

# How a digital place value chart could foster substantial understanding of the decimal place value system

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*When using digital technology in the mathematics classroom, teachers should always consider whether the tool supports the students' learning process in a way "traditional tools" do not allow. In this sense, the aim of this paper is to discuss theoretically how an interactive place value chart could foster substantial understanding of the decimal place value system in the context of decimal fractions, by analysing the artefact's potential for learning. Here, special attention is dedicated to the mathematical, structural aspects of the place value system included in the software. This theoretical discussion builds the foundation for future empirical investigation that will be outlined at the end of the paper.*

**Keywords:** Decimal fractions, place value system, interactive technology, semiotic resources.

## INTRODUCTION

Different studies emphasize that students build several misconceptions concerning decimal notation (e.g., Heckmann, 2006; Steinle & Stacey, 2004). The majority of these misconceptions indicate a lack of understanding of the decimal place value system which is fundamental to understand our decimal notation of numbers and which therefore plays an important role in learning mathematics.

Besides practical learning approaches based on activities with concrete material (e.g., Montessori) also research on human cognition and especially on mathematical thinking emphasizes that knowledge formation does not evolve only in human mind, but rather through activities concerning the body, the interaction with an artefact and linguistic and symbolic resources (Radford, 2005, p. 111).

In this sense a digital, interactive place value chart (designed by Kortenkamp) offers special opportunities to actively explore the place value system for a substantial understanding of our decimal number notation in a way traditional tools (e.g., paper-pencil place value charts, base-ten-blocks) do not allow.

To discuss this theory, I will first show how the mathematical structure of the place value system is build up and how this is included in the digital tool, so that a first analysis of the mathematical content within the tool is given in this paper. Based on this I will furthermore reason why especially this tool could support students' understanding of the place value system in comparison to other materials. As a last part I will present my research approach to investigate empirically how this tool might support students' understanding of the place value system regarding decimals. This project, called DeciPlace, is a cooperative project between Ulrich Kortenkamp (University of Potsdam) and Angelika Bikner-Ahsbals (University of Bremen).

## AN INTERACTIVE PLACE VALUE CHART TO FOSTER THE CONCEPT OF PLACE VALUE CONCERNING DECIMAL FRACTIONS

The expansion of the number system from natural to rational numbers often gives rise to difficulties in the learning process in relation to fractions but also concerning decimals.

Different research projects emphasize that students build several misconceptions on decimal notation (e.g., Heckmann, 2006; Steinle & Stacey, 2004). These misconceptions often lead to inadequate strategies, for example to order decimals by size like "longer-is-larger" or "shorter-is-larger" (Steinle & Stacey, 2004). Heckmann (2006) considers transmissions from nat-

ural numbers and fractions (especially if fractions are taught before decimals) as the main causes for misconceptions (ibid, pp. 77–89). This suggests the assumption that some students have a non-viable understanding of our decimal place value system or do not use it, so that they are not able to arrange decimals according to size correctly (ibid, p. 51).

That is why it is recommended to focus explicitly on the place value system while learning the concept of decimals (Heckmann, 2006, pp. 52, 562ff.; Padberg, 2009, p. 166; Steinle & Stacey, 2004, p. 548).

### Place value

The decimal place value system is based on four properties that are important to understand (Ross, 1989, p. 47):

- 1) Positional property. The quantities represented by the individual digits are determined by the position they hold in the whole numeral.
- 2) Base-ten property. The values of the positions increase in powers of ten from right to left.
- 3) Multiplicative property. The value of an individual digit is found by multiplying the face value of the digit by the value assigned to its position.
- 4) Additive property. The quantity represented by the whole numeral is the sum of the values represented by the individual digits.

Within the mathematical structure of the place value system defined by these properties, the decimal fraction is a particular structure that is determined by a fixed value represented by a number in decimal notation. In order to preserve the decimal fraction and its structure, only transformations like bundling and de-bundling are allowed in the place value system.

Although learners already know the decimal place value system and its properties, at least unconsciously, while ordering natural numbers and calculating with them, they normally do not pursue this concept in relation to decimals. Furthermore the extension of the place value system from natural numbers to decimal numbers is not self-explanatory (Padberg, 2009, p. 167), since there are not only analogies but serious differences between both (ibid, p. 170). Therefore the emerging extensions and general properties of the place value

system should be addressed adequately when teaching decimals to avoid inappropriate transfers from natural numbers and to foster a suitable understanding of our decimal place value system related to decimals.

### The interactive place value chart by Kortenkamp and its benefits

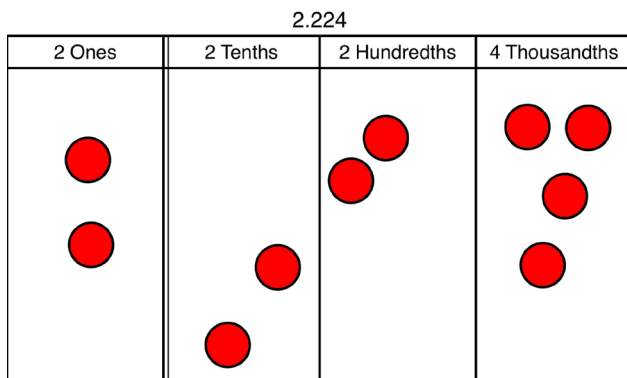
One of the main reasons for difficulties in mathematics learning is the fact that mathematical objects and structures are not directly accessible through our sensory organs. Nevertheless we can represent mathematical objects and structures through symbols as well as through visuals and we can gather experience engaging in mathematical activities on concrete material to get access to mathematical concepts (Gersten et al., 2009; Schipper, 2003).

That is why it is recommended to use visuals and concrete activities to develop an adequate understanding of place value (e.g., Heckmann, 2006, p. 577).

A digital, interactive place value chart (designed by Kortenkamp, see Ladel & Kortenkamp, 2013) allows the combination of concrete acting and iconic and symbolic representation of numbers. The following a priori analysis of the artefact examines the possible functions of the interactive place value chart and their corresponding mathematical meanings within the place value system. The main functionalities of the interactive place value chart are the representation of a number by tokens in a place value chart and the variation of the number's representation by changing the quantity within place values moving tokens, while keeping the number invariant.

At the upper row of the place value chart the symbolic representation is described through the particular place values (Tens, Ones, Tenths, Hundredths, etc.), structured in columns. Additionally, the represented number can be displayed in standard notation. The bottom row as the main part shows tokens representing the quantity of each place value within the whole number (see Figure 1).

A number can be created by adding tokens in the given place value columns by tapping the screen. Mathematically, this signifies an increase of the quantity in a particular place value by the quantity of added tokens. Here, the relation between particular place values and the number in standard notation can be explored addressing indirectly the *positional*



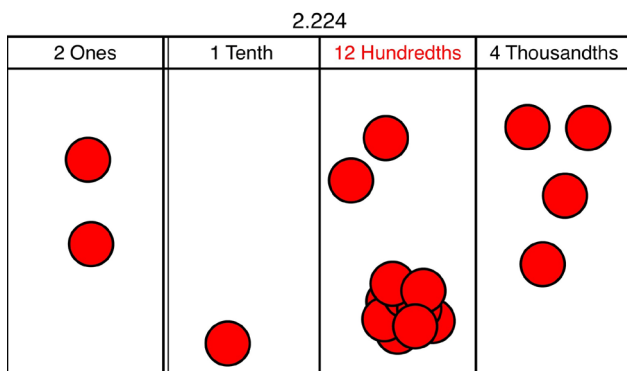
**Figure 1:** The digital place value chart

and the *additive property* of the place value system. Additionally, the function of the decimal zero is illustrated by a blank column of tokens.

The fundamental characteristic of this interactive place value chart concerns the invariance of the number, while varying its representation within the particular place values. Specifically, the user can slide tokens directly on the touchscreen, either within the same column or to other columns. Here, the former action can be done to structure the token’s arrangement, e.g. to facilitate counting the tokens within a column. For the latter activity of moving a token to another column there are three different possibilities of the artefact’s reaction, which are guided by the mathematical structure of the decimal place value system:

- a) When the user slides one token to one column rightwards, the token multiplies into ten tokens within the new column. If the user moves for example one token of tenths to the hundredths, in the hundredths-column there will emerge 10 tokens (see Figure 2).

Here, the artefact represents the transformation of the representation through de-bundling ac-



**Figure 2:** Changing the number’s representation within the place value chart by moving tokens

ording to the *base-ten-property* of the decimal place value system. The artefact furthermore follows the *multiplicative property* of the place value system, which implies that the quantity in each place value has to be multiplied with the according place value that decreases in powers of ten from left to right, so that the quantity in place values has to increase in powers of ten while de-bundling from left to right.

- b) When the user slides one token to one column leftwards, there are two cases:
  - i) If there are at least ten tokens in the initial column, nine tokens go along with the slid token and the ten tokens merge to one token in the new column. Here, the artefact automatically bundles according to the *multiplicative* structure and the *base-ten-property* of decimal place value.
  - ii) If there are less than ten tokens in the initial column, the token slides back to its initial position, because the condition for bundling only groups of ten is not given.

To sum up, the analysis of the relation between artefact signs and the underlying mathematical meaning shows that the interactive place value chart follows the particular structure and corresponding properties of the decimal place value system by performing and illustrating bundling and de-bundling activities.

As an additional functionality, the number of place values as well as the basis of place values can be changed, so that the chart can address further number systems (basis 2–16) and can be used on different levels of learning.

How this interactive place value chart might support substantial learning activities related to decimal place value in comparison to other learning materials for place value systems will be the focus of the following passage.

**The interactive place value chart in comparison to other materials**

To explore the structure of decimal fractions and the allowed rules within the decimal place value system, students need a tool that follows the rules keeping the structure invariant. In contrast to “traditional”

place value charts (paper-pencil or with real tokens), the interactive place value chart allows and even performs bundling and de-bundling activities (see above). Ladel and Kortenkamp (2013) emphasize that this allows to “operate with the tokens *while keeping the represented number unchanged*” (p. 191, italic in the original), whereas in traditional place value charts the manipulation of tokens means a change of the number and therefore a change of the underlying structure. In this sense the “modern approach emphasizes the human activity and is ruled by the object (i.e. the numbers), not the artefact” (ibid). That means that the use of traditional place value charts requires to reflect and understand the artefact, so that the artefact and its function become additional issues to understanding. In contrast, the interactive place value chart follows and even illustrates the structure and the rules of the place value system, so that it could be helpful especially for low-performing learners.

Compared to the base-ten-blocks as well as linear-arithmetic-blocks materials, the interactive place value chart does not illustrate particular place values by different sizes and aspects, but by its position within the chart. Therefore the positional property of the place value system is emphasized. This can be seen as a preparation for the symbolic notation through digits, because the digits of a number equally do not differ in aspect, instead their position within the number is essential (positional property). This preparation is important for disconnecting from concrete activities on the tool to understand and symbolise decimal numbers (Scherer & Moser-Opitz, 2010, p. 85; Schipper, 2003, p. 223). Hasemann (1995, p. 13) points out that exactly this transition from the concrete situation to formalising and symbolising is the main difficulty of understanding mathematics for many students.

Another chart that is used to illustrate the structure of our decimal number system is the Gattegno Chart (see Figure 3), where “each column has a *digit name* associated with it (*one, two, three, ..., nine*) and each row has a *value name* associated with it (*thousand, hundred,*

*-ty, [nothing]*). Thus, a number name can be thought of as made up from *digit names* and *value names*” (Hewitt, 2005, p. 45, italic in the original). With respect to this structure the Gattegno Chart aims to “[help] students to learn to say and write number names” (ibid, p. 44) as well as it is meant to “[provide] a basis for work on place value, decimals, addition and subtraction, multiplication in powers of 10 and standard form” (ibid). Various tasks are suggested for exploring the chart’s structure to invent calculating methods (see, e.g., Hewitt, 2005). However, these calculating methods are rather based on the chart’s structure than on conceptual understanding of the decimal place value system. The properties of our decimal place value system could be addressed by specific tasks within this chart, but in contrast to the interactive place value chart they are not illustrated directly within the table. Moreover the Gattegno Chart is focused on the symbolic representation of numbers in contrast to the interactive place value chart, where the iconic representation includes in particular the illustration of the multiplicative and base-ten property.

### OUTLOOK INTO THE EMPIRICAL RESEARCH TO BE CONDUCTED

With this analysis of the mathematical structure of the place value system included in the interactive place value chart and the theoretical considerations on its benefits, the basis for an empirical investigation of the artefact’s impact on students’ learning is provided. On this basis my research approach will be to investigate empirically how students interact with the digital place value chart in the process of conceptualising the place value system as a main factor for constructing a fruitful concept of decimals.

In learning processes and their profound analysis, the use of artefacts seems to play a crucial role, especially while investigating the interactive place value chart, because the culturally construed structure of the place value system is already included in this artefact. Recognising this structure “may occurs [sic] for the

0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
1	2	3	4	5	6	7	8	9
10	20	30	40	50	60	70	80	90
100	200	300	400	500	600	700	800	900

Figure 3: An example of the Gattegno Chart

expert automatically and unconsciously” (Mariotti, 2012, p. 28), but students firstly have to explore and get to know this structure within the place value chart through the use of the tool (ibid.). That is why Radford emphasizes that artefacts in the mathematics classroom “are not merely devices that provide stimuli for cognitive development” (Radford, 2014, p. 360), but “they are part of cognition, which we see simultaneously as ideational and material” (ibid).

With the neuropsychological model “gestures as simulated action” from Hostetter and Alibali (2008) we can moreover understand why also the dynamics and relations between different semiotic resources (gestures, inscriptions, language, etc.) might be important in the analysis of learning processes. Hostetter and Alibali (2008) state that “gestures emerge from the perceptual and motor simulations that underlie embodied language and mental imagery” (p. 502). That is why this approach leads to suppose that for example not only gestures and language, but also gestures and activities on artefacts in a learning situation are intertwined with each other and so both have to be considered in the analysis also concerning their relationship.

Recent studies about the role of gestures in mathematics learning revealed that especially high-performing learners use gestures while “generalising” a mathematical concept, so that it seems as if they bridge the transfer from concrete acting to mathematical thinking through gestures (e.g., Goldin-Meadow & Beilock, 2010).

Regarding the interactive place value chart we can note that the sliding-actions needed to operate with the tokens originate in concrete sliding-actions of real tokens, but as well they are gesture-based because of the chart’s touchpad-surface. Furthermore the place value chart emphasizes the spatial property of the place value system within the chart, which also can be experienced by the sliding-actions on the touchpad. The virtuality of the digital place value chart, its gestural basis of sliding-actions and the spatial property seem to be suitable to transfer the chart and the sliding-actions into the gesture space as a virtual and dynamic representation, that can similarly be used to reason (verbally) about the place value system and its properties in relation to decimals.

Equally it would be possible that students transfer the virtual place value chart into an inscription, where

the sliding-action and therefore the bundling- and de-bundling-activities of place value can only be represented statically and fixed in contrast to gestures.

In summary, all semiotic resources, the mathematical concept of the place value system and its impact on the artefact should be part of the analysis of learning processes. Thus the focus of investigation will be on the role of bodily and artefact-mediated action and perception together with linguistic and symbolic activity, so that the following research questions are traced:

- How do students construct decimals with the interactive place value chart? What conditions foster or hinder this process?
- How do students use semiotic resources interacting with and disconnecting from the interactive place value chart, and what functions do these resources accomplish within the process of learning?

To examine the mentioned research questions video-recorded interviews with student-pairs in grade 5 will be conducted before these students are formally introduced to decimal fractions. The students will be on different levels of their mathematical development and come from an inclusive setting within a comprehensive school (Oberschule) in Bremen. Within these interviews the students receive tasks dealing with the properties of the place value system in relation to decimal fractions, where they can use the interactive place value chart.

The final goal of my research project will be to develop further the tool of the interactive place value chart and to design a learning environment that fosters substantial understanding of the place value system regarding decimal fractions taking into account different semiotic resources. Therefore, the tasks as well as the design of the tool will be developed further during the process of data collection based on the emerging results of analysis. That is why the realisation and the analysis of the single interviews will be organised in an alternating way.

Concerning the design of the tasks it is important to allow a non-predetermined use of different semiotic resources besides the tool (gestures, inscriptions, language, etc.), so that it will be possible to investigate

the impact and function of different, spontaneously used resources on the students' process of understanding. Here, the semiotic bundle (Arzarello, 2006, p. 267) as a refined notion of semiotic system provides a suitable framework for the analysis of semiotic resources. This framework of semiotics combines the enlarged notion of semiotic system stated by Ernest (2006) and the Vygotskian approach of psychological processes (Arzarello, 2006, pp. 278f.). This means that besides the classic semiotic resources like words and inscriptions also gestures and artefacts are included in this modern notion of semiotic system (Radford, 2002, footnote 7). Thereby the Vygotskian approach allows "a deeper understanding of its [the semiotic system's] dynamics" (Arzarello, 2006, p. 279). That this enlarged semiotic approach of semiotic bundle could be fruitful in the analysis of learning processes has already been shown in particular studies. Different research projects on the role of gestures in mathematical learning processes (e.g., Behrens, Krause, & Bikner-Ahsbahs, 2014; Krause, 2015 in preparation; Sabena, 2007) revealed for example that within the whole semiotic activity, gestures play an important role in the students' process of generalisation. In this sense Radford describes the role of gestures by supplying "the unperceivable general with something concrete" (Radford, 2005, p. 115).

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