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# The Adaptive Controlled Stratification Method Applied to the Determination of Extreme Interference Levels in EMC Modeling with Uncertain Input Variables

Mourad Larbi, Philippe Besnier, *Senior Member, IEEE* and Bernard Pecqueux

**Abstract**—This paper deals with electromagnetic compatibility simulations at early design stage of equipment or systems development. In this context, expensive simulations based on rigorous modeling are performed, including numerous uncertain variables. The most important configurations are those associated to extreme values of the observed quantity. In this paper we introduce a variance reduction technique to accelerate the estimation of an extreme quantile of the output probability distribution. The approach is based on using a simple model (at a low computational cost) to identify relevant realizations of uncertain variables in strata partitioning the output space of the model. Application of the method is detailed on a rather simple cable system in order to estimate an extreme quantile level of an interfering current. We show that extreme current values are obtained at a reduced computational cost compared to a standard empirical quantile estimation.

**Index Terms**—computational electromagnetics, crosstalk, electromagnetic interference, electromagnetic susceptibility, quantile estimation, uncertainty propagation, variance reduction.

## I. INTRODUCTION

One of the main challenges for electromagnetic compatibility (EMC) modeling consists in giving, in a probabilistic sense, the better solution according to a given risk of interference. This estimation depends on many input parameters, including various unknown ones that may be considered as random in a first place. These evaluations require sophisticated models at different scales of a system (electromagnetic field distribution, interaction with cables, electronic circuits) taking a lot of modeling resources including in terms of software and computation time.

Obtaining with efficiency the moments of the response probability distribution of a system stressed by a source of interference, has been the scope of research for some researchers in the recent years. Different techniques have

been introduced such as polynomial chaos, stochastic collocation, unscented transforms and design of experiments [1]–[8]. Experimental design techniques aim at building metamodels of the interest response. Such metamodels were also used in a series of works on space-mapping technique introduced by [9] for antenna applications. Most of these methods were developed to identify the main trend of the response. They are therefore not suitable for the assessment of extreme events probability. Since only a few techniques deal with the estimation of extreme values [10], [11], we recently suggested the use of a reliability approach which aims at approximating the failure domain that corresponds to the probability of exceeding a threshold value [12]. These techniques are well suited to determine the sensitivity of observations with respect to many random variables or combination of them.

This paper is dedicated to an alternative approach called controlled stratification (CS) which was recently proposed and enhanced in the field of applied statistics [13]. This approach has two main features of interest for EMC applications. First, it aims specifically at accelerating the process of estimating an extreme quantile of the output probability distribution, which could represent values at risk for an EMC point of view. Second, this method aims at using a rigorous but time consuming computation method in a less intensive way, through the control of a much less sophisticated model, but still correlated, in a certain way, to the original one. Once such a model exists, it helps in elaborating a sampling pattern that will accelerate in a very significant manner the estimation of the probability of extreme events with the same uncertainty as a standard Monte Carlo simulation (MCS) with a large sample size would provide.

This paper is composed as follows. First, the detailed principle of the CS method is presented in Section II. A specific attention is paid to the definition of a correlation coefficient between the simple and the rigorous calculation method. Furthermore, this section introduces a refined algorithm called the adaptive controlled stratification (ACS) that potentially enhances the efficiency of the method. Section III presents the case study and Section IV and V are application sections highlighting the properties of the CS method and its ACS version. The considered scenario is that of a propagation of interference within a network

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of electrical wires above a ground plane. In both sections, the rigorous method is based on a Method of Moments (MoM) Maxwell's equations solver. The simple model is based on the transmission line theory (TLT) in Section IV or on a coarse mesh for the numerical solution of MoM solver in Section V. Finally, Section VI illustrates that the amount of extreme values obtained from the CS method allows fitting a Generalized Pareto distribution.

## II. CONTROLLED STRATIFICATION

### A. Presentation of the Controlled Stratification Method

Readers not acquainted with probability theory and statistics, may refer to [14], [15].

Suppose that  $Y = f(\mathbf{X})$ , where  $\mathbf{X}$  of size  $q$  is a random vector describing the uncertainties of input parameters,  $Y$  is the random response of a rigorous but expensive computer code  $f$ . We also assume that a simple model  $f_s$  exists:  $Z = f_s(\mathbf{X})$ . The response  $Z$  of this substitute model is a rough approximation of  $f$ .

The CS method proposes a strategy allowing to use the simple model  $f_s$ , to estimate extreme values with a reduced number of calls to the expensive computer code  $f$ .

1) *Quantile Estimation by Empirical Method:* Let us consider that we want to estimate the quantile of order  $\alpha$  of the output  $Y$ , denoted  $y_\alpha$ . Generally, the quantile estimation is performed via the estimation of the cumulative distribution function (cdf), which leads then to an estimator of the quantile  $y_\alpha$ . Let us define a  $n$ -sample  $(Y_1, \dots, Y_n)$ , which is a set of  $n$  independent and identically distributed (i.i.d) random variables with the pdf  $p(y)$  of  $Y$ . The empirical estimator (EE) of the cdf  $F(y)$ , denoted  $\hat{F}_{EE}(y)$ , is obtained as:

$$\hat{F}_{EE}(y) = \frac{1}{n} \sum_{i=1}^n \mathbb{1}_{Y_i \leq y} \quad (1)$$

where  $\mathbb{1}_{Y_i \leq y}$  is the indicator function that equals 1 when  $Y_i \leq y$ , and 0 otherwise. It is asymptotically normal by

$$\sqrt{n}(\hat{F}_{EE}(y) - F(y)) \xrightarrow[n \rightarrow +\infty]{} \mathcal{N}(0, \sigma_{EE}^2), \quad (2)$$

$$\sigma_{EE}^2 = F(y)(1 - F(y)).$$

The quantile estimation of order  $\alpha$  is given by

$$\hat{Y}_{EE}(\alpha) = \inf\{y, \hat{F}_{EE}(y) > \alpha\} \quad (3)$$

which is also asymptotically normal:

$$\sqrt{n}(\hat{Y}_{EE}(\alpha) - y_\alpha) \xrightarrow[n \rightarrow +\infty]{} \mathcal{N}(0, \sigma_{EE}^2), \quad (4)$$

$$\sigma_{EE}^2 = \frac{\alpha(1 - \alpha)}{p^2(y_\alpha)}.$$

This asymptotic result highlights that the variance is higher when an extreme quantile is researched since  $p(y_\alpha)$  is small.

In order to reduce the quantile variance, different approaches such as quantile estimation by control variate (CV), and quantile estimation by CS have been introduced. These techniques are now presented.

2) *Quantile Estimation by Control Variate:* This section presents the variance reduction approach based on using the simple model  $Z = f_s(\mathbf{X})$  as a control variate. Relying on the simple model  $Z$ , it is possible to evaluate the quantiles  $z_\alpha$  or any expectation  $\mathbb{E}[h(Z)]$  of a function of  $Z$  by MCS. An estimator of the cdf  $F(y)$  given by a control variate  $Z$  is written as:

$$\hat{F}_{CV}(y) = \hat{F}_{EE}(y) - C(\hat{h}_n - \mathbb{E}[h(Z)]), \quad (5)$$

where the function  $h : \mathbb{R} \rightarrow \mathbb{R}$  is defined by the user [16] and  $\hat{h}_n = \frac{1}{n} \sum_{j=1}^n h(Z_j)$ . The optimal parameter  $C$  is the correlation coefficient between  $h(Z)$  and  $\mathbb{1}_{Y \leq y}$ . This value is unknown in practice, therefore it is estimated by the parameter  $\hat{C}$ , defined as the slope estimator obtained from a least-squares regression of  $\mathbb{1}_{Y_j \leq y}$  on  $h(Z_j)$ :

$$\hat{C} = \frac{\sum_{j=1}^n (\mathbb{1}_{Y_j \leq y} - \hat{F}_{EE}(y))(h(Z_j) - \hat{h}_n)}{\sum_{j=1}^n (h(Z_j) - \hat{h}_n)^2}. \quad (6)$$

It is shown in [17] that the estimator  $\hat{F}_{CV}(y)$  with the estimated correlation coefficient  $\hat{C}$  can be rewritten by introducing the weights  $W_j$  as

$$\hat{F}_{CV}(y) = \sum_{j=1}^n W_j \mathbb{1}_{Y_j \leq y}, \quad (7)$$

where  $W_j = \frac{1}{n} + \frac{(\hat{h}_n - \mathbb{E}[h(Z)])(\hat{h}_n - h(Z_j))}{\sum_{i=1}^n (h(Z_i) - \hat{h}_n)^2}$  with  $\sum_{j=1}^n W_j = 1$ .

If we choose  $h(Z) = \mathbb{1}_{Z \leq z_\alpha}$ , then  $\mathbb{E}[h(Z)] = \alpha$  and  $\hat{h}_n = \frac{N_0}{n}$  with

$$N_0 = \sum_{j=1}^n \mathbb{1}_{Z_j \leq z_\alpha} \text{ and } W_j = \frac{\alpha}{N_0} \mathbb{1}_{Z_j \leq z_\alpha} + \frac{1 - \alpha}{n - N_0} \mathbb{1}_{Z_j > z_\alpha}. \quad (8)$$

The estimator (7) has been shown by [18], as equivalent to the maximum likelihood estimator for probabilities. Since the estimators of Monte Carlo are convergent [16], we obtain

$$\sqrt{n}(\hat{F}_{CV}(y) - F(y)) \xrightarrow[n \rightarrow +\infty]{} \mathcal{N}(0, \sigma_{CV}^2), \quad (9)$$

$$\sigma_{CV}^2 = F(y)(1 - F(y))(1 - \rho_I(\alpha)^2),$$

where  $\rho_I(\alpha)$  is the correlation coefficient between  $\mathbb{1}_{Y \leq y}$  and  $\mathbb{1}_{Z \leq z_\alpha}$  given by

$$\rho_I(\alpha) = \frac{\mathbb{P}(Y \leq y, Z \leq z_\alpha) - \alpha F(y)}{\sqrt{F(y)(1 - F(y))} \sqrt{\alpha - \alpha^2}}. \quad (10)$$

The objective is now to estimate the  $\alpha$ -quantile of  $Y$  using the CV estimator  $\hat{F}_{CV}(y)$  of the cdf of  $Y$ . Let the order statistics  $(Y_{(1)}, \dots, Y_{(n)})$ , where the sample is sorted in ascending order with the corresponding weights  $W_{(i)}$  given in (8). Relying on the estimator (7) of the cdf  $F(y)$ ,

the CV estimator of the  $\alpha$ -quantile is

$$\hat{Y}_{CV}(\alpha) = Y_{(l)}, \quad l = \inf \left\{ k, \sum_{j=1}^k W_{(j)} > \alpha \right\}. \quad (11)$$

Using the results from the variance reduction for Monte Carlo techniques [15], this estimator  $\hat{Y}_{CV}(\alpha)$  is also asymptotically normal

$$\begin{aligned} \sqrt{n}(\hat{Y}_{CV}(\alpha) - y_\alpha) &\xrightarrow{n \rightarrow +\infty} \mathcal{N}(0, \sigma_{CV}^2), \\ \sigma_{CV}^2 &= \frac{\alpha(1-\alpha)}{p(y_\alpha)^2} (1 - \rho_I^2(\alpha)), \end{aligned} \quad (12)$$

where  $\sigma_{CV}^2$  is the reduced variance and  $\rho_I(\alpha)$  is the correlation coefficient between  $\mathbb{1}_{Z_i \leq z_\alpha}$  and  $\mathbb{1}_{Y_i \leq y_\alpha}$ :

$$\rho_I(\alpha) = \frac{\mathbb{P}(Y \leq y_\alpha, Z \leq z_\alpha) - \alpha^2}{\alpha - \alpha^2}. \quad (13)$$

We can see that the variance of  $\hat{Y}_{CV}(\alpha)$  is reduced by a factor of  $(1 - \rho_I^2(\alpha))$  with respect to the variance of the empirical estimator  $\hat{Y}_{EE}$  given in (4). This underlines that the higher is the correlation coefficient  $\rho_I(\alpha)$ , the more the quantile variance  $\sigma_{CV}^2$  is reduced [13].

Finally, the estimator  $\hat{\rho}_I(\alpha)$  of the correlation coefficient is then given at the top of the following page by (14), with  $\hat{h}_n = \frac{1}{n} \sum_{j=1}^n \mathbb{1}_{Z_j \leq z_\alpha}$ .

### 3) Quantile Estimation by Controlled Stratification:

Both simple model  $Z = f_s(\mathbf{X})$  and rigorous model  $Y = f(\mathbf{X})$  provide response realizations. The simple model may lead to significant deviation of the response with respect to that of the rigorous model. However, it is required that its sensitivity to input realizations, remain similar to that of the rigorous model, especially for extreme response realizations of  $Y$ . This is quantified, only a posteriori, with the correlation criterion given by (14).

Thereby, we introduce the CS technique consisting in combining the simple model and the rigorous model for an extreme quantile estimation. More precisely, the simple model is used to stratify the output space of  $Z = f_s(\mathbf{X})$  in  $r$  particular intervals  $I_1, \dots, I_r$  called strata. Once these intervals are identified, realizations of  $\mathbf{X}$  are chosen in each of them and exploited to assess the rigorous model  $Y = f(\mathbf{X})$ . By doing so, we increase the number of adequate realizations for the estimation of the researched quantile [13].

Let  $r + 1$  quantiles  $-\infty = z_{\alpha_0} < z_{\alpha_1} < \dots < z_{\alpha_r} = \infty$  related to the probabilities  $0 = \alpha_0 < \alpha_1 < \dots < \alpha_r = 1$ . We define by strata the intervals  $]z_{\alpha_{j-1}}, z_{\alpha_j}]$ . Using the formula of total probability, the cdf of  $Y$  can be given by

$$F(y) = \sum_{j=1}^r \mathbb{P}(Y \leq y | Z \in ]z_{\alpha_{j-1}}, z_{\alpha_j}]) \cdot (\alpha_j - \alpha_{j-1}). \quad (15)$$

Thus, the estimation of  $F(y)$  requires the computation of the following conditional probabilities

$$p_j(y) = \mathbb{P}(Y \leq y | Z \in ]z_{\alpha_{j-1}}, z_{\alpha_j}]). \quad (16)$$

This leads to introduce a sequence of integers  $N_1, \dots, N_r$  such as  $\sum_{j=1}^r N_j = n$ . At the beginning, we draw  $N_j$

realizations of input random variables  $(X_i^{(j)})_{i=1, \dots, N_j}$  to obtain  $N_j$  responses  $Z_i^{(j)}$  in the stratum  $]z_{\alpha_{j-1}}, z_{\alpha_j}]$ . Then, we compute for these input realizations, the responses of the rigorous model  $Y_i^{(j)}$ . Once these responses are obtained, the quantities  $p_j(y)$  are estimated by

$$\hat{p}_j(y) = \frac{1}{N_j} \sum_{i=1}^{N_j} \mathbb{1}_{Y_i^{(j)} \leq y}. \quad (17)$$

Finally, the estimator  $\hat{F}_{CS}(y)$  of the cdf  $F(y)$  is obtained from:

$$\hat{F}_{CS}(y) = \sum_{j=1}^r \hat{p}_j(y) (\alpha_j - \alpha_{j-1}). \quad (18)$$

This estimator is unbiased, i.e.  $\mathbb{E}[\hat{F}_{CS}(y)] = F(y)$  and its variance is:

$$\text{Var}[\hat{F}_{CS}(y)] = \sum_{j=1}^r \frac{(\alpha_j - \alpha_{j-1})^2}{N_j} (p_j(y) - p_j(y)^2). \quad (19)$$

When the number of strata  $r$  is defined, we introduce positive real numbers  $\beta_j$  such as  $\sum_{j=1}^r \beta_j = 1$  and  $N_j = [n\beta_j]$ , where  $[a]$  is the integer closest to  $a$ , then the estimator  $\hat{F}_{CS}(y)$  is asymptotically normal by

$$\begin{aligned} \sqrt{n}(\hat{F}_{CS}(y) - F(y)) &\xrightarrow{n \rightarrow +\infty} \mathcal{N}(0, \sigma_{CS}^2), \\ \sigma_{CS}^2 &= \sum_{j=1}^r \frac{(\alpha_j - \alpha_{j-1})^2}{N_j} (p_j(y) - p_j(y)^2). \end{aligned} \quad (20)$$

If  $Z$  and  $Y$  are independent then  $p_j(y) = F(y)$  and the variance of  $\hat{F}_{CS}(y)$  becomes

$$\text{Var}[\hat{F}_{CS}(y)] = \sum_{j=1}^r \frac{(\alpha_j - \alpha_{j-1})^2}{N_j} (F(y) - F(y)^2). \quad (21)$$

Taking  $N_j = (\alpha_j - \alpha_{j-1}) \cdot n$ , it can be noticed that the variance of the estimator (21) would be identical to that of the empirical estimator:  $\text{Var}[\hat{F}_{CS}(y)] = \frac{1}{n} [F(y) - F(y)^2] = \text{Var}[\hat{F}_{EE}(y)]$ . In this case, the CS technique is therefore useless.

Otherwise, if  $Z$  and  $Y$  are positively correlated, this leads to  $\forall y \in ]y_{\alpha_{j-1}}, y_{\alpha_j}]$ :

$$\begin{aligned} \text{Var}[\hat{F}_{CS}(y)] &= \frac{(\alpha_j - \alpha_{j-1})^2}{N_j} (p_j(y) - p_j(y)^2) \\ &\leq \frac{(\alpha_j - \alpha_{j-1})^2}{4N_j}. \end{aligned} \quad (22)$$

Fixing  $\alpha_j = \frac{j}{r}$  and  $N_j = \frac{n}{r}$  and if the outputs  $Z$  and  $Y$  are strongly correlated, the variance of the estimator  $\hat{F}_{CS}(y)$  becomes  $\text{Var}[\hat{F}_{CS}(y)] = \frac{1}{4rn}$ . We notice that the variance is reduced by a factor  $r$  with respect to that of the empirical estimator (2). This underlines that the variance reduction can be quite large when the variables  $Z$  and  $Y$  are well correlated.

Moreover, we show now that it is relevant to raise the realization number in the distribution tail of the output  $Z$  in order to have a better estimation of an extreme quantile  $y_\alpha$  of the output  $Y$ . Let us suppose that we are interested

$$\hat{\rho}_I(\alpha) = \frac{\sum_{j=1}^n (\mathbb{1}_{Y_j \leq y} - \hat{F}_{EE}(y)) (\mathbb{1}_{Z_j \leq z_\alpha} - \hat{h}_n)}{\sqrt{\sum_{j=1}^n (\mathbb{1}_{Y_j \leq y} - \hat{F}_{EE}(y))^2} \sqrt{\sum_{j=1}^n (\mathbb{1}_{Z_j \leq z_\alpha} - \hat{h}_n)^2}} \Bigg|_{y=\hat{Y}_{CV}(\alpha)} \quad (14)$$

in estimating the distribution tail of the cdf  $F(y)$ , in the zone where  $F(y) \approx 1 - \varepsilon$  with  $0 < \varepsilon \ll 1$ . To present the technique benefit, we define  $r = 4$  strata with  $\alpha_1 = 1/2$ ,  $\alpha_2 = 1 - 2\varepsilon$ ,  $\alpha_3 = 1 - \varepsilon$ , and we take  $N_j = n/4$  ( $j = 1, \dots, 4$ ) realizations in each stratum. This strategy locates  $\frac{n}{2}$  realizations in the distribution tail, where  $Z > z_{1-2\varepsilon}$ . Two situations may be assumed:

- 1) If  $Z$  and  $Y$  are independent, this strategy means that the variance given by (21) is  $\text{Var}[\hat{F}_{SC}(y)] \approx 2\varepsilon/n$ . This leads to a raising variance with respect to the empirical estimation:  $\text{Var}[\hat{F}_{EE}(y)] = \varepsilon/n$ .
- 2) If  $Z$  and  $Y$  are largely correlated, the variance given in (22) is then reduced with respect to the empirical estimation since  $\text{Var}[\hat{F}_{CS}(y)] = \varepsilon^2/n$ . In this situation, the variance reduction is very significant.

From the estimator  $\hat{F}_{CS}(y)$  of the cdf  $F(y)$ , the estimator of the quantile of order  $\alpha$  given by CS, denoted  $\hat{Y}_{CS}(\alpha)$  writes as:

$$\hat{Y}_{CS}(\alpha) = \inf\{y, \hat{F}_{CS}(y) > \alpha\}. \quad (23)$$

This quantile  $\hat{Y}_{CS}(\alpha)$  is asymptotically normal

$$\sqrt{n}(\hat{Y}_{CS}(y) - y_\alpha) \xrightarrow[n \rightarrow +\infty]{} \mathcal{N}(0, \sigma_{CS}^2), \quad (24)$$

$$\sigma_{CS}^2 = \frac{\sum_{j=1}^r \frac{(\alpha_j - \alpha_{j-1})^2}{\beta_j} (p_j(y_\alpha) - p_j(y_\alpha)^2)}{p(y_\alpha)^2}.$$

In summary, the benefit of the CS method relies on the correlation between simple and rigorous models, and the allocation strategy chosen in the strata.

### B. Adaptive Controlled Stratification (ACS)

As mentioned above, the allocation strategy in the  $r$  strata impacts directly the efficiency of the quantile estimation  $y_\alpha$  of  $Y$ . Thus, we present in this section an adaptive strategy allowing to identify a possibly better allocation of realizations of input random variables for the quantile estimation  $y_\alpha$ . The choice of this allocation depends on probabilities  $p_j(y)$  that we have to estimate. This adaptive technique is described as follows:

- 1) Apply the CS technique with an a priori choice of  $\tilde{n} = n^\mu$ ,  $\mu \in ]0, 1[$  and a number of allocation  $\beta_j$  in the strata; Then, we estimate the conditional probabilities  $p_j(y)$  by

$$\tilde{p}_j(y) = \frac{1}{[\beta_j \tilde{n}]} \sum_{i=1}^{[\beta_j \tilde{n}]} \mathbb{1}_{Y_i^{(j)} \leq y}.$$

- 2) The estimation of the optimal allocation denoted  $\tilde{\beta}_j$

is then given by

$$\tilde{\beta}_j = \frac{(\alpha_j - \alpha_{j-1}) [\tilde{p}_j(y) - \tilde{p}_j(y)^2]^{1/2}}{\sum_{k=1}^r (\alpha_k - \alpha_{k-1}) [\tilde{p}_k(y) - \tilde{p}_k(y)^2]^{1/2}}$$

- 3) Use the  $n - \tilde{n}$  last simulations in allocating them in each stratum so that we get the estimation of the optimal number  $[n\tilde{\beta}_j]$  for all  $j$ .
- 4) The estimation of  $p_j(y)$  and  $F(y)$  is then given by

$$\hat{p}_j(y) = \frac{1}{[\tilde{\beta}_j n]} \sum_{i=1}^{[\tilde{\beta}_j n]} \mathbb{1}_{Y_i^{(j)} \leq y},$$

$$\hat{F}_{ACS}(y) = \sum_{j=1}^r \hat{p}_j(y) (\alpha_j - \alpha_{j-1}).$$

The estimator  $\hat{F}_{ACS}(y)$  is asymptotically normal:

$$\sqrt{n}(\hat{F}_{ACS}(y) - F(y)) \xrightarrow[n \rightarrow +\infty]{} \mathcal{N}(0, \sigma_{ACS}^2),$$

$$\sigma_{ACS}^2 = \left( \sum_{j=1}^r (\alpha_j - \alpha_{j-1}) [p_j(y) - p_j(y)^2]^{1/2} \right)^2. \quad (25)$$

The a priori choice of the positive numbers  $\beta_j$  does not affect the ACS method convergence but a good choice of these one speeds it up.

### C. Discussion about the Choice of a Good Simple Model

The correlation between simple and rigorous models is a key point of the CS method efficiency. As already mentioned, the simple model has to reproduce the same sensitivity related to input realizations providing extreme values. More precisely, a set of input realizations having response quantities above the quantile of order  $\alpha$  of the simple model, has to provide (i.e. with a high probability) output realizations above the associated quantile of the rigorous model. For this reason, a simple model is said to be well correlated to the rigorous one, if it can reproduce the same trend related to input variables than the rigorous one, for the estimation of rare events.

In practice, this level of correlation is quantified through the operator defined in (14), once computations are performed for the same sample, with the two models. This correlation estimation is only carried out to control the quality of the simple model, and to confirm that it will speed up the identification of extreme response realizations. Once a good simple model is identified, this step does not need to be performed. In a real case study, only a rough estimation of this correlation is available due to expensive computation time of rigorous models. However, in Sections IV and V, for the only purpose of validation, we will use a large sample size to estimate that correlation.

Identification of relevant simple models for EMC purposes is out of the scope of this paper but a few tracks may be followed. A physical model based on a 1D or 2D approximation for a complex 3D computation is a possibility. An engineer model of a complex situation is another track (for example a circuit model of interference propagation on a chip of a printed circuit board). A coarse mesh approximation is also another candidate as illustrated in Section V.

### III. CASE STUDY

A case study is used throughout the rest of this article to highlight the application of the CS/ACS techniques.

The electromagnetic interference problem chosen deals with a crosstalk issue within an interconnected wire system above a perfect electric conductor (pec) ground (Fig. 1). Wire n°1 is composed of a first section of length  $L_1$  parallel to (Ox) until the junction, and a second section of length  $L_3$  deviating of an angle  $a_1 = 30^\circ$  with respect to (Ox). This wire is situated at a height  $h_1$  above the pec ground being fed by an electromotive force  $e = 1$  V. Wire n°2 also consists of a first section of length  $L_1$  parallel to (Ox) until the junction. At the level of the junction, wire n°2 is then divided into two branches (with a perfect electric connection): a branch of length  $L_3$  deviated of  $a_1 = 30^\circ$  with respect to (Ox), and a second branch veered to  $a_2 = 45^\circ$  with respect to (Ox) in the opposite direction. This wire is located at a height  $h_2$  above the pec ground. Wire n°3, situated at a height  $h_3$  above the pec ground, is also composed of two sections: the first one of length  $L_1$  is parallel to (Ox), the second one of length  $L_2$  makes an angle  $a_2 = 45^\circ$  with (Ox). Wire n°4 is at a height  $h_4$  above the pec ground. It is composed of only one section of length  $L_2$  carrying out an angle  $a_2 = 45^\circ$  with (Ox).

Furthermore, wire n°1 is loaded by two resistances  $R_{11}$  and  $R_{12}$  at its ends while wire n°2 is directly linked to the pec ground at its ends. Wire n°3 is loaded by two impedances  $R_{31}$  and  $R_{32}$ , and wire n°4 is connected to two loads  $R_{41}$  and  $R_{42}$ . Wires n°1 and n°2, n°2 and n°3, n°3 and n°4 are separated by the distances  $d_{12}$ ,  $d_{23}$ ,  $d_{34}$ , respectively (Fig. 1).

In this crosstalk problem, the purpose is to compute the extreme induced current  $I_{42}$  at the opposite end of the wire n°4, at the input of a device which has an impedance  $R_{42} = 75 \Omega$ . Thus, we aim at estimating the quantile of order 95% denoted  $I_{0.95}$  verifying  $P_f = \mathbb{P}(\max_{\Delta f} I_{42} \geq I_{0.95}) = 0.05$ , where  $\Delta f = [70-110 \text{ MHz}]$  is the widened FM frequency band.

The impact of uncertain input parameters on the induced current  $I_{42}$  is studied from the following random variables:

- $R_{11}$  and  $R_{31}$  are uniform random variables between 1  $\Omega$  and 10  $\Omega$ ;
- $R_{12}$  and  $R_{32}$  are uniform random variables between 1 k $\Omega$  and 10 k $\Omega$ ;
- $L_1$ ,  $L_2$  and  $L_3$  are variables uniformly distributed respectively between 0.9 m and 1.1 m, 1.8 m and 2.2 m, 1.35 m and 1.65 m;

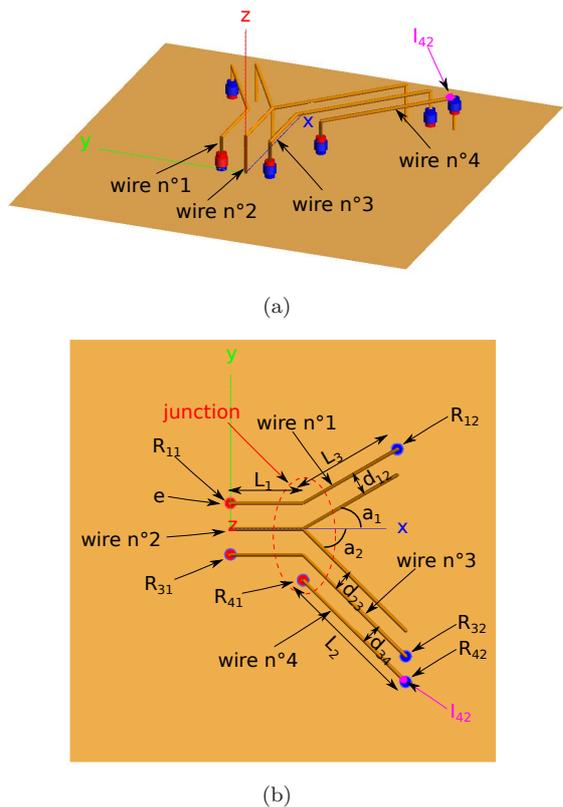


Fig. 1. Overview (a) and top view (b) of the four wires above a perfect electric conductor ground.

- $h_1$ ,  $h_2$ ,  $h_3$ , and  $h_4$  are variables uniformly distributed between 12 mm and 18 mm, and varying independently from each other;
- $d_{12}$ ,  $d_{23}$  and  $d_{34}$  are uniform random variables between 4.8 mm and 7.2 mm, and varying also independently from each other.

Furthermore, the diameter of each wire is 1 mm and the load  $R_{41}$  has an impedance of 75  $\Omega$ .

In total, this crosstalk problem deals with 14 random variables. In the following, an application of the CS technique and the ACS method will be used in order to estimate an extreme interference level represented by the quantile  $I_{0.95}$  of the induced current  $I_{42}$ .

### IV. APPLICATION TO THE CASE OF A FULL-WAVE SIMULATION WITH A TRANSMISSION LINE MODEL AS A SIMPLE MODEL FOR THE CONTROLLED STRATIFICATION

In this section, the simple model used is a computer code called PERL, which is based on the transmission line theory (TLT) using the Baum-Liu-Tesche (BLT) equation formalism. As rigorous model, we have chosen FEKO®, a computer code relying on a method of moments (MoM) solver applied to the integral form of Maxwell's equations. In the following, the two computer codes will be denoted TLT and MoM.

#### A. Controlled Stratification

1) *Estimation of the Correlation between Simple and Rigorous Models:* In this section, the simple model used relies on the TLT. It disregards the effect of vertical wires, the presence of other modes than the transverse electromagnetic (TEM) mode (i.e. no radiation), and neglects the coupling between the three sets of coupled wires of the network. Regarding the rigorous model, it is based on a full wave numerical simulation (MoM). TLT computes the induced current in around 0.4 seconds, while MoM calculates it in around 35 seconds (using a fine mesh of wires  $\frac{\lambda}{27} \approx 10$  cm, where  $\lambda$  is the wavelength).

In order to emphasize the levels of the induced current  $I_{42}$  obtained by TLT and MoM, we represent them for the same random realization of the 14 input variables. Selecting a first random combination of input variables, we plot the induced currents  $I_{42}$  over the [70-110 MHz] frequency band with TLT (curve with the red asterisks, see Fig. 2(a)) and with MoM (curve with the blue squares, see Fig. 2(a)). The maximum induced current is slightly higher in MoM than in TLT with a resonance occurring at almost the same central frequency ( $\approx 94$  MHz). The lower induced current computed by TLT may be partly explained by the role of the vertical wires, which are not taken into account.

We first estimate reference-like quantiles from empirical estimations (3) with a very large sample. The 95% quantiles of TLT ( $\hat{Z}_{EE}(0.95) = 0.3560$  mA) and of MoM ( $\hat{Y}_{EE}(0.95) = 0.4521$  mA) computed from 7000 realizations by MCS are rather different. Looking at Fig. 2(a), we notice that the maximum levels of the induced current  $I_{42}$  computed by TLT and MoM for that particular combination of input variables, i.e.  $z_{max} = 0.5104$  mA and  $y_{max} = 0.5395$  mA, are higher than their 95% quantiles. This highlights that the correlation level  $\hat{\rho}_I(0.95)$  (14) between the two computer codes is not negligible.

Performing a new simulation with TLT and MoM for another random configuration of input variables, we see that the currents  $I_{42}$  evaluated by TLT (curve with the red asterisks, see Fig. 2(b)) and by MoM (curve with the blue squares, see Fig. 2(b)) are totally different with respect to their quantiles. On the one hand,  $z_{max} = 0.3099$  mA is smaller than its 95% quantile,  $\hat{Z}_{EE}(0.95) = 0.3560$  mA. On the other hand,  $y_{max} = 0.4678$  mA is larger than its 95% quantile,  $\hat{Y}_{EE}(0.95) = 0.4521$  mA. This emphasizes that the correlation coefficient  $\hat{\rho}_I(0.95)$  (14) is not equal to 1. In order to measure the general tendency between TLT and MoM around their quantiles, an estimation of the correlation coefficient  $\hat{\rho}_I(0.95) = 0.52$  has been obtained by 1000 realizations from MCS. It is worth emphasizing that the differences between maximum current magnitudes obtained from the two models may be considerable when estimating extreme interference events. These differences may be mainly explained by the simplifying assumptions used in the transmission line theory at frequencies for which resonances occur.

2) *Uniform Allocation Strategy:* Let us now apply the CS method. We propose to select 50 realizations by strata from the simple TLT model. For this, we evaluate the

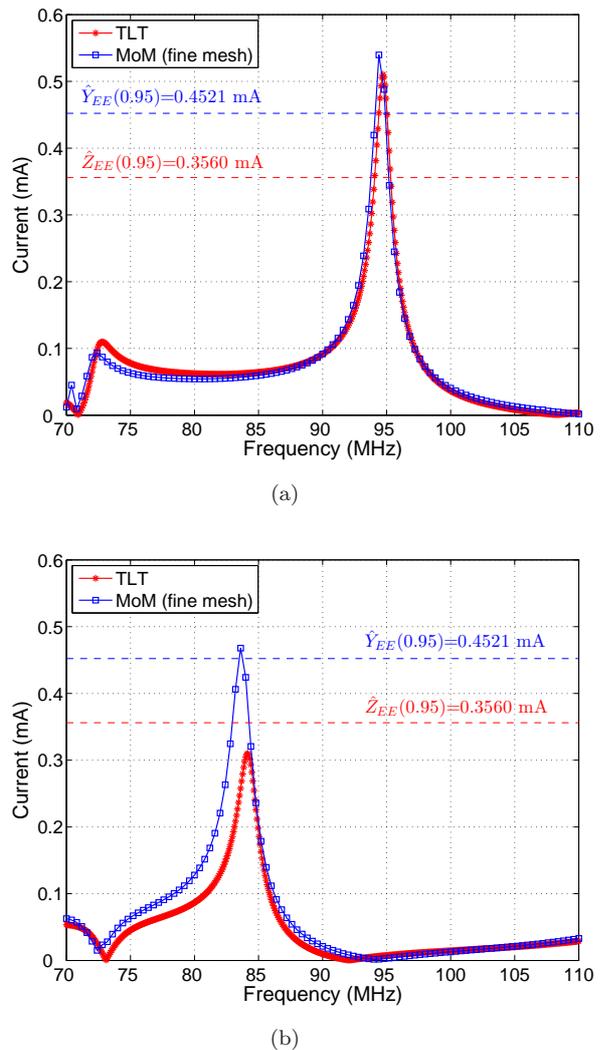


Fig. 2. Evaluation of the induced current  $I_{42}$  within the [70-110 MHz] frequency band. Two random configurations of input variables are considered in (a) and (b) for the computation of the currents  $I_{42}$  with the simple TLT model (curve with the red asterisks) and with the rigorous MoM model (curve with the blue squares).

quantiles of TLT from MCS with 7000 realizations of  $\mathbf{X}$ :

$$\begin{cases} \alpha = 0.5 \longrightarrow z_\alpha = 0.1639 \text{ mA} \\ \alpha = 0.9 \longrightarrow z_\alpha = 0.2981 \text{ mA} \\ \alpha = 0.95 \longrightarrow z_\alpha = 0.3560 \text{ mA.} \end{cases}$$

Suppose that the quantile of order  $\alpha = 0.95$  has to be estimated with no more than  $n = 200$  calls to the rigorous MoM model. Thus, relying on  $n = 200$  realizations of MoM, a first estimation of the quantile of order  $\alpha = 0.95$  provides  $\hat{Y}_{CS}(0.95) = 0.4672$  mA. The quality of this first estimation is quite good since it is rather close to the reference empirical quantile given by 7000 realizations from MCS:  $\hat{Y}_{EE}(0.95) = 0.4521$  mA. Let now examine the correlation level between TLT and MoM. To do so, we plot in Fig. 3 the maximum current values obtained in the 4<sup>th</sup> stratum, where  $Z > z_{0.95} = 0.3560$  mA. The histogram shows the occurrence probability of the

current values observed in the 4<sup>th</sup> stratum of the response space  $Y$ . From Fig. 3, we check that a very significant number of extreme current values is reached: among the 50 current values computed, 33 of those are higher than the reference quantile  $\hat{Y}_{EE}(0.95) = 0.4521$  mA. This is worth emphasizing that a simple model, moderately correlated to the rigorous one, is able to determine relevant input realizations providing extreme current values. This feature will be exploited in Section VI for fitting an extreme value distribution.

Since  $\hat{Y}_{CS}(0.95) = 0.4672$  mA is only an estimation, it would be convenient to check for its own statistical uncertainty. It is normally not affordable but our rather simple case study enables to perform many  $\hat{Y}_{CS}(0.95)$  estimations to evaluate its standard deviation.

Carrying out 10000 quantile estimations  $\hat{Y}_{CS}(0.95)$  (using  $n = 200$  realizations of MoM for each estimation), we obtained a mean  $\hat{\mu}_{Y_{CS}(0.95)} = 0.4532$  mA and a standard deviation  $\hat{\sigma}_{Y_{CS}(0.95)} = 0.0265$  mA. Performing 10000 quantile empirical estimations  $\hat{Y}_{EE}(0.95)$  with also  $n = 200$  realizations of MoM for each estimation, we found a mean  $\hat{\mu}_{Y_{EE}(0.95)} = 0.4550$  mA and a standard deviation  $\hat{\sigma}_{Y_{EE}(0.95)} = 0.0305$  mA. It is worth noting that 10000 quantile estimations were used in order to estimate accurately the standard deviation of the quantile. It validates the fact that the standard deviation of the CS method  $\hat{\sigma}_{Y_{CS}(0.95)}$  is smaller than the one from an empirical estimation. To obtain a standard deviation of the same order (i.e.  $\hat{\sigma}_{Y_{CS}(0.95)} = 0.0265$  mA) from an empirical estimation, we would have carried out  $n = 260$  realizations of MoM. In this case, the CS method brings a gain of a bit more than a quarter of simulations of a rigorous model. However, we have to take into account the addition of computation time required for the simple model. Nevertheless, our goal in this study is not to save computation time for the quantile estimation associated with that particular and simple test case. It is rather to highlight the reduction of the number of calls to the rigorous model assuming that the simple model would be almost free in computation time.

3) *Importance of Allocation Strategy*: In the previous section, the CS technique has been applied using an uniform allocation strategy with 50 realizations in each of the 4 strata. Let us apply again the CS method with a new allocation strategy in order to obtain a better quantile estimation. Thus, we select a proportional allocation strategy with  $N_j = (\alpha_j - \alpha_{j-1}) \cdot n$ , for  $j = 1, \dots, 4$ , realizations in each stratum, which represents 100, 80, 10 and 10 realizations respectively in the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> stratum. Adopting this allocation strategy, and carrying out 10000 CS quantile estimations, we get a mean  $\hat{\mu}_{Y_{CS}(0.95)} = 0.4504$  mA and a standard deviation  $\hat{\sigma}_{Y_{CS}(0.95)} = 0.0278$  mA. Thus, we notice that this strategy is worse than the uniform one since the standard deviation is higher.

We instead choose another allocation strategy with a larger allocation in the tail of the distribution, i.e. 100, 60, 20 and 20 realizations respectively in the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>

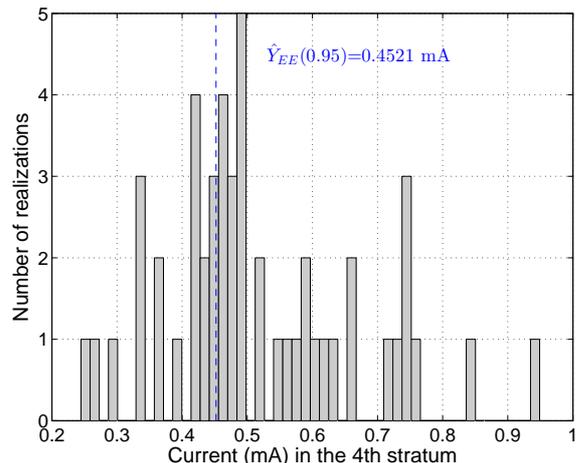


Fig. 3. The histogram represents the number of current values obtained in the 4<sup>th</sup> stratum. Among 50 current values computed, 33 values are higher than the reference quantile  $\hat{Y}_{EE}(0.95) = 0.4521$  mA.

and 4<sup>th</sup> stratum. Using 10000 CS quantile estimations, we obtain then a mean  $\hat{\mu}_{Y_{CS}(0.95)} = 0.4547$  mA and a standard deviation  $\hat{\sigma}_{Y_{CS}(0.95)} = 0.0250$  mA. This time, we observe a better quantile estimation compared to the uniform one insofar as the standard deviation has been reduced. It would have been possible to get a standard deviation of the same order using  $n = 280$  realizations with an empirical estimation. The gain of this allocation strategy is around 40% of simulations if we neglect the computation time required by TLT.

This highlights the fact that this allocation strategy allows a more accurate quantile estimation reducing its standard deviation. This leads to the application of the ACS technique in the following section, which allows to estimate the optimal allocation strategy in each stratum.

### B. Adaptive Controlled Stratification

The aim of this section is to estimate the best allocation strategy in each stratum using the ACS technique from Section II-B.

First, we apply the CS technique with an a priori choice of  $\tilde{n} = 100$  simulations with an allocation of 25 realizations in each of the 4 strata. Then, estimating the optimal allocation in each stratum  $\hat{\beta}_j$ , for  $j = 1, \dots, 4$ , we are called to add 0, 67, 12 and 21 realizations respectively in the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup> and 4<sup>th</sup> stratum. Once these simulations are carried out, we obtain a quantile estimation of  $\hat{Y}_{ACS}(0.95) = 0.4272$  mA from  $n = 200$  realizations. Then, we quantify the standard deviation of the ACS technique as we performed for the CS method. Using 10000 ACS quantile estimations with an a priori choice of 25 realizations in each of the 4 strata, we found a mean of  $\hat{\mu}_{Y_{ACS}(0.95)} = 0.4396$  mA and a standard deviation  $\hat{\sigma}_{Y_{ACS}(0.95)} = 0.0267$  mA. Thus, we obtain a similar estimation than the one from the CS method with 50 realizations per strata. The ACS method does not bring

a significant contribution compared to the CS, possibly because the quality of the simple model is quite weak.

In the following section, we use a better simple model (i.e. better correlated to the rigorous one) in order to verify the contribution of the CS, and the ACS techniques compared to the EE one.

#### V. APPLICATION TO THE CASE OF A FULL-WAVE SIMULATION WITH ANOTHER FULL-WAVE SIMULATION WITH A COARSE MESH AS SIMPLE MODEL FOR THE CONTROLLED STRATIFICATION

A coarse mesh is a rather intuitive choice as a simple model and may be adequate since calculated at a much lower cost. Beyond that intuition, the more important feature is the correlation criterion between both models.

In this section, the case study is still the one depicted in Section III. The only difference is therefore the new simple model. Indeed, the simple model used is the same computation code FEKO using MoM with a coarse mesh of wires, i.e.  $\frac{\lambda}{5.5} \approx 50$  cm. This simple model will be denoted  $\text{MoM}_{\text{coa}}$  in the following.

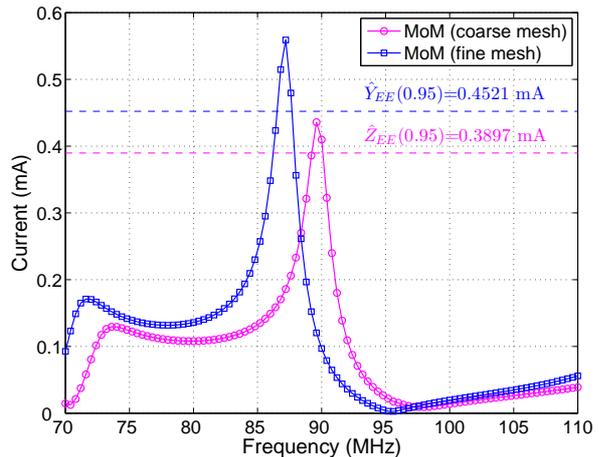
##### A. Estimation of the Correlation between Models

As in Section IV-A.1, we want to illustrate again the difference between simple and rigorous models.

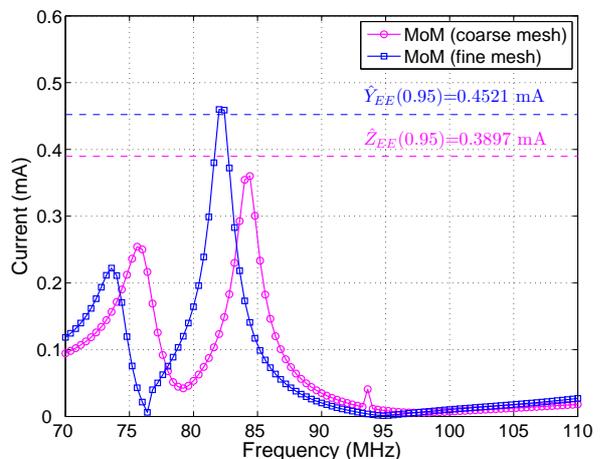
Therefore, for the same input realizations, we represent the levels of the induced current  $I_{42}$  as a function of frequency with  $\text{MoM}_{\text{coa}}$  (curve with the pink circles, see Fig. 4(a)) and with MoM (curve with the blue squares, see Fig. 4(a)). Once again, we can note a similar trend between the 2 curves but there are differences in terms of peak current magnitudes and frequencies computed by  $\text{MoM}_{\text{coa}}$  (i.e.  $z_{\max} = 0.4359$  mA) and MoM (i.e.  $y_{\max} = 0.5591$  mA). Note that for this simulation a coarse mesh with 49 segments of wires was used by the simple  $\text{MoM}_{\text{coa}}$  model, and a fine mesh with 150 segments of wires was used by the rigorous one MoM.

Moreover, we also evaluated the 95% quantiles of  $\text{MoM}_{\text{coa}}$  ( $\hat{Z}_{EE}(0.95) = 0.3897$  mA) and of MoM ( $\hat{Y}_{EE}(0.95) = 0.4521$  mA) from 7000 realizations by MCS. Once again, the 95% quantiles are different. However, we can see in Fig. 4(a) that the maximum current levels of the two curves  $z_{\max} = 0.4359$  mA and  $y_{\max} = 0.5591$  mA, respectively computed by  $\text{MoM}_{\text{coa}}$  and MoM, are larger than their 95% quantiles. This configuration shows again a certain correlation level (14) between the two simulations.

Carrying out simulations for another random configuration of input variables, the currents  $I_{42}$  obtained by  $\text{MoM}_{\text{coa}}$  (curve with the pink circles, see Fig. 4(b)) and MoM (curve with the blue squares, see Fig. 4(b)) are this time rather different with regard to their 95% quantiles. On the one hand,  $z_{\max} = 0.3599$  mA (wires were meshed into 49 segments) is lower than its 95% quantile,  $\hat{Z}_{EE}(0.95) = 0.3897$  mA. On the other hand,  $y_{\max} = 0.4598$  mA (with a mesh of wires of 147 segments) is larger than its 95% quantile,  $\hat{Y}_{EE}(0.95) = 0.4521$  mA. To quantify the similarity between  $\text{MoM}_{\text{coa}}$  and MoM around their quantiles,



(a)



(b)

Fig. 4. The induced current  $I_{42}$  is evaluated within the [70-110 MHz] frequency band. Two random configurations of input variables in (a) and (b), are used to compute the currents with the simple  $\text{MoM}_{\text{coa}}$  model (curve with the pink circles) and with the rigorous MoM model (curve with the blue squares).

we estimated the correlation coefficient  $\hat{\rho}_I(0.95) = 0.76$  by 1000 realizations from MCS. In this case, the correlation level between simple and rigorous models is quite high. This should allow to identify many relevant realizations with the simple model. We therefore expect a more significant improvement of the estimation of the 95% quantile than in Section IV.

##### B. Controlled Stratification and its Adaptive Variant

As previously, we apply the CS method with an uniform allocation strategy.

Let us choose 50 realizations by strata from the simple  $\text{MoM}_{\text{coa}}$  model. Using 7000 realizations from MCS, we identified the quantiles defining the strata of the output

$$Z: \begin{cases} \alpha = 0.5 \longrightarrow z_\alpha = 0.1750 \text{ mA} \\ \alpha = 0.9 \longrightarrow z_\alpha = 0.3277 \text{ mA} \\ \alpha = 0.95 \longrightarrow z_\alpha = 0.3897 \text{ mA}. \end{cases}$$

Performing 10000 quantile estimations  $\hat{Y}_{CS}(0.95)$  (with  $n = 200$  computations of MoM at each estimation), we obtained a mean  $\hat{\mu}_{Y_{CS}(0.95)} = 0.4547$  mA and a standard deviation  $\hat{\sigma}_{Y_{CS}(0.95)} = 0.0202$  mA. By comparison with an empirical estimation, it would have required  $n = 400$  computations of MoM at each estimation to obtain a standard deviation of the same order. In this case, the CS technique with a uniform allocation strategy allows to save 50% of the computation time if we do not count the simulation time used by MoM<sub>coa</sub>. We notice that the CS quantile estimation is better using the simple MoM<sub>coa</sub> model, instead of the TLT model (see Section IV, the standard deviation of the CS method is larger, i.e.  $\hat{\sigma}_{Y_{CS}(0.95)} = 0.0265$  mA). This example confirms that the larger is the correlation coefficient  $\hat{\rho}_I(0.95)$  (14), the more efficient is the CS technique.

Having at our disposal a good simple model, we apply the ACS technique with  $n = 200$  realizations to verify its additional contribution compared to the CS method. Note that in Section IV, the ACS technique was not better than the CS method, possibly because the simple model used was quite poor.

As above, we apply first the CS technique with an a priori choice of  $\tilde{n} = 100$  realizations with an allocation of 25 realizations in each of the 4 strata. In a second step, we estimate the optimal allocation (among the remaining 100 realizations) in each stratum  $\tilde{\beta}_j$ , for  $j = 1, \dots, 4$ . Once this allocation is identified, we add the realizations required in each stratum and estimate the quantile  $\hat{Y}_{ACS}(0.95)$ . Reproducing 10000 ACS quantile estimations, we found a mean  $\hat{\mu}_{Y_{ACS}(0.95)} = 0.4452$  mA and a standard deviation  $\hat{\sigma}_{Y_{ACS}(0.95)} = 0.0157$  mA. To get a similar standard deviation, it would have required around  $n = 600$  realizations. The gain of the ACS method is 3 times in terms of computation time (without taking into account the computation cost of the simple model). This confirms that the ACS method can be very efficient using a simple model highly correlated to the rigorous one for the estimation of extreme events.

### C. Discussion about the Performances of the Two Simple Models

The CS technique for the case study has been performed with a simple model based on the TLT, and another one based on a MoM solver with a coarse mesh.

However, even if the estimations of quantiles were pretty good with the TLT model, results were improved using the simple MoM<sub>coa</sub> model better correlated to the rigorous one, according to the definition of  $\hat{\rho}_I(\alpha)$  (14). The fact that the TLT model is less correlated to the rigorous one could be explained by some approximations of the TLT, which neglects the effect of vertical wires, the radiation of wires and ignore the coupling between the three branches of

the network. On the contrary, MoM<sub>coa</sub> still considers them. Having used a coarse mesh of  $\frac{\lambda}{5.5} \approx 50$  cm (while the rigorous model is meshed at  $\frac{\lambda}{27} \approx 10$  cm), this allowed to identify more accurately, some output realizations in the last stratum, and therefore to estimate more efficiently the 95% quantile.

## VI. EXTREME VALUE DISTRIBUTION

The set of extreme realizations above the quantile  $\hat{Y}_{CS}(0.95) = 0.4672$  mA may be used as goodness-of-fit test for a Generalized Pareto distribution in order to get further insight about extreme events probability distribution [19].

This is here applied as an example to one particular set of realizations of Section IV-A.2.

Taking the current values greater than the quantile  $\hat{Y}_{CS}(0.95) = 0.4672$  mA (i.e. the location parameter) obtained in the 3<sup>rd</sup> and the 4<sup>th</sup> strata during the CS quantile estimation, we fit a Generalized Pareto distribution. Among 100 values in the last two strata, 35 values are larger than  $\hat{Y}_{CS}(0.95) = 0.4672$  mA and are used to fit a Generalized Pareto distribution by the maximum-likelihood technique [20], see Fig. 5(a). The shape  $\xi = 0.23$  and the scale  $\eta = 0.12$  mA parameters of the Generalized Pareto distribution have been obtained with a 95% confidence interval:  $\xi \in [-0.18; 0.64]$ , and  $\eta \in [0.06; 0.18 \text{ mA}]$ .

The Generalized Pareto distribution fits reasonably well the empirical extreme values distributions. Note also that the CS method may help finding very rare events such as a response at almost 1,6 mA. However, we cannot deduce any probability of occurrence for such an extreme value. A reasonable statement is to restrict the analysis of the Generalized Pareto distribution to assess a higher quantile probability, as long as there are enough realizations above that quantile, say typically 10. Since there are 35 realizations above the estimated 95% quantile, this limitation applies to  $(35-10)/35$ , i.e. about the 70% quantile of these extreme realizations. This corresponds to the 98.5% quantile of the parent distribution  $F(y)$ .

Fig. 5(b) shows the cdf of the extreme current values distribution compared to the one obtained from the Generalized Pareto distribution. It confirms that the Generalized Pareto distribution hardly follows the empirical distribution above the 71% quantile i.e. the validity limitation of the fitting. The 70% quantile of these extreme realizations estimated from the Generalized Pareto distribution is 0.6365 mA. By comparison with a direct empirical estimation of the 98.5% quantile (obtained by 7000 realizations from MCS) gives  $\hat{Y}_{EE}(0.985) = 0.5722$  mA. These values are very consistent highlighting that we may look at even more extreme quantile estimation, starting from a CS sampling technique.

## VII. CONCLUSION

This paper introduces the CS method and its variant the ACS technique applied to EMC analysis. This approach proposes a variance reduction for estimating the quantile

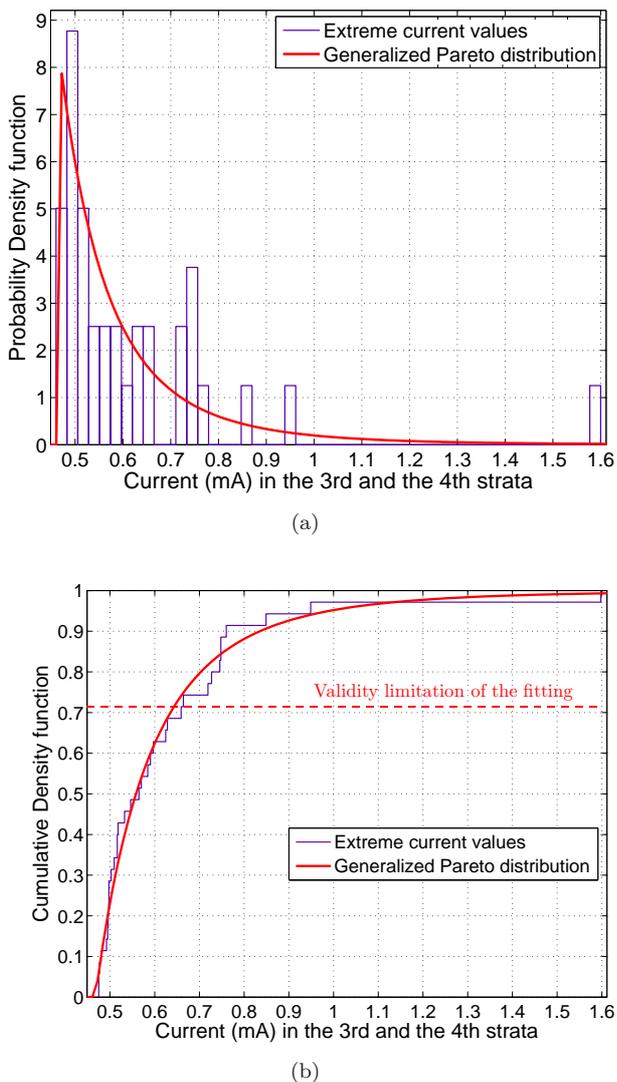


Fig. 5. Fitting of (a) the pdf and (b) the cdf of the Generalized Pareto distribution on the extreme current values identified by the simple TLT model in the 3<sup>rd</sup> and the 4<sup>th</sup> strata.

of order  $\alpha$  ( $\alpha$  close to 1), when the output of the computer code used is expensive in terms of computation time. This method relies on a simple model which is an approximation (supposed at a low computational cost) of the expensive computer code to select relevant realizations in order to estimate extreme events in the context of uncertain input variables.

The quality of this estimation depends on the ability of the simple model to replicate the same behaviour in terms of input variables than the rigorous model according to a specific definition of their correlation.

Relying on a simple model based on the TLT, and on a rigorous model solving rigorously Maxwell's equations, we estimated an extreme quantile of an interfering current in a cable bundle reducing the computation time (compared to an empirical estimation). Moreover, using a simple model allowed to fit an extreme value distribution in order to estimate even more extreme quantiles. The same study was also applied using a computer code with a coarse mesh,

even if this latest was rather costly. Since this coarse mesh simulation is more correlated to the fine mesh simulation, the reduction in terms of number of calls to this last one, has been more significant. The object of future work may consist in researching simple models, allowing to accelerate the computation time of rigorous models for the estimation of extreme events in EMC problems.

One of the main challenges about this CS method is to find adequate simple substitute models for various EMC modeling scenarios. We have seen that coarse versus fine mesh of the spatial discretization for solving Maxwell's equations is a good candidate. Other models like circuits models or analytical models including approximations could be eligible as well as metamodels built up from different statistical techniques. Identifying such models is a track for future research.

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