Developing future mathematics teachers' ability to identify specific skills needed for work in GeoGebra

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Our research aims at some aspects of the teachers' TPACK, namely their ability to identify specific skills needed for work in GeoGebra and to develop these skills in their pupils. Future mathematics teachers were acquainted with specific skills via selected tasks to be solved in GeoGebra. The data consist of the pre-service teachers' written solutions and proposals for teaching with GeoGebra and transcripts of discussions. The data were analysed in a qualitative way. Explicit work with the skills is needed for pre-service teachers to realise their importance for pupils' work in GeoGebra. While their technology knowledge was good, their proposals for teaching were mostly static and provided pupils with step by step directions. Implications for educating future teachers to teach effectively with GeoGebra are given.

Keywords: Dynamic geometry, TPACK, pre-service teachers, GeoGebra skills.

INTRODUCTION

In our work as educators of pre-service teachers, we strive to find ways of developing their knowledge and skills so that they use ICT tools in their future practice productively. The programme for future teachers naturally includes mathematical courses (developing their content knowledge), a technology course (developing their technology knowledge or TK) and a general didactics course (developing their pedagogical knowledge). All these strands should meet in a mathematics education course. In this article, we will report on a part of a design experiment (Cobb et al., 2003) aimed at designing a part of a regular mathematics education course focused on ICT. To meet the time constraint within the course, we decided to explore the merits and limitations of one ICT tool only. We chose GeoGebra (or GG) as it mutually connects different representations of mathematical concepts (geometric, algebraic, tabular, graphic) and can assist pre-service teachers during all their mathematical learning. Two areas of mathematics were selected for investigation; geometry (synthetic and analytic) and, thanks to GG's multi-representational nature, functions.

THEORETICAL FRAMEWORK

To work with ICT tools in mathematics, differences between computer and theoretical mathematics must be observed, i.e., issues caused by the representation of mathematical concepts, operations, objects, etc. on the computer screen. For dynamic geometry software, Laborde (1998) points out a necessity to understand differences between theoretical-geometric and computer-graphic worlds (e.g., the necessity to construct such figures whose properties are preserved when the objects are moved – i.e., robust constructions). Robová (2013) presents what she calls *Specific Skills* for work in GG. For the purpose of this article, we chose only those pertinent to functions: I. making functions visible (on the screen), II. changing visual appearance of graphs, III. interpreting points on objects, IV. interpreting numerical results, V. using dynamic features of GG, VI. using graphic styles. Skills I. to IV. are part of technology content knowledge (TCK) while V. is between TCK and technology pedagogical knowledge (TPK) and VI. is part of TPK .

We suggest that the teachers' awareness of the Specific Skills and their ability to design teaching which takes them into account belong to their technological pedagogical content knowledge (or TPACK).

> [TPACK is] the basis of good teaching with technology and requires an understanding of the representation of concepts using technologies; pedagogical techniques that use technologies in constructive ways to teach content; knowledge of what makes concepts difficult or easy to learn and how technology can help redress some of the problems that students face; knowledge of students' prior knowledge and theories of epistemology; and knowledge of how technologies can be used to build on existing knowledge and to develop new epistemologies or strengthen old ones. (Mishra & Koehler, 2006, p. 1029)

There is a growing body of literature on developing TPACK. For example, Balgalmis, Shafer and Cakiroglu (2013) claim that "focusing on the mathematical concept more than technology and using technology when it is really necessary were the basic criteria for effective technology based lesson" (p. 2534). Bowers and Stephens (2011) suggest that rather than elaborate specific types of knowledge for TPACK, the best we can do is to "engage prospective teachers in technology-enhanced mathematical explorations with the explicit goal of discussing the ways in which technology enabled them to describe relationships among objects on the screen that could not have been developed without the tools employed" (p. 291). They describe a course on the Geometer's Sketchpad; they let pre-service teachers explore the software by solving problems and then they discussed the model of TPACK with them. Sfard's (2008) commognitive perspective of learning is of particular importance here: "*Thinking* is an individualized version of (interpersonal) communicating." (p. 81). Thus, "teaching can be seen as the practice of orchestrating mathematical discourses" and "learning can be seen as the ways in which students engage in these discourses" (Bowers & Stephens, 2011, p. 287).

Abbitt (2011) summarises performance-based TPACK measures which are based on the idea that "by examining the design and planning process, it is possible to assess the knowledge of a preservice teacher in the TPACK domains" (p. 292). To achieve this goal, Harris, Grandgenett and Hofer (2010) developed Technology Integration Assessment Rubric for analysing lesson plans. The rubric is to be used with pre-service teachers during a teacher preparation programme and thus it is specifically useful for us. According to the rubric, the plan is assigned 1 to 4 points in four measures: 'Curriculum Goals & Technologies' (curriculum-based technology use), 'Instructional Strategies & Technologies' (using technology in teaching/learning), 'Technology Selection(s)' (compatibility with curriculum goals and instructional strategies), 'Fit' (content, pedagogy and technology together).

Bowers and Stephens (2011) describe a rubric for assessing the quality of pre-service teachers' teaching plans made after 18 lessons of work with the Geometer's Sketchpad. They determined for each plan:

> the degree to which the student demonstrated TK, TCK, TPK, or TPACK with the assumption that these types of knowledge are additive. [...] If a student demonstrated a good use of the technology to examine a particular content area but did not include any particular presentation affordances, such as use of colour or scripting tools, then he or she was characterized as having knowledge at the level of TCK, but not TPACK. (p. 293)

Both rubrics, slightly modified, were used in our research. Finally, we made use of a Tool Competence Model for symbolic calculators suggested by Weigand (2011) which specifically deals with functions: Static Mode (producing static representations), Dynamic Mode (creating dynamic representations), Multiple Mode (using the ICT tool as a multiple representational tool).

METHODOLOGY

The research questions were the following: (1) Are pre-service teachers with good technology knowledge aware of the Specific Skills? (2) How do pre-service teachers connect their content knowledge, pedagogical content knowledge and technology knowledge to prepare proposals for teaching with the help of GG?

By good technology knowledge, we mean that pre-service teachers were able to work in GG, they could orientate themselves in its workspace, they knew where to find required tools and how to use them. In view with literature, rather than presenting them with ideas of effective use of GG, we prepared *worksheets with tasks* in which they had to use Specific Skills which emerge from the differences between computer and theoretical mathematics. It was hoped that by confronting pre-service teachers with such tasks, the

Specific Skills will be brought to their attention and they will realise their importance for their future pupils, too. Two worksheets concerned geometry and two concerned functions. The first set of worksheets on each topic comprised less demanding tasks, while the second set put the pre-service teachers' in pupils' role (the tasks were mathematically more demanding). There was a space after each task in which any skills needed for its solution in GG should be written.

The *first (pilot) stage* of the design experiment took place in spring 2013 (see Robová & Vondrová, 2014) with 19 future lower and upper secondary mathematics teachers, within their second mathematics education course. One of the results was that pre-service teachers should meet more tasks in which technology fails as they are "forced" to reason mathematically then. If the task leads them towards using technology (the solution via software seems to be obvious), they might forget to check the appropriateness of the solution by mathematical means. Thus, we modified worksheets accordingly. It also transpired that we underestimated the importance of discussing the emerging issues. Thus, two discussion periods were to be organised in the second stage in order to make the Specific Skills more visible for pre-service teachers with the hope that they will use them in their teaching proposals. Finally, the quality of the pre-service teachers' proposals for teaching was not very high. Bowers and Stephens' study (2011) suggests that such proposals should be subject to class discussion and the basis for bringing out pre-service teachers' own metacognitive processes (e.g., by asking them to speak about the development of the proposals). Thus, a written peer review was to be included in the next stage.

Main study

Twenty three future lower and upper secondary mathematics teachers participated in the study. It took place during a mathematics education course taught by the second author (Table 1). In the two discussions, the course teacher asked general questions first "what knowledge was needed in order to solve the tasks" and then chose questions on the basis of the pre-service teachers' reactions. The audiotaped discussions focused on specific skills and on merits and drawbacks of GG. The pre-service teachers' written work was collected. Seven pre-service teachers' work on the screen was captured by Camtasia and saved as a videorecording.

After completing 4 worksheets, the students were to prepare a *Project*, that is, a proposal for teaching with the support of GG. The project was to include tasks which would lead to pupils' autonomous investigation of a topic in GG, solutions to the tasks, GG figures, the goal(s) of activities, pupils' prior mathematics and technology knowledge, their expected problems and a suggestion of their remedy, etc. The pre-service teachers submitted their Projects via a Moodle module Workshop. Afterwards, they were randomly assigned their peers' Projects to evaluate.

This paper only focuses on the part of the study dealing with functions. Worksheet 1 on functions comprised tasks on determining the domain and range of functions, their monotony, zero points and x and y-intercepts. They were quite easy to solve as the aim was to make the pre-service teachers aware of the Specific Skills without having to concentrate on the mathematical aspect too much. Worksheet 2 tasks also referred to the Specific Skills but they were mathematically more demanding. Examples of tasks are given in (Robová & Vondrová, 2014).

Analysis of data

In the analysis of the solutions to worksheet tasks, it was followed (research question 1): the pre-service teachers' TK, content knowledge, the Specific Skills. When in doubt, Camtasia recordings were used to get more information. For the analysis of the pre-service teachers' Projects (research question 2), the modified Harris, Grandgenett and Hofer's (2010) and Bowers and Stephens' (2011) rubrics were used as well as

Session 1 **4 lessons**	**Session 2** **4 lessons**	**Session 3** **4 lessons**	**Home study**	**Home study**
Individual work: Geometry worksheet 1, 2	*Individual work:* Geometry worksh. 2 *Discussion 1* *Individual work:* Function worksheet 1	*Individual work:* Function worksh. 2 *Discussion 2* *Assigning project proposals*	*Individual work:* Project proposals either on geometry, or function	*Individual work:* Peer review of project proposals

Table 1: Organisation of the study

Weigand's Tool Competence Model. As the tasks were to lead to pupils' autonomous work, we also used the measure of hypothetical pupils' role in gaining new knowledge (from A1 – pupils are given step by step instructions, to A4 – it is up to pupils to decide how they will solve the tasks). The two authors coded the projects independently and discussed their coding until 100% agreement was reached.

Two of Harris, Grandgenett and Hofer's (2010) measures were not used in our coding: 'Curriculum Goals & Technologies' (as all the Projects were aligned with the curriculum) and 'Technology Selection(s)' (GG was prescribed). Instead, we used 'Types of Goals' measure which concern explicitly stated goals: Content Goals (related to specific mathematical topics), Skill Goals (related to the development of some skill), General Competence Goals (outside of mathematics). The two remaining measures were used: 'Instructional strategies and technologies' was coded from I1 (technology use *does not support* instructional strategies) to I4 (*optimally supports*); 'Fit' was coded from F1 (content, instructional strategies and technology *do not fit together* with the instructional plan) to F4 (*fit together strongly*).

While Bowers and Stephens' (2011) pre-service teachers' projects consisted of one sketch, our pre-service teachers proposed several tasks for more than one lesson. Thus, we had to assess what feature prevailed in the Projects. Moreover, we did not fully embrace the authors' assertion that the elements of TPACK are additive (they coded pre-service teachers' projects either TK, or TCK, or TPK, or TPACK), thus introducing a kind of hierarchy which, in our opinion, is not in the original model of TPACK. Unfortunately, the authors do not explain what kinds of projects were coded as TPK and we had to use our own modification. Instead of TPK and TPACK codes, we used TPACK1 and TPACK2. The former means that the pre-service teacher demonstrates an understanding of using technology for more informative, quick and effective teaching of mathematics. Tasks proposed make understanding easier but do not lead to argumentation. The latter means that the pre-service teacher realises the potential of the software for developing mathematical reasoning up to the level of argumentation and proof; it goes beyond what can be observed on the screen. This is, we believe, Bowers and Stephens' code TPACK.

Each Project was peer reviewed in writing by 1 to 3 pre-service teachers. In their analysis, we followed to what extent they commented on the phenomena which we identified as important for TPACK (and used for the analysis of Projects): E1 – a shallow evaluation, a pre-service teacher does not comment on obvious deficiencies or merits of the Project, his/her comments are general; E2 – he/she comments on some aspect only, such as the content; E3 – he/she comments on both positive and negative features, uses most measures (Tables 2 and 3); E4 – expert evaluation.

RESULTS

Research question 1 (pre-service teachers' solutions to worksheet tasks)

All pre-service teachers demonstrated good TK. Some, though, did not connect technology and content knowledge sufficiently. For example, when they did not get a result via predefined tools in GG or when no direct tool is available, they did not use other tools of GG to solve the task but made paper calculations. On the other hand, in some cases TCK was evident; e.g., as for *f*: $y = \sqrt{3x - x^3}$, GG does not find *x*-intercepts and some pre-service teachers used a function with the same zero points instead to solve the task: *g*: $y = 3x - x^3$. In terms of pupils' required skills, the pre-service teachers mostly recorded content knowledge, and some TCK (such as "to input an equation of a function", "to determine intercepts", etc.). They least *commented* on the fact that GG rounds off numbers and thus its numerical results must be critically evaluated (for irrational or periodic numbers).

In terms of the Specific Skills, nearly all participants demonstrated the use of skills I., III. and V. Ten of them failed in skill II. *Input a bounded domain*. The most problematic was skill IV. *Interpret numerical results;* seven pre-service teachers did not record 1,7320 as $\sqrt{3}$, thirteen left 0,33333 instead of 1/3 and sixteen did not recognise $\sqrt{2}/2$ in its decimal expansion (or did not use their mathematical knowledge to solve the task without the help of GG to see the result in the exact form).

The group discussion confirmed the above results. The pre-service teachers were able to solve the tasks and commented on many of the differences between computer and theoretical mathematics as they manifested themselves in the tasks (e.g., the point of discontinuity for *f*: $y = \frac{x^2 + x - 2}{x - 1}$ is not depicted in the graph). The necessity to recognise decimal expansions of

common real numbers was not mentioned and when it was brought out by the teacher in the discussion, it transpired that some pre-service teachers did not use this skill for evaluating results GG produced for them, leaving them in decimal expansions. This corroborates our above analysis.

Research question 2 (pre-service teachers' Projects)

Table 2 shows what Specific Skills the pre-service teachers mentioned (or not) in their Projects. Unsurprisingly, the skill to input the equation of the function was the most frequent one. However, the opposite relation, i.e., the fact that GG can display an equation of the function for a given graph (and when the graph is moved, the equation changes), was only given once and once used but not mentioned. Most pre-service teachers used dynamic features and noted a necessity to work with a slider.

Consistently with the above results, the skill to interpret numerical results was only mentioned once and most alarmingly, in five pre-service teachers' Projects the skill was needed but not demonstrated. Almost half of the pre-service teachers used such functions in their Project where this skill was not needed at all and we cannot say whether it was a chance or intention – they might have realised a possible problem and wanted their pupils to avoid it rather than confront it.

Table 3 shows that the quality of Projects varied. It was not our intention to rank them but to assess as many features as possible. But still, differences between pre-service teachers can be seen. On the one hand, student F3 reached very good evaluations in all measures, her Project was of a high quality. On the other hand, student M4's project was quite poor, he only demonstrated TK and did not make any use of the potential of GG. Let us now look at individual measures.

If a pre-service teacher provided some goals for their Projects, they were only about content, that is, what mathematical knowledge they want to develop. No competence outside of mathematics was mentioned nor a skill such as "to experiment", "to make hypotheses based on the observation of what is happening on the screen", etc. It is quite important as skills like that are stressed in Czech curricular documents. The instructional strategy as proposed in Projects was mostly supported by technology. It concerned pupils' individual work with the teacher helping them. The marks for the measure to which 'content, instructional strategies and GG fit together within the instructional plans' were mostly average. The same can be seen from the sixth column: only a minority of Projects included dynamic features of GG (mostly using sliders for functions – "observe what happens on the screen – how the graph changes – when the slider

Specific Skills in the Projects (I. to VI., see above) Fi – females, Mi –males	Explicitly stated	Not stated, but used	Not used but required by the task
I. Input the equation of the function	F2, M1, F3, M2, F4, F5, M4, F6	M3, M5	
I. Display the equation of the function for a graph	F3	M1	
II. Input a suitable range of coordinates	F1	F4	
II. Input a bounded domain	F2, F5, F6		
II. Input multiples of pi on the axes	F3	F2	
III. Determine x and y intercepts	F2, F4	M4	
III. Determine intercept of objects via tools of GeoGebra	M1, F4		
IV. Interpret numerical results produced by GeoGebra	F3		F2, M1, M2, F4, F6
V. Work with a parameter – slider	F1, F2, F3, M2, F4, F5, F6	M3, M5	
VI. Work with graphic styles of objects (including hiding objects)	M1, F3, F6	F1, F2, M3, F4, F6	
VI. Use texts and symbolic records in GeoGebra		F4	

Table 2: Specific Skills for tasks in Projects on functions (n = 11)

is moved"). In other words, the proposed tasks could be solved without the software and GG only made their solutions more illustrative and quicker.

Student M4's Project was coded as TK only, he does not seem to grasp the ways technology can support the development of mathematical knowledge. Most of the Projects were rated as TCK which means that the pre-service teachers could use GG productively for developing their own mathematical knowledge but did not concentrate on the ways tools of GeoGebra can support their pupils' learning. No Project included tasks which would lead pupils towards the level of argumentation and proof. Just to compare: in Bowers and Stephens' study (2011) 3 out of 21 pre-service teachers were coded as TPACK and 9 pre-service teachers as TK only.

Even though the pre-service teachers were asked to devise teaching in which pupils will work autonomously, the Projects of about half of them were too instructive. They gave pupils tasks which lead them step by step towards the desired goal and provided them with explicit instructions what they should notice and how to continue work.

The pre-service teachers' peer reviews of the Projects were mostly quite superficial and did not comment on even very obvious drawbacks. In the vast majority, the pre-service teachers evaluated the mathematical content only. Their comments were often general ("the project will contribute to pupils' understanding", "the tasks are well chosen", "the use of GeoGebra is effective"). The Projects of students F3 and M2 were evaluated by us as the best ones in most measures. This was confirmed by the analysis of their reviews of their peers' Projects. They wrote detailed and specific evaluations and commented on most of the measures as given in Tables 2 and 3.

DISCUSSION AND CONCLUSIONS

As in the pilot study stage, we conclude that it is far from straightforward that the pre-service teachers will connect their technology, content and pedagogical knowledge without deliberate support from the educator. The presented study remedied some of the deficiencies of the pilot study. We organised more discussion about the TCK and TPACK aspects and included a peer evaluation process. Still, the quality of the pre-service teachers' Projects did not meet our expectation. A good example is the skill to interpret numerical results produced by GeoGebra. The pre-service teachers solved many problems which required this skill, the skill was emphasised in the whole group discussions and still, some of them did not use this skill when it was required and many did not seem to realise its importance for their future pupils. Some of them might not have good understanding of real numbers and do not see that a decimal number with an infinite expansion can sometimes be written in a precise way as a fraction or square root. More content knowledge is needed.

Fi – female Mi – male	**Types of goals**	**Instr. strate-gies and techn.**	**Fit**	**TK, TCK, TPACK1**	**Static Mode, Dynamic Mode**	**Pupils' auto-no-my**	**Peer eval.**
F1 Absolute value	Content	I2	F2	TCK	SM	A1	E3, E2
F2 Goniometric functions	Content	I3	F3	TCK	SM	A1	E1, E1
M1 Power functions	Content	I2	F2	TCK	SM	A2	E1, E1
F3 Sine and cosine	Content	I4	F4	TPACK1	DM	A3	E1
M2 Fractional linear function	Not given	I3	F3	TPACK1	DM	A3	E1, E2
M3 Quadratic function	Not given	I3	F3	TCK	DM	A3	E1, E1
F4 Quadratic function	Content	I2	F2	TCK	SM	A2	E1, E2
F5 Logarithmic function	Not given	I2	F1	TCK	SM	A3	E1, E1
M4 Transformations of graphs	Content	I2	F2	TK	SM	A1	E3, E1, E1
M5 Power functions	Content	I3	F3	TCK	DM	A3	E1, E1
F6 Quadratic functions	Content	I2	F2	TCK	SM	A2	E1, E1

Table 3: Overview of measures used for analysing Projects (n = 11)

One of the implications of our study is that a good choice of tasks must be accompanied by a more explicit discussion about the features of GG use in teaching (represented, e.g., by the measures in Tables 2 and 3). It can "guide students' own metacognitive processes as they reflect on their learning and development efforts" (Bowers & Stephens, 2011, p. 301). This discussion, however, should be in person. In Bowers and Stephens' (2011) study the worst results were reached by pre-service teachers who chose the completely online version of the course: "This could suggest that these students did learn how to use the program, but did not engage in the discursive practices of using technology to probe either their own, or their future students' thinking in the conceptual ways that were discussed during in-person sessions." (p. 294) Students in the blended course had better results and the best were reached by students in the "in person" course. Thus, the discussion should be organised during and after the work on the worksheets and after writing the Projects, commenting on the peers' Projects and seeing the feedback from the peers. The instructions for the peer evaluation must be specific in order to provide pre-service teachers with more guidance by drawing their attention to measures from Tables 2 and 3. We see the peer review as an important part of the development of TPACK.

Due to the number of participants, the results cannot be generalised. The geometry part of the study awaits the analysis and its results together with the presented results will inform the next round of the design experiment in spring 2015.

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