a vertical asymptote as well. Finally, they decided to submit the third trial. They appear to have started working with a firm conjecture in place regarding the functions with two horizontal asymptotes, and therefore constructed successfully the appropriate functions after a small number of

educated trials. They used the dynamic MLR environment merely as a control, to provide feedback about their results. The conversation transcript (Figure 5) shows that their trials were accompanied by correct mathematical explanations. By contrast, the second pair used the MLR as a tool for empty trial and error experimentation. They made 16 trials (Figure 6), all of them involving polynomial or rational functions, which cannot lead to a function with two different horizontal asymptotes. Their conversation transcript (Figure 6) implies that they had no ideas or direction, and guessed without method. Shira suggested cheating the computer, and Ayala tried to recall the rules of her teacher for finding asymptotes. They did not have a ready-made example, which may have helped them reach a correct answer, and the tools were of no help in constructing such an example. They could not meet the challenge and therefore did not submit a function. The dynamic MLR environment provided the second pair with feedback regarding their incorrect answers. We may carefully state that the MLR tools in construction e-tasks can help those who are close to the correct answer, but when the correct answer is too far removed, the tool may encourage trial and error behavior. When the students constructed the correct function, they submitted the screenshot with the appropriate value table, as shown in Figure 4. In this case, other justifications, beyond the construction itself, are redundant. Although we report here only on the asymptote task, the evidence is consistent with other construction e-tasks included in the pilot experiment, but not reported here because of lack of space.

In the pilot experiment we saw repeatedly that the correct construction was accompanied by appropriate value tables, mathematical explanations, and educated trials. We therefore cautiously suggest that a correct final answer, if produced in a compound environment such as the MLR, eliminates the need for manually checking all the solution stages, enabling automatic checking of the solution. Tracking the solution process may be important to assess a partial solution, however, when the submitted construction is partially correct. The experimental setting would allow triangulation of human checks of scanned paper submissions, tracks of intermediate stages and simultaneous computerized video records.

ACKNOWLEDGEMENT

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ENDNOTES

1. JISC 2009- <http://www.jisc.ac.uk/assessment.html>, accessed: December 2009.

2. Clearly, a value of a function cannot be infinite ($\pm\infty$). But in the GeoGebra software it is the way to indicate that the value goes to infinity. This may explain the “ ∞ ” shown in the value table.

3. <http://cms.education.gov.il/EducationCMS/Units/MazkirutPedagogit/Matematika/VaadatMkzoa/BaaretzBaolam/>.