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# The use of hands and manipulation touchscreen in high school geometry classes

Marcelo Bairral<sup>1</sup> and Ferdinando Arzarello<sup>2</sup>

1 UFRRJ, Seropédica Brazil, [mbairral@ufrj.br](mailto:mbairral@ufrj.br)

2 Università di Torino, Torino, Italy, [ferdinando.arzarello@unito.it](mailto:ferdinando.arzarello@unito.it)

*Touchscreen dynamic environments user interfaces employ a specialized interaction model on screen. In this paper, we analyse students' manipulation to explore and justify their geometrical reasoning on a free online touch device: the Geometric Constructor (GC) software. We discuss data from a teaching experiment with Italian High School students. The experiment was videotaped. Based on this we observe two domains (constructive and relational) regarding the development of geometrical thinking on GC. Students' manipulation on constructive domain is basically done to make construction and it contributes to exploration and to arise conjecturing. Indeed, manipulation in relational domain can suitably support and improve students' justifying and proving performances.*

**Keywords:** Touchscreen device, GC software, constructive domain, relational domain, dragging to approach.

## INTRODUCTION

The emergence of multi-touch devices - such as iPods, iPhones and iPads - will promote new impact and challenges in learning and instruction in general, and in mathematics in particular. Although in Mathematics Education some touch devices have been developed (for instance, Geometer Sketchpad Explorer, Geometric Constructor, GeoGebra app, Sketchometry and Math Tappers apps) research is still scarce concerning mathematical learning through touchscreen manipulation.

In our current research project<sup>1</sup> we are interested in the way of manipulation of tablet resources as iPad. Particularly, how ways of touchscreen manipulation can improve students' geometrical thinking. In this

paper we are addressing issues regarding the question: during the process of solving geometric problems using the software GC which domain (constructive or relational) of manipulation touchscreen could be fruitful to improve student's strategies for justifying and proving? We assume (i) that manipulation on tablet is different from a mouse click and (ii) that mathematics used by students to solve a geometrical task in a paper-and-pencil environment is different from what they use in a touchscreen device.

## Gesture and touchscreen manipulation

The role of gesture, particularly the touchscreen, in supporting mathematical reasoning in technological context is an emerging field of research in mathematics education (Arzarello et al., 2013; Nicholas, 2013). Regarding their usage, environment mobile touchscreen user interfaces employ a specialized interaction model.

Interaction through current mobile touchscreens basically occurs with the computer recognizing and tracking the location of the user's input within the display area. In other words, interactivity occurs in response to two dimensions of the input action (Yook, 2009; Park, 2011). This enables six basic finger actions for input: tap, double tap, long tap (hold), drag, flick, and multi-touch (rotate). According to Sinclair and Pimm (2014), these types of manipulations "describe specific configurations and actions of the finger(s) on the screen and they are different from those discussed in the mathematics education literature in two ways: they involve contact with a screen and they perform an action" (p. 210).

Even though we are not looking only for ways of touch that represent mathematical concepts (for instance, rotation) we agree with Boncoddo and colleagues (2013) that a particular way of manipulation may

<sup>1</sup> In Brazil, the research project is granted by Capes (Ministry of Education).

serve as an important function of grounding mathematical ideas in bodily form and they may also communicate spatial and relational concepts. Specifically for geometrical thinking, inspired in (Hostetter & Alibali, 2008), we consider important to stress that, in touchscreen devices, manipulations are based on visuospatial images, linguistic factors influence gestures and ways of touchscreen are communicatively intended.

Adopting an embodied cognition perspective in our research we highlight reciprocal connections between ways of touchscreen and cognition. Contrary to what happens in clicking, manipulating touchscreen interface implies a continuity of action, the spatiality of the screen, the movement simultaneousness and movement combination and, depending on the resource device, the feedback speed. On the following Figure, we observe one student trying to explain one of the properties of the isosceles trapezoid. He uses hands to represent the sides that are not parallel.

**Touchscreen manipulation in dynamic geometric devices**

Inspired in (Tang et al., 2010) we assume that touchscreen manipulation is not the same as mouse clicks (Arzarello et al., 2014; Bairral et al., 2015). There are differences if we use a usual PC, where dragging is produced with the help of a mouse, or we use a touch-

screen of a tablet, where we can use our fingers to move the figures, and there are differences if we can use more than one finger (as in multi-touch environments) or only one finger.

As we have had a first shift and improving passing from paper and pencil environments to DGS with drag and drop activities (e.g., Cabri Géomètre, Sketchpad, etc.), now we have a further shift and improvement with the transition to multi-touch environments (e.g., Geometric Constructor, SketchPad Explorer, Sketchometry) and to the variety of simultaneous fingers' actions they allow.

When we manipulate the screens in our device with touchscreen interface, we perform a set of movements. Some manipulations we perform have specific mathematics cognition, like when we want to enlarge (or reduce the size of) a picture in some image editor (Paintbrush), or when we perform this through a touchscreen manipulation.

On such occasions we “pull” the image diagonally, upwards or downwards, or we “click” on one of its vertices, so both dimensions (width and height) are reduced or enlarged proportionately. If we do not perform this type of movement, i.e., if we manipulate only one dimension, the image will come out deformed.

Nevertheless, although all these manipulations are based on one mathematical concept (method of the diagonal as a way to generate similar figures), they are not necessarily the same in cognitive terms (the action of enlarging without deforming), epistemological (the simultaneous changing in different parts of the shape) and spatiality (work and manipulating area on the screen). By the way, we still have to go further on these singularities.

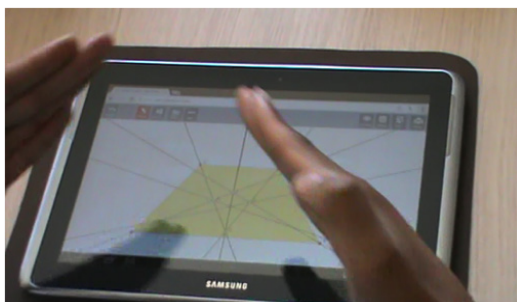


Figure 1: Student construction on GC

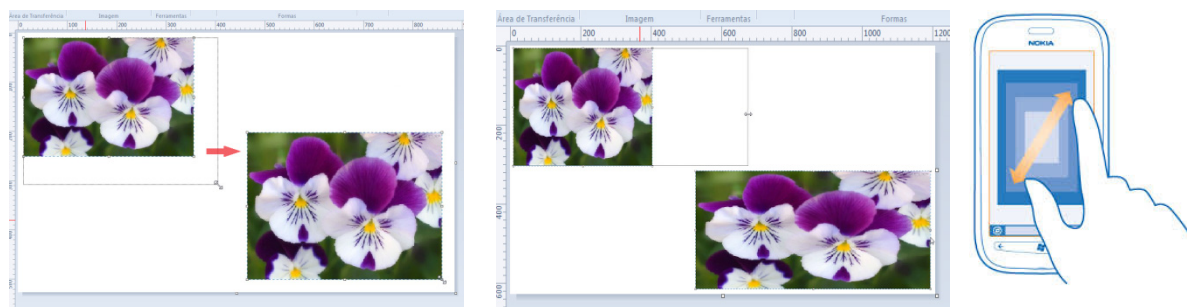


Figure 2: (a) Illustration of an enlargement in a drawing program; (b) distortion in a drawing program; (c) enlargement through sliding on the screen (Bairral et al., 2015)

### Domains of manipulation and geometric learning in dynamic touchscreen devices

Touchscreen dynamic environments user interfaces employ a specialized interaction model on screen. In agreement with Arzarello and colleagues (2002), within this type of environment the interaction also concerns deeply perceptual aspects, which involve not only the objects (e.g., drawings) but also the physical perceptions of students, their motions, gestures, languages, etc. and the artefacts that they use as mediating instruments. Perceptual aspects which must be analysed concern many components, i.e. visual phenomena, motion, kinaesthesia, inner time(s); on the other hand, the most typical theoretical features are the structured mathematical objects, their invariant properties, conjectures, theorems, proofs.

Even though in the relational domain students also construct geometric objects we observed (Arzarello et al., 2014) that it is in this particular domain where they show more interacting and reflecting about the construction.

According to Arzarello and colleagues (2014), a cognitive process within a GC device could be seen in two interrelated domains of manipulation: the construction domain, where students basically refer either to tap and hold, which are the basic actions, or to isolated ways of constructing geometric objects (point, line, circle, shape, etc.) with a touch interface. What we call relational domain is a combination of this constructional and the performed touchscreen actions, which include drag, flick, free or rotate.

While in a construction domain student act as discrete observation (focused on some specific construction or constructed object or even doing some touch on the screen) in the relational domain their manipulation seemed more focused on their questioning, conceptu-

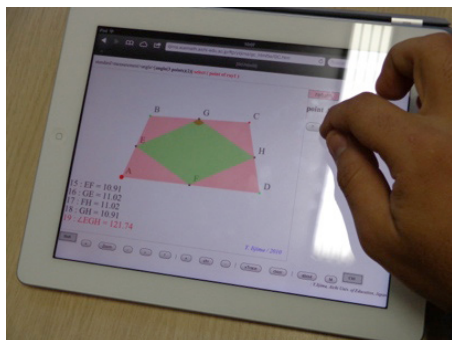


Figure 3: Student construction on GC

al understanding and other emergent demands concerning their manipulation as a whole construction.

### METHODOLOGICAL ASPECTS OF THE STUDY

We are conducting teaching experiments (TE) with High School (Brazilian and Italian) students and Brazilian prospective mathematics teachers. In this paper, we discuss data from one TE: five High School students (16–17 years old) at Liceo Volta (Turin, Italy) working on software Geometric Constructor (GC). All of them had previous experience with dynamic geometric environments (DGE). Each session took about two hours long and it was videotaped. In each session the students worked out on proposed tasks.

#### Geometric constructor features

The choice of GC software is because, as far as we know, it is the only software which incorporates all the potentialities of usual DGE in a fully touch-screen device. By ‘potentialities’ we mean two main facts (Arzarello et al., 2014): (i) the possibilities of using more than one digit (multi-touch) on the screen to interact with the software and (ii) the possibility of making constructions and not only explorations. As far as we know, at the moment there are very few types of mathematical software that satisfy both these features.

Some of the haptic devices on the market (for instance, GeoGebra app and FreeGeo) satisfy (ii) but not (i): in fact, they allow users to move only one point each time, which makes them very similar to environments where dragging is done with the mouse. A very few, for example Sketch-explorer, satisfy (i) but not (ii). GC satisfies both<sup>2</sup>. Using GC we may construct basic geometrical objects (points, segments, lines, circles), measure them, drag and make traces of geometrical objects and so on. The Student using different colors to edit the construction and measuring internal angles from the quadrilateral EGHF for the Varignon theorem task<sup>3</sup>.

- 2 It has been designed by Professor Iijima Yasuyuki (Aichi University of Education, Japan<sup>2</sup>) and we used its version in English.
- 3 The Varignon Theorem proposed task: In quadrilateral  $ABCD$ , the middle points ( $E$ ,  $F$ ,  $G$  and  $H$ ) on each side have been drawn, forming quadrilateral  $EFGH$ . What characteristics does  $EFGH$  have? What happens if  $ABCD$  is a rectangle? What if it is a square? What if it is any quadrilateral? Demonstrate.

**The proposed and analyzed task:**

**Constructing square<sup>4</sup>**

Build a quadrilateral ABCD. On every one of its sides build a square external to the quadrilateral with one side coinciding with the side of the quadrilateral. Consider the centers of the squares that have been built: R, S, T, U. Consider the quadrilateral RSTU: what can you observe? What commands do you use in order to verify your conjecture?

**Data analysis**

Due to continuity of motion and spatiality on the screen we consider that with touchscreen devices analysis should be about paths of interaction rather than points of interaction. Further, it would be mathematically inappropriate (in most cases) to reduce data of a trace to a single point, as we observe in device without touch action. The analytical process was done in two main steps: (1) identification of each type of manipulation (Arzarello et al., 2014; Park et al., 2011; Yook, 2009) and (2) construction of timeline to gain in-

formation of the global cognitive movement throughout interaction on GC software. Based on videotaping the timeline illustrates the ways of touchscreen and shows geometric aspects from students' interaction on the GC software (Arzarello et al., 2014, p. 47). For the first step we adopted Yooks' (2009) framework as summarized in the following Chart 1.

**RESULTS**

In the following two Charts (2a and 2b), we show part of a timeline elaborated by students' solving the task with the software GC performing four types of basic actions<sup>5</sup> (tap single, scale, hold single and hold multi).

Although in order to make a construction (point, line, angle, circle etc.) the user has to use the software icons, we observed all the manipulation on the screen. We didn't consider touch on the icon as an example, for instance, of the tap or hold touchscreen. Rather, in some interval of time we could observe more than

Action		Type	Motion
Basic	Refers to tap and hold which are the basic ways of interacting with a touch interface.	Tap (single) Tap (double) Hold (single) Hold (multi)	Closed
Active <sup>1</sup>	It is a combination of the basic action and the performed finger action, which includes drag, flick, free, or rotate.	Drag Flick Free Rotate	Open

**Chart 1:** Yook framework quoted by Park (2011, p. 23)

Basic actions	0:00–0:30	2:06–2:56	3:10–3:15	3:43–4:54	4:55–6:01	6:36–6:37	7:06–7:08	15:11–15:30
<b>Tap (single)</b>								
Flip								
Move								
Push								
<b>Scale</b>								
<b>Tap (double)</b>								
Scale								
<b>Hold (single)</b>								
<b>Hold (multi)</b>								

**Chart 2a:** Part of the timeline illustrating basic actions

4 This activity was thought as a task to introduce curiosity among students for the next task (Napoleon Theorem).

5 To fix the timeline on the CERME template, we cut down some time interval.

Active actions	0:00–0:30	0:30–0:50	1:28	1:46–1:54		3:15–3:20		6:05–6:09		8:31 / ... / 15:02		15:35–16:55
Drag free												
Drag approach												
Flick												

**Chart 2b:** Part of the timeline illustrating active actions

one way of touch, but we selected some in which the exemplified type has predominance.

Due to the nature of the task (with open construction and exploration) we identified the predominance of touchscreen types on the relational domain and basically touch such drag (free or approach) and flick. The rotate didn't occur in this task. As we can see on the Chart 2b the usage of drag to approach was dominant.

As we observed in a previous analysis (Arzarello et al. 2014) the dragging to approach works as a refreshing, a quite stabilizing and reflecting area for deep understanding of the geometric properties that emerge from the manipulation on drag free or other way of touchscreen. This type of manipulation seems to be an appropriated moment to improve justification and proving.

According to Arzarello and colleagues (2014), manipulation in the constructive domain seemed to be focused on only predetermined motion, whereas motion through relational manipulations is open in the sense that it can generate more unpredictable processes. We still have to research further on the issue of open motion.

Manipulation on construction domain seems focused on only predetermined motion although motion through relational manipulations provides motion open in a sense that they can generate more unpredictable processes. By the way, we still have to go further on the issue of open motion and on the issue of the two domains of manipulation on GC software.

To summarize the reflection above, we illustrate on Chart 3 how we are relating the two domains of touchscreen with geometrical thinking and the motion through touchscreen. Although students dealt naturally with the device, their manipulation apparently was related with the software constraints (or advantages) or with the proposal task.

## FINAL REMARKS

As simultaneous touchscreen manipulation of spots on the screen brings about implications of an epistemological order, it also adds complexity to our cognitive structures. This particular feature was observed by one of the students in our research. According to him, *“in a very complex figure, moving several elements at the same time can become a bit difficult”*. Besides this cognitive implication, the use of touchscreen devices in the teaching of mathematics brings about transformations in didactic and epistemological realms, and educational research is still lacking.

Another relevant issue to consider is the way using a multi-touch-screen allows changing the task design in a substantial way. More precisely, multi-touch screen devices allow designing geometrical problems in a different way from the usual one, which would be very difficult within non-multi-touch screens environments. For example, within multi touch screens it is possible to ask two students, who use the same screen, to play mathematical games, where each of them pursues antagonist aims: exploiting the strategy they use to win they can so enter into the mathematical property upon which the game has been built (Arzarello et al., to appear).

We identified the touch “to approach” as a predominant way in this type of environment. This sort of touchscreen should be seen as a cognitive tool to empower learners conjecturing and exploring for argumentation during the process of solving the task. This allows us to ascertain that the drag-approach allowed by the multi-touch environment can suitably support and improve students’ justifying (exploring) and proving (conjecturing) performances.

We think that manipulation that promotes open motion (relational ways of touching) can be appropriate to provide new epistemological challenges regarding geometric knowledge and different ways of proving. Since the drag to approach is a relational action, it

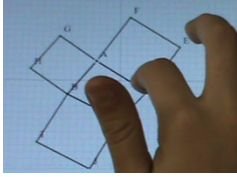

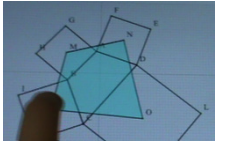
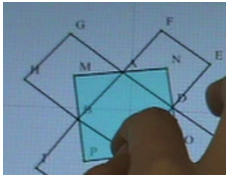
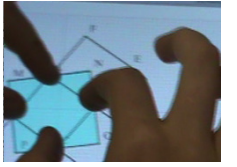
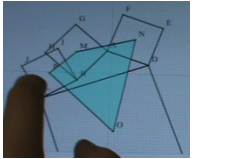
Video time	Screen example	Students geometrical thinking on GC			
		Type of touch-screen manipulation	Geometric strategy by touchscreen	Nature of motion	Domain of manipulation
2:30–2:40		Tap (single or double)	Student constructing square tapping with one finger and making the construction point by point	Closed motion, predetermined (specific goal, basically construction)	Discrete construction and isolated observation (perception). Usually students make constructions for exploration.
6:30		Hold (single)	Student making a zoom at one point		
0:03		Drag free	After having constructed the last square on each side of the quadrilateral ABCD, a student drags freely point P to see what happen with the shapes	Open motion, but focused on emergent conceptual demand of the task	Related construction and global observation for analyze conjectures and geometric properties and shapes. In this domain manipulation on screen is predominant.
5:35–5:45		Drag-approach	Student approaching MNOP to a rectangle to analyze how the shapes become constructed on the side		
			Student using 4 fingers (2 from each hand) for dragging 4 points simultaneously and shaping them as a square		
5:18–5:28		Free	Student moving freely point C		

Chart 3: Students' geometrical thinking on GC device

seems to be an appropriated moment to improve justification and proving within mathematics classrooms using touchscreen devices. But we would say that, depending on the aim of the teacher, the nature of the task is important and the teacher may let students work freely on the task, using naturally their own way of touch.

A new organization of lessons and of the nature of proposed mathematical tasks (didactic), a view on the touchscreen manipulation that is different from mouse dragging (cognitive), and attention to the

changes in mathematics when simultaneously moving different points in a figure (epistemological) are examples of changes and will be an object for reflection on our results in CERME9 (TWG16).

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## ENDNOTE

1. According to Yooks’ (2009) framework, the four active actions can be associated to multi hold manipulation.