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# Typology of incoherence due to the didactic transposition in Quebec textbooks introducing abstract algebra

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## INTRODUCTION

As shown in the Quebec Ministry of Education curriculum, the first notions of abstract algebra are presented in the linear algebra class, at CEGEP level (students of 17 to 19 years old). After a census of the 13 French CEGEP of Montreal region, which offer pre-university preparation for mathematics, we found out that four textbooks (regularly re-edited) are used by the majority of teachers as guidance for this class (Ouellet, 2002; Amyotte, 2009; Charron & Parent, 2005; Papillon, 1993, 2012). Unlike high schools, CEGEP are free to choose the didactic material they propose to their students; textbooks do not have to be approved by a Ministerial commission. However, for the sake of consistency, editors try to respect ministerial guidance and organise the textbooks following objectives given by the ministry. Notably, amongst those programme objectives (MELS, 2010, p. 5), we found that the most relevant to the class of linear algebra students were:

- identify a certain amount of ideas in link with the subject to compare, classify and evaluate them;
- link pertinent ideas in a logical order;
- building a coherent argument / reasoning / proof (loose translation).

### Goal of the research

It therefore seems relevant for us to inspect to what extent the organisation of didactic sequences of these textbooks permits the attainment of the ministerial objectives. In reference to the aforementioned ministerial objective, we also want to study what are the valid domains of reasoning in the different textbooks.

We propose the outline of a typology of incoherence revealed following the textbook analysis.

## FIRST ANALYSIS AND RESULTS

Our analysis revealed that, in terms of style in which linear algebra is taught (Dorier, 1997), the way usually chosen by Quebec authors consists of building on previous notions, already introduced to the students, to present their generalisations [1]. However, when the notion with a high level of abstraction is presented, very few references are shown with a low level abstraction. We can observe here a first breach in the presentation since these references could serve as examples and these passages could allow to work on the comparison and classification of ideas as mentioned in the ministerial objectives. More generally, by making a crossing between the typology of abstraction levels and the notion of didactic transposition (as in Chevallard, 1985), three categories of incoherence were identified:

- 1) Breach in the generalisation process: This includes elements such as the circular generalisations (lack of distinction between the didactic transposition levels) and the lack of examples (either prior to introduce generalisation, or to verify the coherence of the previous knowledge of low level of abstraction in this generalised theory). We also include here breaches in the deductive structure (presentation order of notions.)
- 2) Unnecessary generalisation and presentation of mathematical concepts of low relevance within the deductive sequence: This includes elements such as the introduction of concepts which are never revisited or in only in an artificial and

technical way, solely to achieve technical ends. This category can also generate breach in the deductive structure.

- 3) False concept and creation of didactic obstacles (in the sense defined by Brousseau, 1998): This includes elements such as the introduction of mathematical concepts by failing to mention their range of validity. These concepts turn out to be incorrect once they are evaluated out of this implicit range. For example, to consider a vector and a scalar quotient only makes sense in real vectorial spaces.

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## ENDNOTE

1. In reference to a term used by Aristotle (and notice by Cleary, 1985), notions from low level of abstraction are generally used as foundation for the presentation of high level abstraction notions.