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Student understanding of linear independence of functions

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We present findings regarding student understanding of linear independence of vector-valued functions, a concept common in linear algebra and differential equations. Data were collected from written homework questionnaires and a paired interview, prior to any formal instruction about the definition of linear independence of functions. Grounded theory was used to categorize student approaches, resulting in five main categories: Function Combination, Focus on t Values, Focus on Graphs, Focus on Scalars, and Previous Rule. Results suggest that students' transition from linear independence of vectors in Euclidean space to linear independence of functions in function space is not trivial and that fostering an object view of function may facilitate students more readily operating with functions as elements of a vector space.

Keywords : Linear algebra, linear independence, function space, Euclidean space.

Linear algebra and differential equations are important courses for mathematics and engineering students, and research shows that students tend to struggle with these courses (e.g., Dorier, 2000). While a growing body of literature exists about student understanding of linear independence of vectors in \mathbb{R}^n , few empirical studies report on student understanding of linear independence of functions. In one report, Harel (2000) contended that students might struggle to determine $A = \{x, x^2, x^3, x^4\}$ is linearly independent if they “have not formed the concept of function as a mathematical object, as an entity in a vector space” (p. 181). The focus of this study is on how students make sense of linear independence of vector-valued functions, a concept common in linear algebra and differential equations. In particular, we investigate students' initial notions of how linear independence in Euclidean space might extend to linear independence in function spaces.

THEORETICAL FRAMEWORK AND METHOD

This study is framed by Cobb and Yackel's (1996) emergent perspective, which coordinates the individual cognitive perspective of constructivism and a socio-cultural perspective based on symbolic interactionism. Within our analysis, we assume that learners acquire knowledge from their daily experiences, that prior conceptions affect interaction with new ideas, and that knowledge structures are contextually dependent.

Data were collected during the Fall 2012 semester in an honors linear algebra and differential equations course for first-year mathematics or engineering students. Data sources include a semi-structured pairwise interview and written responses to two homework surveys collected prior to students' encounter with the formal definition of linear independence of functions. This timing allowed us to capitalize on a unique opportunity to investigate how students initially make sense of linear independence of vector-valued functions with only their prior understanding of vectors in \mathbb{R}^n . Questions analyzed in this study, written exactly as they were presented to students, are given in Figure 1. Grounded theory (Strauss & Corbin, 1990) was used to analyze student responses, which led to five categories of student approaches.

SUMMARY OF RESEARCH RESULTS

We analysed two aspects of student responses: their problem solving approach and their answer for if the given functions were linearly independent (LI), linearly dependent (LD), or something else. For the first aspect, we found student approaches could be sorted into five main categories: *Function Combination*, *Focus on t Values*, *Focus on Graphs*, *Focus on Scalars*, and *Previous Rule*. The *Function Combination* approach, for example, which is typified by a primary focus

1. Consider the functions $f(t) = 2$ and $g(t) = \sin t$ with $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$. Would you say these functions are linearly dependent or independent for all $t \in \mathbb{R}$? Explain.
2. Consider the functions $F(t) = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$ and $G(t) = \begin{bmatrix} \sin t \\ 0 \end{bmatrix}$ with $F: \mathbb{R} \rightarrow \mathbb{R}^2$ and $G: \mathbb{R} \rightarrow \mathbb{R}^2$. Would you say these functions are linearly dependent or independent for all $t \in \mathbb{R}$? Explain.
3. Consider the functions $F(t) = \begin{bmatrix} t \\ 1 \end{bmatrix}$, $G(t) = \begin{bmatrix} t^2 \\ 2 \end{bmatrix}$, and $H(t) = \begin{bmatrix} t^3 \\ 0 \end{bmatrix}$ with $F: \mathbb{R} \rightarrow \mathbb{R}^2$, $G: \mathbb{R} \rightarrow \mathbb{R}^2$, and $H: \mathbb{R} \rightarrow \mathbb{R}^2$. Would you say these functions are linearly dependent for all $t \in \mathbb{R}$? Explain your reasoning.

Figure 1: Questions analysed about linear independence of vector-valued functions

on whether or not the functions could be written as a linear combination of one another, supported the correct answer (LI) across the three questions. The *Focus on t Values* approach, which never supported a correct answer in this data, is characterized by first evaluating the functions at specific t values and then making conclusions about vectors in \mathbb{R}^n rather than about functions in a function space. For the second aspect, we categorized student answers for each question. For instance, for Question 2, twelve students stated correctly LI, six stated LD, and four stated LI at some t values and LD at others. A description of all results, with examples, can be found in the poster presented and discussed at the conference at megan-wawro.com/presentations.

We acknowledge that the students' ways of reasoning captured in both their approach and final answer may have been influenced by the formulation of the questions in Figure 1. For instance, the phrase "for all $t \in \mathbb{R}$ " was included to align with the formal definition for linear independence of functions the instructor had used; however, this may have led to more approaches aligning with the *Focus on t Values* category than if the questions had not included that phrase. Similarly, students who stated linearly independent at some discrete t values but dependent at others may have been influenced by the inclusion of that phrase.

This preliminary work illustrates that students' transition from linear independence of vectors in \mathbb{R}^n to linear independence of functions in function spaces is not trivial. The *Function Combination* approach, however, not only led to correct answers but also seemed within reach for these novice students; as such, it deserves consideration as a fruitful approach to be leveraged as students encounter linear independence of functions. This further suggests an importance of fostering an object view of function so that students can more readily operate with them as elements in a vector space.

REFERENCES

- Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist*, 31, 175–190.
- Dorier, J.-L. (Ed.). (2000). *On the teaching of linear algebra*. Dordrecht, The Netherlands: Kluwer.
- Harel, G. (2000). Three principles of learning and teaching mathematics: Particular reference to linear algebra – Old and new observations. In J.-L. Dorier (Ed.), *On the teaching of linear algebra* (pp. 177–190). Dordrecht, The Netherlands: Kluwer.
- Strauss, A., & Corbin, J. (1990). *Basics of Qualitative Research: Grounded Theory Procedures and Techniques*. Newbury Park, CA: Sage Publications.