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It is recognized that the process in which an artefact becomes an instrument for a user, denoted as instrumental genesis, is a complex process. The aim of this paper is to identify elements of the process of instrumental genesis when students are dealing with scales and scaling issues in a dynamic mathematics software environment. This study involves four upper secondary school teachers and their classes. By observing the students' instrumented techniques while working with tasks designed with a hypothetical instrumental genesis in mind, some key elements are identified.

Keywords: Instrumental genesis, scaling of axes, dynamic software environment.

INTRODUCTION

Several decades ago, researchers recognized the affordances provided by graphical technologies, in particularly in the field of functions and graphs (e.g., Goldenberg, 1988; Leinhardt, Zaslavsky, & Stein, 1990). For instance, in comparison to the corresponding work with paper and pencil, they emphasized the ease, and thereby the speed of changing the scales of the axes to obtain several different views of a graph. However, some difficulties have also been identified relating to the issue of scales and scaling of axes (Hennessy, 1999; Mitchelmore & Cavanagh, 2000; Yerushalmy, 1991).

The availability of different kinds of technology in mathematics classrooms is increasing and more and more students are provided with a computer of their own (Valiente, 2010), which entails new possibilities for the integration of technology in mathematics education. However, there is a need for students to learn how to use the technology appropriately so that it becomes an instrument for them. Several researchers use the notion of *instrumental genesis* to describe this process by which an artefact [1] becomes an instrument for a user (e.g., Artigue, 2002; Drijvers & Gravemeijer, 2005; Trouche, 2004). However, there is agreement that the complexity of this process has been underestimated and may have contributed to the recognized difficulty of integrating technology into mathematics teaching and learning (Artigue, 2002).

To address the challenge of integrating technology into the mathematics classroom, Trouche (2005) introduces the notion of *instrumental orchestration*, which also takes account of the social dimension of the instrumental genesis within a classroom. Although many researchers associate instrumental orchestration primarily with the organisation of classroom interaction (e.g., Drijvers, Doorman, Boon, Reed, & Gravemeijer, 2010), the original notion also involves the customization of an artefact in order to create a particular task environment (Ruthven, 2014). This paper concerns the customization of a dynamic mathematics software environment, especially the tools associated with scales and scaling. The tasks were designed with a particular instrumental genesis in mind. The aim of this paper is to pinpoint some elements affecting the process of instrumental genesis in relation to scale and scaling issues within a dynamic mathematics software environment, in this case *GeoGebra*.

INSTRUMENTAL APPROACH

One important aspect of this approach is the process of *instrumental genesis* (Verillon & Rabardel, 1995). Through this process an artefact becomes an instrument for a user. An artefact is an object, material or abstract, available to the user and aimed at perform-

ing a certain type of task. For an artefact to become an instrument for a user, there need to exist a meaningful relationship between them (Drijvers & Trouche, 2008). In this way, "...the instrument consists of both the artefact and the accompanying mental schemes..." (p. 367), developed by the user.

It is recognized that the process of instrumental genesis has two directions, one towards the artefact (*instrumentalisation*) and one towards the user (*instrumentation*) (Trouche, 2004). The user shapes the artefact by his/her knowledge and former method of working while the artefact shapes the subject by its constraints and potentialities. However, the fact that the process of instrumental genesis is a rather complex and thereby a time-consuming process has been underestimated (Artigue, 2002). One reason for this, Artigue argues, is the predominant role as a pedagogical tool given to technology.

> Suggesting that instrumentation may be a complex and costly process does not fit visions that consider technology mainly as an easy tool for introducing students to mathematical contents and norms defined independently from it. (2002, p. 253)

Regarding what could be considered as an artefact depends on the situation under consideration. For instance, a symbolic calculator could be considered as a collection of artefacts (Trouche, 2004). Accordingly, this provides students with the possibility to develop several types of instrument while working with this kind of technology. Besides the artefacts, the kinds of instrument being developed depend on the students and the accompanying tasks (Maschietto & Soury-Lavergne, 2013). Hence, it is important for a task designer to be aware of, among other things, the potentialities and constraints of an artefact (Artigue, 2002; Trouche, 2004).

Concerning the mental schemes students develop through the instrumental genesis process, researchers (e.g., Drijvers & Gravemeijer, 2005; Trouche, 2004) distinguish between two categories, *usage schemes* and *instrumented action schemes*. The usage schemes are basic and relate closely to the artefact while instrumented action schemes focus on actions upon objects such as graphs or formulas. In this way "Instrumented action schemes are coherent and meaningful mental schemes, and they are built up from elementary usage schemes by means of instrumental genesis." (Drijvers & Gravemeijer, 2005, p. 167). However, it is not always obvious how to distinguish between these kinds of scheme; it might be a matter of level of capability of the user (Drijvers & Gravemeijer, 2005).

Drijvers and Gravemeijer (2005) argue that instrumented action schemes involve both technical and conceptual aspects. Even if it is the development of conceptual knowledge that is the most interesting, it is the technical activities that are visible and thus the observable part which can be the object of investigation. The technical activities that are developed through instrumental genesis are denoted as *instrumented techniques* (Artigue, 2002) or just *technique* (Lagrange, 1999). In this way, it is the technique that is the "...gateway to the analysis of instrumental genesis." (Drijvers & Gravemeijer, 2005, p. 169).

SCALES AND SCALING WITH TECHNOLOGY

Several decades ago, researchers were already discussing the influence that new technology would have in the field of functions and graphs (e.g. Leinhardt et al., 1990). This section introduces some issues relating to scales and scaling of axes.

In traditional paper-and-pencil work, for example, textbook tasks, graphs often are presented as static diagrams with the coordinate axes scaled in an appropriate way (Zaslavsky, Sela, & Leron, 2002). When working with a graphical technology, on the other hand, the scaling of axes is often left for students, which has proved to cause them some difficulties (Hennessy, 1999; Mitchelmore & Cavanagh, 2000). Further, Leinhardt and colleagues (1990) assert that students' ability to deal with scaling of axes is often taken for granted and argue that "...the construction of axes requires a rather sophisticated set of knowledge and skills." (p. 43). As an example of elements of instrumental genesis observed in a CAS environment, Artigue (2002) discusses "framing schemes":

> When students use function graphs in a computer environment (or a graphic calculator), they are faced with the fact that a function graph is "window-dependent" and they have to develop specific "framing schemes" in order to cope efficiently with this phenomenon. (p. 250)

Mitchelmore and Cavanagh (2000) argue that one reason for students' limited understanding of scaling might be their lack of experience in dealing with graphs where the axes are unequally scaled. In line with this, Goldenberg (1988) discusses students' preference for "symmetric scaling", i.e., *x*- and *y*-axis are equally scaled. He refers to an example where students who had received an appropriate view of a graph, still changed the scales to obtain symmetric scaling. In this way, the students received a visual appearance obscuring important features of the graph. To receive a better visual appearance, they changed the scales of the axes by the same factor, i.e. they used a zoom operation. Goldenberg (1988) argues that one reason for this might be that students' intuition about scale changes is closely connected to real-world experiences: "... our almost automatic approach is to change both scales by the same factor..." (p. 36). However, since usually different units on the axes are required to see the graph in an appropriate way, Goldenberg (1988) stresses the importance for students to deal with unequally scaled axes.

Mitchelmore and Cavanagh (2000) found that students in their study showed limited understanding of the zoom operation. The students used the zoom operation as a magnifying glass but "...were unable to link the operation of zooming with any change in the scale of the graphs displayed in subsequent viewing windows." (Cavanagh & Mitchelmore, 2003, p. 14). As an example, they refer to a case were students who zoomed in on the vertex of a parabola were surprised to see a linear shape of the graph.

Concerning how unequal changes of the scales of the axes (i.e. not zooming) impact on the visual appearance of the graph, Yerushalmy (1991) emphasizes the importance of understanding the difference between the properties of a function and its picture in a graphical view. In her study, students showed difficulties when interpreting the same function graphed in two different scale systems, since the visual appearance differed (Yerushalmy, 1991).

Godwin and Sutherland suggest graphing software as a mean to provide students with experiences of this kind since it provides "The ability to change the scale easily and hence the 'frame of view' of a function..." (p. 134). Further, Hennessy (1999) emphasizes the importance for students not only to realize what will change but also what remains constant as the scales change.

METHOD

This study is embedded within a form of design research project involving two researchers and four upper secondary school teachers with one class each. The focus of the project was on a sequence of three lessons, taught over the course of a school year, in which some use was made of *GeoGebra* to tackle tasks concerning functions and graphs. The researchers and teachers had meetings before and after each lesson. In total, three worksheets, one for each computer lesson, were designed. Although the responsibility for the design was the researchers', the teachers provided valuable information regarding the participating students' capabilities and their current practices.

Each worksheet consists of a sequence of related tasks, denoted as a *task sequence, TS*. The first and the third worksheets concern exponential functions, while the second worksheet is mainly about linear relationships. This paper focuses on issues concerning scale and scaling which are mainly addressed in the task sequences for the first and second lessons. A more comprehensive report of the project will be provided in another paper by the author.

The overarching aim of the task sequences is to enhance both students' conceptual knowledge about functions and graphs and the scope for the dynamic mathematics software to become an instrument for the students (Verillon & Rabardel, 1995). Therefore, on the worksheets the task sequences were intertwined with computer instructions. As noted earlier, the way in which this was done constitutes an instrumental orchestration of the task environment.

The participating students were tenth grade students with no previous experience of working with either dynamic software or graphical calculators. Altogether, 85 students participated in the study. The students were to work in pairs with *one* computer per pair. The purpose of this is that the computer screen should provide a shared object for discussions between students.

The empirical data reported in this paper were mainly collected during the two relevant lessons with the four classes. Each lesson lasted about 60 minutes. In each class one pair of students was video recorded and all teacher-student interactions during the lessons were audio recorded using a microphone attached to the teacher. When necessary in the analysis process,

copies of the written responses from the students were used.

In the analysis process, the video recordings were the primary source since these data made it possible to observe students' instrumented techniques which are the observable part of the instrumented action schemes (Drijvers & Gravemeijer, 2005). The audio recordings provided further insight into the frequency of some student responses.

The scope of this paper is restricted to only dealing with aspects concerning scale and scaling in relation to the process of instrumental genesis. Thereby the tools addressed in this paper are those associated with the coordinate system, and particularly the tools needed to obtain appropriate visual graphical appearances.

RESULTS

This section introduces some prototypical observations regarding how students deal with scales and scaling of axes. The first task in the first task sequence (TS 1) was intended to be a routine paper-and-pencil task which introduces the context of TS 1:

> The height of a sunflower is 50 cm when it is measured for the first time (June 1). After that the sunflower grows so that it becomes 30 % higher each week. Calculate the height of the sunflower one week after the first measurement.

The first task is followed by computer instructions and the students are expected to use the software to enter the points known so far. Regarding the first point (0,50), we decided to provide students with both conceptual and technical guidance as follows:

> When the first measurement is performed, $x = 0$ (since 0 weeks have passed) and $y = 50$. The corresponding point in a coordinate system is (0,50).
>
> Insert this point by entering *(0,50)* into the "*Input Bar*": Input: **(0,50)**
>
> *NOTE!* To be able to see the point you must adjust the scale on the *y*-axis. This can be done by "dragging" the *y*-axis. (first mark )

Since we anticipated that several students might be confused when they cannot see the point in the Graphics View [2], we decided to add the note above to draw attention to the scaling of the *y*-axis. However, this note proved to be insufficient since utterances like "Should we not get a point there?" appeared several times both in the audio and video recordings. The video recordings indicate that the reason for this might be students' eagerness to start using the computer and not to spend time reading the instructions. Consequently, the teachers often had to draw students' attention to the *y*-axis making them aware of its scale. Further, since the students tended to use the zoom operation when changing the scales, the teachers frequently had to demonstrate the possibility to only adjust the *y*-axis.

Next, the students were expected to enter the point corresponding to their calculation in Task 1, i.e. the point (1,65). To make students aware of the possibility of adjusting the *x*-axis, i.e. adjusting one axis at time, to obtain an appropriate graphical view, we added the following note:

> If appropriate, adjust the scale on the *x*-axis!

However, this note seemed to be ignored by most of the students. Maybe the note came too early, and by this stage the students could not see the point of it. The empirical data reveal that even students who had adopted the technique of adjusting one axis at a time tend to focus on the *y*-axis and not use this possibility when it comes to the *x*-axis. Both the audio and video recordings reveal that the teachers frequently had to draw students' attention to the grading of the *x*-axis to get a more appropriate visible view of the coordinate system.

The second task sequence, TS 2, introduces a context problem where the students are encouraged to formulate a linear function formula to insert into *GeoGebra*:

> To get money for a class trip, the Class 9b at Sugar School decided to rent a table and sell candy at the market place. The rent for a table is 100 SEK. They purchase candy for 40 SEK per kg. Determine a formula that describes how the total cost depends on the weight in kg of the candy purchased.

As in TS 1, the *y*-axis has to be adjusted to see the object, in this case a graph, in the coordinate system. Therefore, we decided to give the following note:

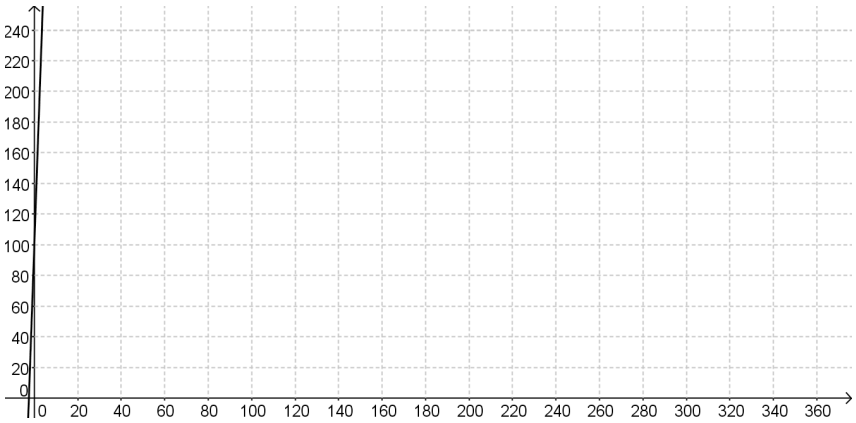

Figure 1: One example of a Graphics View obtained by students

NOTE! To be able to see the graph (the line) there might be a need to adjust the axes.

As in TS 1, both audio and video recording show that several students could not see the object due to the scaling of the *y*-axis. In their attempts to find out the reason why they cannot see the graph, one pair of students discussed whether they have to type a sign (*) for multiplication between 40 and *x*. Thus, they did not reflect on the scales. Reminded by the teacher of the scaling of the *y*-axis, they used the zoom operation, and obtained a graphical view like the one in Figure 1.

The empirical data reveal that graphical views similar to the one in Figure 1 were frequently obtained by students. Actually, all the video recorded pairs obtained a similar view. This, in turn, caused difficulties when they were encouraged to attach a point at the graph and (a) describe how they could use the point to check their calculation in the preceding task, in this case moving the point so that its *x*-coordinate becomes 10 and read off the corresponding *y*-value and (b) decide the value of *x* for a specific value of *y*. With a graphical view like the one in Figure 1 it turned out to be impossible to use the Graphics View to read off the corresponding value of *y* when of $x = 10$ without changing the scale of the *x*-axis. One of the video recorded pairs of students expressed their confusion:

Student 1: It looks like we have done wrong.
Student 2: mm
Student 1: Go back!
Student 2: We check a little more...lock from the start. There is nothing wrong with the equation.

Having convinced themselves that the formula is correct and that they understand what the point represents, they felt that they got stuck and asked the teacher for help. Notably, they did not reflect on the scales of the axes. Reminded of the possibility of changing the *x*-axis, these students solved the task.

The audio recordings also revealed this kind of confusion on several occasions. Consequently, the teachers often had to remind and instruct the students how to scale the *x*-axis to receive integer marks and to obtain a better visual appearance of the graph.

However, it was observed how one of the video recorded pairs tackled the problem caused by inappropriate scaling of the *x*-axes (see Figure 1), by only observing the Algebraic View while solving the task. They moved the point along the graph until the *x*-coordinate of the point shown in the algebraic view became 10.

Although the participating students when entering the first two task sequences seemed to lack experience of adjusting one axis at a time, to obtain an accessible graphical view, the video recordings indicate an enhanced ability among the students to perform this when tackling the third task sequence.

CONCLUSION AND DISCUSSION

The aim of this paper was to identify elements affecting the process of instrumental genesis concerning appropriate scaling of the coordinate system in a dynamic mathematics software environment. During their work with the tasks, the participating students encountered situations which required rescaling of the axes; to see an object in the coordinate system and/or to obtain an appropriate visible appearance of the object(s). The following closely related elements of instrumented action schemes were recognized:

1) Knowing how to change the scale of the axes, both by using the zoom operation and by adjusting one axis at a time.

2) Realizing when it is necessary to adjust the *y*-axis to see an object, for example, points or a graph.

3) Realizing when it is appropriate to change the scale of the *x*-axis to obtain a better visible picture of the objects.

It is possible to distinguish between the technical and conceptual character of the elements (Drijvers & Gravemeijer, 2005). While the first item primarily requires technical capabilities the other two items primarily demand conceptual knowledge. In this study, the mathematical knowledge required to be able to rescale the axes in an appropriate way is about the range and domain of functions representing real world situations. Concerning the instrumented techniques of changing scales of the axes, the result indicates that students are disposed to employ the technique of zoom operation, which aligns with the findings already reported by Goldenberg (1988) more than two decades ago. This sometimes gave rise to obstacles when the visual appearance of an object, for example, a graph, made it hard or even impossible to solve problems graphically.

One reason for students' preference for using the zoom operation technique might be that students already are familiar with this technique from the use of other screen based technologies, for example, smart phones. In comparison, they did not adopt the technique of adjusting one axis at a time so easily. The reason for this might be that students lack of prior experiences of this kind of technique.

Another reason might be the features of this kind of tool in the particular software under consideration, i.e., *GeoGebra*. While the zooming tool, and the associated zoom operation technique, is readily available, the technique of scaling one axis at a time is more demanding, probably because the associated usage schemes involve knowledge of tools with limited accessibility in *GeoGebra*.

A further observation made in this study is that students tend to neglect the written instructions on using the computer, at least when they do not see any immediate need for them. When, later on in the task sequence, they encountered problems due to their inattention to the instructions, they asked the teacher for help. The reason why students tend to disregard such instructions may be that they do not usually read instructions while using digital technology.

To summarize, the findings highlight various obstacles which arose in the course of instrumental genesis under the instrumental orchestration of *GeoGebra* use provided by the teaching sequence and task environment as originally designed. These findings suggested ways in which that orchestration might be redesigned in order to better support students' development of the desired instrumentation schemes. In particular, they highlight the potential importance of taking account of students' previous experiences both regarding the use of technology in mathematics and the use of every-day technology, e.g., smart phones. Furthermore, these findings emphasize how students draw on existing instrumentation schemes, developed in relation to what they perceive as similar tools, when they start to work with a new tool; instrumental orchestration needs to plan for such transitions, taking account both of the continuities and discontinuities between tools.

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ENDNOTES

1. In this paper, the terms of 'tools' and 'artefacts' are used interchangeably.

2. We assume that students have obtained an equally scaled coordinate system showing *y*-values between 0 and 16 and *x*-values between 0 and 28.