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Revival of classical topics in differential geometry

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We present a reflection on the usage of modern algorithms and technology, in order to revive topics of classical Differential Geometry. This enables students to acquire new mathematical knowledge and provides new skills for applied science.

Keywords: Differential geometry, 1-parameter families of curves, envelopes, technology.

MOTIVATIONS

Classical topics in Differential Geometry like the study of 1-parameter families of curves and their envelopes have been abandoned in the past for various reasons: the classical theory may not be rigorous enough, it has too many particular cases, it may be so rich that it cannot be forced into a rigorous pedagogy, nothing ensures that all the “pathological” cases have been included in a catalogue, etc. (Thom, 1962).

Nevertheless, this topic has a great interest, both in mathematics and in modern applied science. It can be studied via a blended activity, i.e. pencil and paper work together with CAS work. Moreover it has many applications in science and engineering – caustics and wave fronts, i.e., Geometrical Optics and Theory of Singularities (Arnold, 1976), robotics and kinematics, rigid motion in 2-space and in 3-space, collision

avoidance, etc. (Pottman & Peternell, 2000). This topic is also connected to the theory of billiards.

EXAMPLES

Let be given a family of plane curves by an equation $f(x, y, c) = 0$, where c is a real parameter. An envelope of the family, if it exists, is a curve tangent to every curve in the family. It can be shown that this envelope is the solution set of the system of equations $f(x, y, c) = 0$, $\frac{\partial f}{\partial c}(x, y, c) = 0$. Figure 1a shows the envelope of the family of lines given by the equation $x + cy = c^2$ (it is the parabola whose equation is $x = -y^2/4$). Figure 1b shows the envelope of the family of circles with radius 1 and centre on the parabola whose equation is $y = x^2$ (here the result has two components).

STRUCTURE OF A WORKING SESSION

We use infinitesimal methods for exploration and derivation of the equations, then algorithms based on Gröbner bases computations, first to solve the system of equations, and then to transform the obtained parametric representation into an implicit one.

This is a core issue in other fields (Pech, 2007). A similar, but non identical, scheme gives the mathematical educational frame for other topics in Differential

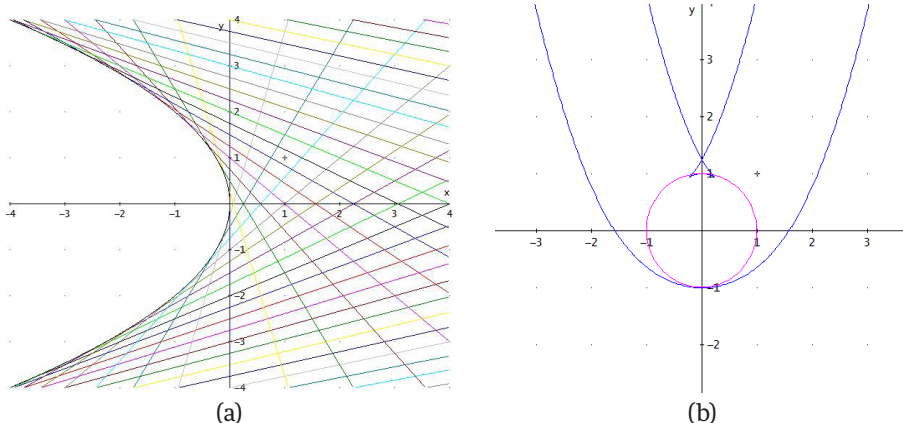


Figure 1: Exploration of envelopes

Geometry, giving an extension of the curriculum (Dana-Picard, Mann, & Zehavi, 2014).

CORE ISSUES

Such a study provides an opportunity to discover new topics beyond the scope of the regular curriculum, sometimes together with applications to practical situations. New computation skills with technology may be developed, in particular for the experimental aspect of the work (e.g., exploring the existence of cusps, as in Figure 1b). For this, the availability in the software of a slider bar is a central issue. Moreover, ability to switch between different registers of representation may be improved, within mathematics itself (parametric vs implicit) and with the computer (algebraic, graphical, etc.). An interesting instrumental genesis may appear (Artigue, 2002) and mind-and-machine interaction is a central feature of the work.

An envelope may not exist (e.g., for a family of lines where the coefficients of the equations are affine functions of the parameter). These issues have been observed by the authors in sessions for in-service teachers at the Weizmann Institute of Science.

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