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What can be learned from online public-generated mathematical content? A case-study of the comments on a viral mathematical video

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The study explores the phenomenon of “viral mathematics”, providing insights about the availability and the structure of mathematical content in public-generated media (in particular in comments on videos). The study offers the research methodology, relying on original mix of qualitative, quantitative and data mining paradigms. The findings show that communication using scientific argumentation can occur in comments on a recreational mathematical video; the growth of public interest in mathematical themes can be triggered by the viral video; and the arguments used by the comment-makers contain mathematical derivations, terminology and views on science. Public-generated content can be harnessed as a unique open educational resource and a new source for engaging mathematical problems.

Keywords: Public-generated content, viral video, web cloud analysis, argumentation, divergent series.

INTRODUCTION

My first account with the video in question occurred in February 2014. My 10th grade students were intrigued and asked for my opinion about the “one plus two plus three plus four video”. The challenge was accepted and I designed the subsequent lesson to discuss the mathematical ideas in the video. For me it was an example of blurring the boundaries between the classroom and out-of-school learning. In turn, I was intrigued by the thought that a You Tube video succeeded to engage my students in university level mathematics. At that time, the video “Astounding: $1 + 2 + 3 + 4 + \dots = -1/12$ ” had about two and a half million views and about four and a half thousands comments, which defines it as a viral mathematical video. In general, can public interest in such a video facilitate learning of mathematics? What can students learn from the comments on a video? To

answer the above questions we have to understand the nature of communication through the comments, the structure of the comment and to anticipate its application to education. This research is an exploratory study that used a mixed methodology approach in a relatively new field of learning.

THEORETICAL BACKGROUND

The advent of personal computers, electronic mobile devices and the Internet has increased the blurring between formal (institutional) and informal education in the 21st century. Blogs, wikis, multimedia sharing sites, podcasting, and social networks facilitate individual production and user generated content, harnessing the power of the crowd through an architecture of participation (Andersen, 2007). Hossain and Quin (2012) argue that the interactive features of Web 2.0 technologies enable mathematics teachers and students to create collaborative learning environments inside and outside of the classroom, providing new type of resources and ways of instruction.

Educational resources, their design, sharing and implementation receive a renewal attention from the mathematics education community (see, e.g., Ruthven, 2013 for a comprehensive review). One of the growing tendencies, closely associated with online sharing is a development of open educational resources (OER). Theoretical lens of instrumental approach was used by Trgalova, Soury-Lavergne and Jahn (2011) and Trgalova and Jahn (2013) to study the particular OER (dynamic geometry content repositories) and members of their communities. Kynigos (2014) urges to study new types of OER, which are able to “generate more powerful and relevant sociotechnical communities for the learning of mathematics” (p. 252).

One of the social phenomena related to Web 2.0 is the ‘viral video’: a video clip content which gains wide-spread popularity through the process of Internet sharing (“Viral video”, n.d.). This particular type of media can be harnessed as a unique OER. As shown in 2011 by “The periodic table of videos” channel, educational videos explaining basic notions in chemistry can become viral – garnering millions of views. The impact of the channel awarded its creators with a *Science Prize for Online Resources in Education* (Haran, & Poliakoff, 2011). This explores the potential of a general public interest, manifested in public-generated content, as an educational resource in a mathematical classroom.

Research questions

This study aims to explore the phenomenon of “viral mathematics”, providing insights about the availability and the structure of mathematical content in public-generated media (in particular in comments on videos with a mathematical topic). It raises following research problems. First, little is known about the nature of comments on mathematical video. Whether they consist mainly of expressions such as: “lol” or contain a valuable mathematical component? Second, within the interest on the viral video itself, the public interest in the mathematical concepts and ideas mentioned in the video has to be isolated. Third, the characteristics of comments on the mathematical videos are unclear and have to be typified. To do so, a case study of the comments on the “Astounding: $1+2+3+4\dots = -1/12$ ” video (Haran, 2014) was undertaken.

The research questions, following from the research problems are:

- 1) What mathematical content can be found in public-generated online resources, such as comments on a video?
- 2) Can viral video elevate public interest in the underlined mathematical content?
- 3) What is the structure of comments on a mathematical viral video?

RESEARCH METHODOLOGY AND APPROACH

Viral videos and comments are an unused resource in mathematics education and to date have not been the subject of mathematics educational research.

Therefore, one of the objectives of this study is to propose suitable research methodology, which relies on original mix of qualitative, quantitative and data mining paradigms.

The video “Astounding: $1+2+3+4+5+\dots = -1/12$ ” was published on YouTube on 9/01/2014 and is one of 232 (while writing this article) videos of Numberphile channel. Nearly all of those videos have hundreds of thousands of views. The “Astounding...” video introduced the paradoxical result: $1+2+3+4+\dots = -1/12$ through a manipulation with series (divergent and convergent) and infinity. The creators of the video present the following “proof”. First, the following sums were denoted: $S_1 = 1-1+1-1+\dots$; $S_2 = 1-2+3-4+\dots$; and $S = 1+2+3+4+\dots$. Then the presenter claims and explains that $S_1 = 1-1+1-1+\dots = 1/2$. Based on this result he proves that $S_2 = 1-2+3-4+\dots = 1/4$. Through further manipulations the presenter derives that $S - S_2 = 4S$ and hence $S = -1/12$. In short, the paradoxical result is obtained due to inconsistencies of various natures: different meanings of “=” sign in mathematics; attribution a numerical value to a divergent series; rearrangement of infinite series using the commutative property of addition, etc. Quoting Niels Henrik Abel:

The divergent series are the invention of the devil, and it is a shame to base on them any demonstration whatsoever. By using them, one may draw any conclusion he pleases and that is why these series have produced so many fallacies and so many paradoxes. [1]

To answer the research questions different methodologies were applied:

To answer the first question and to identify the main themes of the discourse in comments on videos, *word cloud* analytics were applied. A word cloud is a special visualization of text where the more frequently used words are highlighted by their proportional size (Feiberg, 2013). Use of word cloud as an analytical tool is still rare in educational research, however it appeared to be a fast and visually rich way to overview the data (McNaught & Lam, 2010). Those features make a *word cloud* especially suitable for preliminary analysis and for validation of previous findings (ibid). Wordle and Tagxego programs were applied to obtain word clouds, containing 50 most frequently used words [2] out of 38452 words

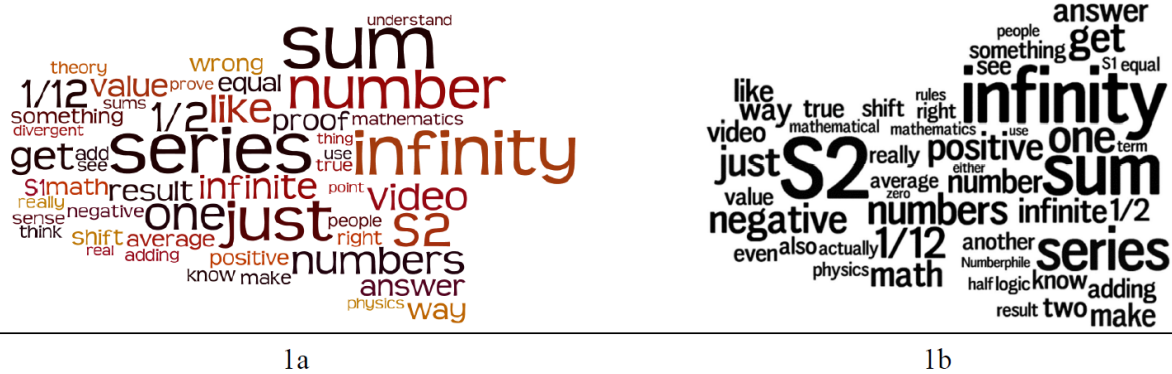


Figure 1: Word clouds containing 50 most frequently used words in all the comments on the “Astounding...” video (2a- out of 38452 words; 2b- out of 5332 words of the sample)

in all the comments to the video. Comparison revealed no significant differences between the word clouds. The further analysis was made using Wordle software (Figure 1).

The data was prepared for analysis in the following way [3]: To open all of the pages of comments I selected “Show more” at the end of each page; to open all replies to each of the comments I selected the “View all N replies” notifications (58); to show all the lines in lengthy comments I selected “Read more” notifications (340). The obtained HTML document was copied to the MS Word using “Keep text only procedure” resulting in a 209 pages document. The utility phrases (e.g., Hide replies; Show less) were deleted from the text file before the data mining stage.

To answer the second question, namely to estimate the impact of the video on public interest to the featured scientific content, the changes in information seeking behavior as expressed by *Google Trends* were used. The methodology, using *Google Trends* tool was proposed and applied by Segev and Baram-Tsabari (2012). The mathematical concepts Grandi series and Cesaro sums were chosen as the subjects of investigation for the several reason:

- Grandi series and Cesaro sums are the key concepts of the video. The Cesàro sums method using the limit of the average of the first n partial sums of the series, as n goes to infinity, enables “summation” of the divergent series, thus Grandi series $1 - 1 + 1 - 1 + \dots$ has Cesàro sums of $1/2$.
- Terms Grandi series and Cesaro sums were not mentioned explicitly in the video and were mentioned in the comments

- Grandi series and Cesaro sums are usually unknown to the lay public and for the non-mathematical academic audience unlike other key concepts of the video (e.g., those present in the university curriculum or related to Riemann or Euler).

The hyperlinks to mathematical sites and blogs in the comments are additional indicators of the video’s impact. The standard Find function of Word with “http” argument was used to find the external links. Each link was labelled according to its content (e.g., Blog, Wiki, scientific paper).

Descriptive statistics of the data sample and qualitative analysis of argumentation in the comments was applied to classify the comments and arguments, to answer the third question. Out of 209 pages of comments 25 pages ($\approx 12\%$ of pages) were randomly selected. The sample contains 85 comments. An additional instrument for establishing the representativeness of the sample is a word cloud, produced from 5332 words of the sample ($\approx 14\%$ of text) (cf. Figure 1a with Figure 1b). The main themes and topics (e.g., sum, infinity, series) are intact. All the comments in the sample was numbered, and coded according to whether they contain the following indicators: presence of mathematical derivations/ scientific terminology; characteristics of scientific enquiry/ epistemology; attitudes toward science or scientists; emotions in support or against the video. In addition, each comment was analyzed in order to locate the essential parts of an argument: a claim, a data and a warrant (Toulmin, 1958/ 2003). For example, comment #80 on the “Astounding...” video: “*Assumption: the sum of an oscillating sequence is its arithmetic mean. Conclusion from assumption: the sum of a set of positive integers*

Mathematical derivations/ scientific content	Characteristics of scientific enquiry/ epistemology	Attitudes toward science and scientists	Emotions	Argument
1	1	0	0	1

Table 1

is negative. You’ve just proved by contradiction that the sum of an oscillating sequence cannot be its arithmetic mean.” was labelled as shown in Table 1.

The claim: the sum of an oscillating sequence cannot be its arithmetic mean, thus the main statement of the video is false; the data: a fragment of the video, where the statement $1 - 1 + 1 - 1 \dots = 1/2$ is proven; the warrant is a proof by reduction ad absurdum. Consequently, the abovementioned comment is an argument, which uses mathematical calculations and terminology and free of emotion. First stage of the research will provide a list of key words, which are the core of scientific communication established in community of commenters on the video. The data collection was completed by simple filtering technique using the key words. The comment-makers use pseudonyms and usually do not share any personal details such as gender, age, occupation and level of expertise in mathematics. However, in some cases it was possible to make grounded suggestions about the comment-maker’s background. For example search phrases: “teacher”,

“class”, “students” revealed math teachers/ students among the comment-makers. I read the mathematical derivations line by line and checked the mathematical terminology used by comment-makers to find the experts in mathematics among them.

RESULTS

What mathematical content can be found in public-generated online resources, such as comments on a video?

The word clouds (Figure 1) show that the comment-makers were involved in scientific discussion, which main topics and themes were connected to a sum of infinite series. The word count confirms this finding. 30 words out of top 50 in the word cloud belong to mathematical/ scientific terminology (general or specific to the content of the video). See Table 2 for the details.

#	Word	Frequency in the comments	#	Word	Frequency in the comments
1	sum	233	16	average	72
2	series	220	*17	shift (technique)	72
3	infinity	213	18	positive	67
*4	S	191	19	negative	59
5	number	184	20	adding	55
*6	S2	161	21	theory	55
*7	1/2	142	22	mathematics	54
*8	1/12	134	23	sums	51
9	numbers	127	24	prove	51
10	infinite	113	25	physics	50
11	value	100	26	equation	50
12	answer	92	27	divergent	48
13	proof	92	28	sequence	47
*14	S1	80	29	summation	43
15	math	76	30	converge	41

Table 1: Word cloud frequency (* indicates the words specific to the video)

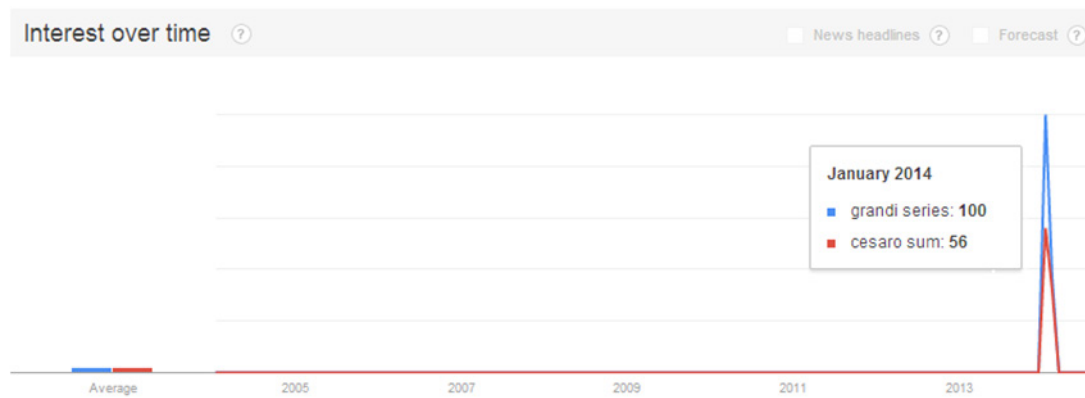


Figure 2: Google trends graph showing a percent of searches on Grandi series and to Cesaro sum

Can viral video elevate public interest in the underlined mathematical content?

Figure 2 indicates the growth of public interest in Grandi series and Cesaro sum (abstract and unknown to the lay public mathematical concepts which are explicitly mentioned in the comments and not in the video). The peak of interest to the concepts in question was registered in January 2014, corresponding to the release date of the video–9/01/2014. It is of note, that there are 37 different hyperlinks pointing to 32 different mathematical sites in comments (Figure 3). Most of the links led to various Web 2.0 platforms: Wikipedia articles about mathematical concepts related to the video (e.g., Divergent series, Riemann zeta function), mathematical blogs, social network, question and answer site or YouTube. Publication of the video has created a teachable moment and new resources for learning were generated/ brought into account.

What is the structure of comments on a mathematical virus video?

The analysis of the sample shows that the comment-makers use mathematical derivations and sci-

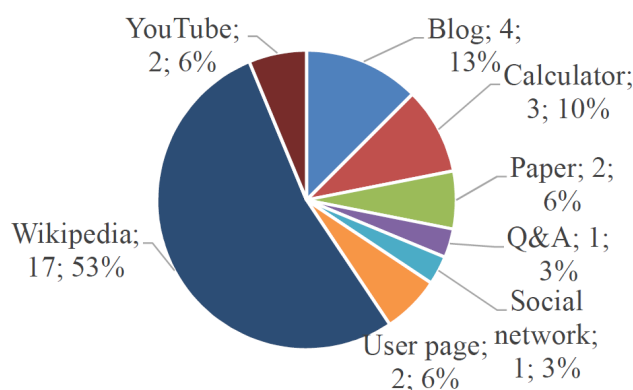


Figure 3: Distribution of hyperlinks in the comments

entific terminology to the broad extent (28% and 36% of comments resp.) The data analysis process revealed that 73% of the comments contained arguments (63 out of 85). These arguments were categorized according to the possible roles that they play within the discourse [4]. In short the most-used categories of argument were: common sense; mathematical laws; mathematical intuition (based on non-infinite mathematics); counterintuitive thinking, proof; and authority. More than half of the comments (52%) in the sample contained indicators of positive or negative emotions toward video and its creators. For example: the fragment of comment #28: “*You know what is more amazing? WHY anyone would ask for the result of $1+2+3+4...$? It is obvious it is infinity!! You must be a genius to think that maybe not.*” Table 3 shows the distribution of the arguments/ emotions in the comments.

The next step was to apply the filtering technique to the text file of all the comments. It revealed data, which the two previous techniques could not disclose. There were evidences of various stakeholders to the “viral math” phenomenon. The math teachers/ lecturers were among the comment-makers. Most of them were concerned with misconceptions which students may develop as a result of watching the video. Comment-maker X wrote: “*My complaint is, this video is leading my students to believe that $1-1+1-1+...$ converges to $1/2$, and is encouraging them to work with limits of sequences without knowing if they converge, which is a huge problem...*” Nevertheless, most of the teachers appreciate the “*arousing interest in discussing the problem*”. The reading of the mathematical derivations line by line reveals mathematicians/ physicists among the comment-makers. Some of the comments were mathematically perfect and could be a part of a textbook, as previously cited in the Methodology section comment #80.

The comment contains:	Emotional expressions supporting / refuting the result	No emotional expressions supporting / refuting the result
Argumentation supporting / refuting the result: $1+2+3+\dots=-1/12$	33	30
No argumentation supporting / refuting the result: $1+2+3+\dots=-1/12$	11	11

Table 3: Argumentative/ emotional component in the sample of the comments on "Astounding..." video

There were some emotionally loaded comments toward the video creators and comments expressing attitudes towards science. The following excerpt of the comments should raise the awareness of mathematics educators, willing to use this resource to the issue of public understanding of the science and its role in society:

- *Mathematicians are frauds (in Russian)*
- *this equation means infinity is different from what we believe, or perhaps our whole concept of mathematics is wrong*
- *Why are you bashing on this channel? Blame the mathematicians who derived this and use it*
- *Unbelievable that scientists make "science" based on fallacies.*

DISCUSSION

This study explores a viral mathematical video and a corpus of corresponding comments as an open resource which was adapted for the needs of (mathematics) education. Trgalova and Jahn (2013) studied more conventional OER (web-based repository specifically designed for the educational needs) and registered a relatively small number of reviews and modifications of the content by users. The authors suggest that lack of tools for communication and collaboration in the repository may decrease users' motivation to contribute. In the list of key ideas of Web 2.0 application to education Andersen (2007) named individual production and user-generated content in first place. The current study, through analysis of the comments, makes a step toward better understanding of the learning in dynamic web environment with active user participation. The findings show that comment-makers on a recreational mathematical video can create, share, and communicate mathematical content using scientific argumentation. The active public participation

through embedded tools of communication and engagement provided an educational component to an open resource.

The arguments used by the comment-makers contain mathematical derivations, terminology and views on science and scientists. The high presence of arguments in the comments advocates the further study of arguments as a promising direction in the research of mathematical communication based on Web 2.0 platforms as file sharing sites, social networks and blogs. This study treats learning from comments as a special case of learning through argumentation. Learning from argumentation has been shown by Asterhan and Schwarz (2007) to lead to students' conceptual gains where the introduction of the dialog of disagreeing discussants proposing reasons for their views about a problem resulted in the students' discussion becoming more argumentative. Lachmy and Koichu (2014) use their analysis of students' arguments to understand the interplay of empirical and deductive reasoning when proving in a computer-enhanced environment. To this end the categories that originated in the comments can facilitate the critical thinking of the students when applying to the public-generated content. For example, the most "innocent" step in the underlying proof is the implicit claim that the sums $1-1+1-\dots$; $1-2+3-4+\dots$; $1+2+3+4+\dots$ can be denoted and the numerical values can be attributed to S_1 , S_2 and S . The argument belongs to a common sense category and must be revisited.

The growth of public interest in mathematical themes triggered by the viral video was shown in this study and is a phenomenon that should be studied intensively as a part of wider vein of research on a public interest in science (e.g., Baram-Tsabari & Segev, 2013).

Despite the study's limitations (it is a case study of the comments on one video), the claim is that this particular type of media can be harnessed as a unique open educational resource. First, it embodies the cy-

ber-social learning opportunities typical of the Web 2.0 environment. Second – it combines the benefits of multimedia learning with mathematically rich content in public-generated comments on the video.

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ENDNOTES

1. From letter (Jan 1828) to his former teacher Berndt Holmböe. In (Morris Kline. *Mathematics: The Loss of Certainty* (1982), p. 170).
2. Except common words of English vocabulary.
3. As far as I know, there is no commercially available solution to the YouTube/ blog commentary's page import to CSV format. One of the possible contributions of the study is to define parameters and variables to be used in the algorithm.
4. The results of the categorization are omitted due to space limitations.