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Students' understanding of marginal change in the context of cost, revenue, and profit

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This paper describes how eight undergraduate students majoring in economics and business studies reasoned about marginal change (marginal cost, marginal revenue, and marginal profit) in the process of deciding how they would advise the management team of an airline about an economic decision involving the addition of another jet plane. To elicit students' understanding of marginal change in an economic context, pairs of students were engaged in a task-based interview. Nearly all of the students were able to reason correctly about marginal change within the immediate context of the task, while four of the students also did so beyond the context presented in the task. Only one student considered the marginal change information in the task as a rate of change.

Keywords: Marginal change, rates of change, business calculus, undergraduate mathematics education, economic decision making.

INTRODUCTION

The role of context in the way students reason about rate of change (average rate of change and instantaneous rate of change) has received considerable attention from researchers interested in the learning and teaching of this concept and students' interpretations of rates of change in various contexts. In particular, there is a large body of research literature on students' understanding of rate of change in a motion context (Beichner, 1994; Bery & Nyman, 2003; Monk, 1992; Nemirovsky, Tierney, & Wright, 1998). Research exists on students' understanding of rate of change in non-motion contexts such as fluid flow, heat flow, temperature, discharging capacitors, and light intensity (Bingolbali & Monaghan, 2008; Carlson, Jacobs, Coe, Larsen, & Hsu, 2002; Doerr, Årlebäck, & O'Neil, 2013; Johnson, 2012; Marrongelle, 2004). However, there

is little research on the context of economic change, which is the motivation for this study.

THEORETICAL FRAMEWORK

This study draws on a multiple representations theoretical framework (Davis, 2007) to examine how students reason about the context of cost, revenue, and profit within a real-world context representation and across other representations. Davis' multiple representation framework contains five representations: (1) tables, (2) algebraic, (3) graphs, (4) spoken language, and (5) real-world contexts. At the center of Davis's framework are real-world contexts. As Davis puts it, "students' investigations are dominated by real-world contexts and students are frequently translating between tables, graphs, and equations, and vice versa" (p. 391). The framework is an adaptation of Lesh's (1979) multiple representation framework. The current study is part of a larger study that used three tasks that situated the context of cost, revenue, and profit in multiple representations, namely graph, table, and text. The current study reports on what students' reasoning about the context of economic decision making, presented as text, revealed about students' understanding of marginal change. Marginal cost refers to the cost per additional unit produced, marginal revenue refers to the revenue generated per additional unit sold, and marginal profit refers to the profit per additional unit produced and sold. Mathematically, marginal change can be calculated using instantaneous rate of change which can be approximated using average rate of change.

LITERATURE REVIEW

A review of the research literature on students' understanding of rate of change in context reveals several things. First, even high achieving students in calculus have difficulties understanding and interpreting

rate information (points of inflection and concavity) in mathematical tasks that are situated in a non-motion context (Carlson et al., 2002). Second, the use of physical models such as the sliding ladder used by Monk (1992) and technology in the form of motion detectors and graphing calculators can enhance students' understanding of rate of change in a motion context (Monk, 1992; Nemirovsky, Tierney, & Wright, 1998). Finally, the research literature reveals that a good understanding of a motion context in physics could enhance students' ability to reason about rate of change when solving calculus problems that have been stripped of context (Marongelle, 2004).

However, research on students' understanding of rate of change in a business and/or economics context is lacking. To the knowledge of the authors, only one study by Wilhelm and Confrey (2003) investigated students' reasoning about rates of change in an economic context. These researchers studied algebra I students' ability to project their understanding of average "rate of change in the context of motion onto the context of money" (p. 887). Wilhelm and Confrey found that some of their participants were able to project their understanding of average rate of change from a motion context to a banking context. However, their study did not examine what students' reasoning about the motion and banking context revealed about students' understanding of instantaneous rate of change and marginal change. The current study seeks to address the gap about what it is that students' reasoning about the context of cost, revenue, and profit reveals about their understanding of marginal change.

Conflating function output values with the average rate of change values for the function considered over subintervals of the function's domain is a well documented difficulty that students have when reasoning about rate of change (Carlson et al., 2002; Monk, 1992; Prince, Vigeant, & Nottis, 2012). It is also known that students have difficulty distinguishing between the amount by which a function changes and the rate at which the function is changing over unit subintervals (Confrey & Smith, 1994; Cooney, Beckmann, & Lloyd, 2010). Research also indicates that students' "understandings of rate in one representation or context are not necessarily transferred to another" (Herbert & Pierce, 2012, p. 455). This study is part of a larger study that investigated students' reasoning about marginal change in three representations, namely graph, table,

and text. This paper reports on students' reasoning about marginal change in the latter representation.

METHODOLOGY

This qualitative case study used task-based interviews (Goldin, 2000) with eight undergraduate students currently enrolled in business calculus. Four pairs of students were engaged in the following non-routine task that was designed to elicit their reasoning about the marginal change information rooted in the task, adapted from Hughes-Hallet and colleagues (2006):

JetBlue is a major airline that currently operates 195 jet planes. The airline serves 84 destinations in 24 states and 12 countries in the Caribbean, South America, and Latin America. The airline is trying to decide whether to add an additional jet plane. The choice that the airline has is between adding this jet plane and leaving things the way they are. The airline's decision is to be made purely on financial grounds.

How should the airline decide on whether or not to add the 196th jet plane?

Setting and respondents

The study was conducted on the campus of a medium sized research university located in the north-eastern part of the United States. The respondents were eight undergraduate students, six females and two males, from the department of economics and the business school who had recently completed a business calculus course as a prerequisite for other required courses in their programs of study. Four of these students were sophomores, one student was a freshman, two students were seniors, and the other student was a junior. Students taking this course are familiar with average rate of change, instantaneous rate of change, and the context of cost, revenue, and profit, hence the reason for recruiting them to participate in the study. Data were collected during a regular semester and the summer following that semester. Three of the four interviews were both audio and video-recorded; one interview was only audio-recorded. Each interview lasted for about 75 minutes. The interview data was transcribed for analysis. Work written by students during the interview was also collected as part of the data.

Data analysis

Transcripts were coded for students' understanding of the marginal change information rooted in the task. In particular, the transcripts were analyzed for students' abilities: (1) to identify and interpret the marginal change information embedded in the task, and (2) to give reasonable advice on how the airline should decide on whether or not to add another jet. Reasonable advice was considered to be one which takes into consideration a comparison of the company's marginal cost, marginal revenue, and marginal profit associated with the jet in question in the process of deciding for or against the addition of another jet plane.

RESULTS

Seven of the eight students who attempted this task were able to reason about marginal change in the context of making an economic decision, that is, advising the management of an airline on whether or not to add another jet plane. Only one student considered the marginal change information rooted in the task as a rate of change. Two of the four pairs of students reasoned beyond the immediate context presented in the task. Following is a discussion of students' reasoning about the marginal change information rooted in the task, first within the immediate context of the problem, and then beyond the immediate context of the problem.

Reasoning about marginal change within the context presented in the task

Two of the four pairs of students reasoned about the marginal cost, marginal revenue, and marginal profit ideas within the context of the problem. The following excerpt illustrates Isabel and Sally's initial response to the question: How should the airline decide on whether or not to add the 196th jet plane?

Isabel: If the rate of change increases with the more jets that they have they should add

another jet but if the rate of change is decreasing they should not add another jet.

Researcher: What do you mean by the rate of change?

Isabel: Like say if they had a graph, if the graph shows their financial cost would look like that [drawing the graph on the left in Figure 1] which means that they should add...and then if they have another graph [drawing the graph on the right in Figure 1] where it is going more like this, then they should not add.

Researcher: What is the curve [asking about the graphs she drew]? What does it represent?

Isabel: Their financial, like how much they are making.

In stating that the airline should add another jet if "the rate of change increases", it appears that Isabel is referring to the increasing profit that the addition of this jet would bring to the airline. It also appears that she is working on the assumption that whenever the airline added a jet in the past their profit always increased (graph on the left in Figure 1). In the graph on the right in Figure 1 which illustrates when the airline should decide against adding another jet plane, Isabel appears to be referring to the decreasing profit that would continue if the jet is added. One may argue that Isabel's focus on the graph shifted her attention away from the context. Isabel, however, did not make any reference to marginal cost, marginal revenue, and marginal profit while talking about her advice to the management. Isabel was the only student to spontaneously talk about rate of change.

In talking about her decision, Sally's reasoning was based on comparing the marginal cost and marginal revenue associated with the additional jet as the following excerpt illustrates.

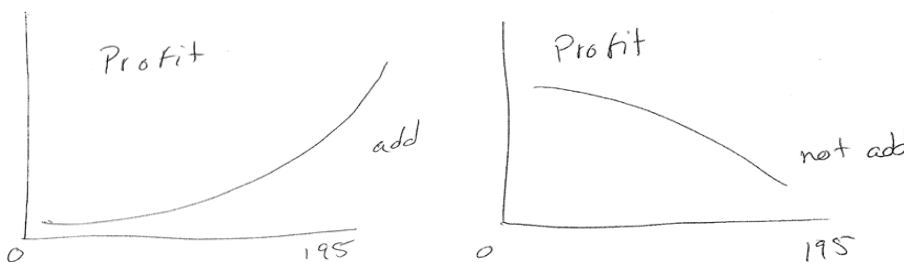


Figure 1: Isabel's graphical illustration of how the management of JetBlue Airline should decide on whether or not to add the 196th jet plane

Researcher: Sally, what's your own view? How should the airline decide on whether or not to add the 196th jet plane?

Sally: I think the profit [referring to Isabel's graphs] works but it would be easier if you do a graph like it was before [referring to a graph from an earlier task that had two curves: a non-linear cost function and a linear revenue function] with the marginal revenue and like the marginal cost. So if this [referring to the graph on the left in Figure 2] I would say 195 just to make it easy, then you would not add with that because for that it would make the revenue smaller than the cost.

Researcher: What is the line?

Sally: It's the marginal revenue.

Researcher: So in that case they should not add?

Sally: Right, but then in that case [referring to the graph on the right in Figure 2] they should add.

Sally appeared to be comparing the marginal cost and marginal revenue associated with the addition of the 196th jet plane. Her left graph in Figure 2 shows a realistic situation for when the jet should not be added: the marginal cost is greater than the marginal revenue generated for any jet plane added beyond the 195th plane. Sally's decision for adding the 196th jet plane in the case of the situation depicted by the graph on the right in Figure 2 is also quite reasonable in that it shows that adding a jet beyond the 195th increases the airline's marginal revenue while adding a jet beyond the 195th decreases the airline's marginal costs and hence it would be in the best interest of the airline to add the 196th jet. The following excerpt illustrates Isabel's reasoning about Sally's graphs (Figure 2) and their final thoughts on the advice they would give to the management of the airline.

Isabel: I see them showing the same thing, the only thing is that you would never know this [pointing at the MR line and MC curve extended beyond 195 in Sally's left graph in Figure 1] because we don't have more than 195 planes, so this is, I don't know but that [pointing at the MR line and MC curve extended beyond 195 in Sally's left and right graphs in Figure 1] wouldn't exist on the graph anyway.

Researcher: Putting your ideas together, you want to give an advice to the management of this airline on what they should do. How can you present your advice?

Sally: If the marginal revenue is higher than the marginal cost for the 196th plane you should purchase it but if it's the other way round you should not.

Researcher: Isabel, how could you present your advice?

Isabel: I agree with her.

Researcher: Would you use exactly the same words she used or?

Isabel: Basically yeah. If it's increasing the profit then purchase it, if it is lessening the gap between the cost and the revenue, then purchase it as well but if it's increasing the gap and the profit is less then don't purchase it.

Sally's advice to the management is based on comparing the marginal cost and marginal revenue. This suggests that Sally is indirectly looking at the marginal profit that adding of the 196th jet would bring to the airline. Isabel, on the other hand, made her decision based on the increasing profit associated with the addition of the 196th jet. When talking about "the gap between the cost and the revenue," Isabel appears to be drawing on Sally's reasoning about marginal change.

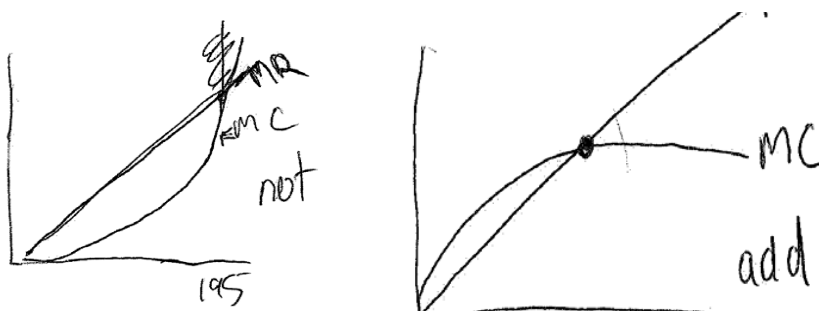


Figure 2: Sally's graphical illustration of how the management of JetBlue Airline should decide on whether or not to add the 196th jet plane

A second pair of students, Paige and Yolanda, reasoned similarly to Sally. Paige said that “they need to see what the potential revenue is from that one plane and then compare with the cost of that plane and see if that would give them more money”. Paige’s advice is based on the marginal cost, marginal revenue, and marginal profit (which she referred to as “more money”) associated with adding the 196th jet. Yolanda said that “they need to see how the profit is at 195 and see how much more one plane makes,” thus reasoning about the marginal profit associated with the addition of the 196th jet.

Reasoning about marginal change beyond the context presented in the task

The other two pairs of students who attempted this task reasoned about the marginal cost, marginal revenue, and marginal profit ideas beyond the context of the problem. In particular, the students’ thinking showed a consideration of the broader economic issues that needs to be considered in trying to decide whether or not another jet should be added. In the case of Beth and Mary, Beth said that the airline’s decision should be “based on how much more profit they will make versus how much they will pay to make the plane”. Beth reasoned about marginal change and in particular she reasoned about the marginal cost and marginal profit associated with the jet in question.

Mary, on the other hand, said that “if the quantity of people that are flying increases then it would be acceptable to get another jet plane but if it’s decreasing there is no need for another one.” After Mary’s response, Beth added that “you should always add the plane because that way you could even add more destinations.” In giving her advice, Beth initially took marginal profit and marginal cost (cost of making the 196th jet plane) into consideration but then she also thought about the expansion of the airline in terms of adding more destinations. Mary’s reasoning was based on the increase or decrease in demand, that is, number of people flying with this airline. Mary’s thinking, however, does not show any evidence of considering the marginal change information rooted in the problem.

Noel and Paul’s advice to the management of the airline on whether or not to add another jet is another example of reasoning beyond the immediate context. Noel said that “if it adds incrementally to the profit of the airline, then truly based on financial grounds you would add that 196th jet.” He went on to add that “they need to know whether there would be capacity on that

plane, actual demand, and cost of other things like fuel and union contracts.” Noel’s decision not only takes into consideration the marginal profit associated with adding another jet, but also several other important economic factors.

Paul, Noel’s partner, said that “they should look at the cost of the plane and the marginal revenue they would get.” He went on to add that “they should look at the cost per average passenger and average revenue per passenger.” Later in the interview, Paul said that “if the marginal profit equals their marginal revenue minus marginal cost equals zero, it makes sense they are still in a capacity to add this jet, if this [the difference between marginal revenue and marginal cost] is negative then it doesn’t make sense to add another jet.” In this situation, Paul showed an understanding of marginal profit as it relates to the marginal cost and marginal revenue associated with adding another jet, thus using his understanding of the mathematical content knowledge (marginal change) to understand the context of the problem.

Summary of students' reasoning about marginal change in context

In summary, the results from this study revealed that: (1) students can correctly reason about marginal change when presented with a problem that is situated in a context [air travel] that is familiar and meaningful to the students, and (2) students are able to reason beyond the immediate information given in a task when it is presented in a familiar situation. A key finding of this study is that students understood marginal change as an amount of change (the difference) and not as a rate of change (the difference quotient). This was supported by the fact that in another task during the interview (not reported in this paper) all but one of the students indicated that the units of marginal change would be dollars instead of dollars per unit.

DISCUSSION AND CONCLUSION

This study investigated what students’ reasoning about a real-world problem in the context of cost, revenue, and profit reveals about their understanding of marginal change. Nearly all the students were able to correctly talk about the marginal change information rooted in the task. However, only one student (Isabel) used the language of rate of change to give her advice to the management.

A major finding of this study is that students reasoned about marginal change as an amount of change and not as a rate of change. This finding would appear to be consistent with results from other studies (Confrey & Smith, 1994; Cooney, Beckmann, & Lloyd, 2012). Distinguishing between the amount of change (a difference) and marginal change (a rate of change over a subinterval of unit length) would appear to be especially difficult for students.

Even at the end of a business calculus course students do not speak of marginal change as a rate of change as it is presented in their calculus course and their textbook (Haeussler, Paul, & Wood, 2010). This is problematic because one major goal of a business calculus course is to help students move from understanding marginal change as an amount of change to understanding it as a rate of change. It might be important for future research to consider using modelling tasks (Doerr & English, 2003; Lesh et al., 2003) to support the development of students' understanding of marginal change as a rate of change.

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