



**HAL**  
open science

# Faculty knowledge of teaching in inquiry-based learning mathematics

Vilma Mesa, Anne Cawley

► **To cite this version:**

Vilma Mesa, Anne Cawley. Faculty knowledge of teaching in inquiry-based learning mathematics. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Charles University in Prague, Faculty of Education; ERME, Feb 2015, Prague, Czech Republic. pp.2194-2200. hal-01288611

**HAL Id: hal-01288611**

**<https://hal.science/hal-01288611>**

Submitted on 15 Mar 2016

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Faculty knowledge of teaching in inquiry-based learning mathematics

Vilma Mesa and Anne Cawley

University of Michigan, Ann Arbor, USA, [vmesa@umich.edu](mailto:vmesa@umich.edu)

*In the context of using Inquiry-Based Learning (IBL) for teaching, 74 instructors teaching a wide range of university mathematics courses and with different levels of familiarity with IBL filled out bi-weekly logs about the challenges they faced and the solutions they found. The analysis of pairs of concerns and solutions expressed over the three-year study reveals that faculty may be drawing from different domains of teacher knowledge for teaching to solve concerns teaching with IBL. Level of familiarity with IBL did not seem to play a role in the types of solutions proposed, except when faculty reported not having a solution. We conjecture that the method may induce faculty's pedagogical awareness, independently of how experienced they are with teaching university mathematics.*

**Keywords:** Inquiry-based learning, teacher knowledge, university mathematics.

Originally proposed as a way to understand how faculty learn to teach mathematics courses using Inquiry-Based Learning (IBL) approaches, the *Learning to Teach Mathematics with IBL Project* followed a group of instructors new to using IBL over a period of three years and documented their experience using IBL. Prior work interviewing a handful of instructors who were new to the method at our university suggested that at the beginning of the semester, their concerns centred on spatial and logistical aspects of managing the class: where do I stand in the room? How much time is it OK to wait before answering a question? As the instructors gained more experience with IBL their concerns shifted towards an interest in understanding students' thinking (Mesa & Cheng, 2008). Such dramatic change over the course of one semester countered some literature available on instructors' knowledge, more notably work done by Nyquist and Sprague (1998), whom having worked with teaching assistants (TAs), identified three dis-

tinct stages of concerns: a self-centred stage in which beginner TAs worried about themselves and students' perceptions of them (am I knowledgeable? do they like me?); a management-centred stage, in which more advanced TAs worried about managing the logistics and the different aspects of the classrooms (making sure students are not engaging in disrespectful behaviours); and a student-learning stage, in which the expert TAs worried mainly about whether students were understanding the material. These are described as stages in the development of TAs' expertise in teaching. Because our own experience did not suggest these stages with the faculty on our campus (Mesa & Cheng, 2008), we proposed this study. The design of this study includes faculty who stated their familiarity with IBL as beginners, novices, advanced, or experts; its longitudinal nature allows faculty to participate two to six consecutive semesters, with some participating only once. New faculty were added each year. We focused on the concerns faculty described and the solutions they proposed to those concerns when they engaged in four areas of teaching: sending students to the board, designing and managing group work, designing assessments, and using assessments. Specifically we investigate the following two research questions: What are the solutions that faculty propose to address concerns in each area of teaching? Does the level of familiarity with the method determine differences in the solutions used? The next section briefly reviews the literature that informs and situates this study; we follow with a brief description of methods. We then present the main findings and their discussion.

## LITERATURE REVIEW

We conceive of *teaching* as the interactions between teachers, students, and the mathematical content and embedded within a particular environment (Cohen, Raudenbush, & Ball, 2003). Strategies are defined here as those activities that teachers organize to facilitate

some of the interactions of teaching (e.g., discussion with other students, technology or manipulative use, group work, etc.). Laursen, Hassi, Kogan, and Weston (2014) refer to IBL as a student-centred approach that “invites students to work out ill-structured but meaningful problems... [and] construct, analyse, and critique arguments... present and discuss solutions alone at the board or via structured small-group work, while instructors guide and monitor this process” (p. 407). Initial research has documented the positive trends on self-reported student gains in the cognitive and social domains (see, e.g., Laursen et al., 2014). Laursen and colleagues (2014) indicate that students in IBL courses report higher cognitive gains than students in non-IBL courses, in terms of mathematical thinking, understanding of concepts, and application of mathematical knowledge. Students in IBL courses, and in particular future teachers, reported higher cognitive gains. In addition, students in IBL courses reported higher gains in terms of confidence, positive attitude, persistence, independence, and collaboration, than students in non-IBL courses. Finally, women in IBL courses reported higher gains than women in non-IBL courses. Thus, if teaching with IBL methods is related to these gains, assisting faculty as they learn to use these methods is crucial. As important as students’ gains are, an important aspect of the process is the teacher and his or her teaching.

The literature on teacher knowledge as it pertains to teaching is extensive at the K-12 level but is sparser in the post-secondary level. Nardi, Jaworski, and Hegedus (2005) interviewed tutors at the University of Oxford over an 8-week period that prompted them to reflect on aspects of their teaching. The researchers identified four types of pedagogical awareness—naïve and dismissive, intuitive and questioning, reflective and analytic, and confident and articulate (p. 293)—which, they propose, reveal a spectrum of awareness about students’ difficulties, strategies to overcome those difficulties, and self-reflection about teaching practices. These researchers claim that instructor awareness can feed into other teaching formats (p. 293), which may suggest a similar categorization for instructors engaged in using an instructional method such as IBL. Other accounts of teaching with inquiry-oriented curriculum (e.g., Speer & Hald, 2008; Stephan & Rasmussen, 2002) point at specific dilemmas that instructors face, in particular navigating the need to stay away from lecturing and moving into more discussion based classes. This literature is informative and allows

us to think that there might be common concerns that faculty have when they teach with IBL methods, revealing key dilemmas that mathematics teachers face in teaching (e.g., Chazan & Ball, 1999). Simultaneously, we think that instructors bring with them knowledge that helps them deal with many of these dilemmas, and that they can actually be very resourceful in solving some of those challenges on their own, depending on the type of work they are called to do, even if their main source of information is their own experience with lecturing (the predominant pedagogical strategy in university mathematics classrooms, Blair, Kirkman, & Maxwell, 2013). This study, which started with an investigation of faculty concerns, allowed us to also investigate a possible association with the familiarity using this method and the solutions proposed to deal with concerns in teaching.

## THEORETICAL FRAMEWORK

We conceive of the work reported here as being under the umbrella of practice-based theorizations of teacher knowledge, as proposed by Ball and colleagues (Ball, Thames, & Phelps, 2008). Their theory starts from observation of practice to derive the types of tasks that teachers frequently engage in when teaching and it seeks to understand the interaction between those tasks of teaching and the specific subject matter one is teaching. It is a further elaboration of Shulman’s (1986) idea that there was a kind of specialized knowledge that was needed for teaching. Ball and colleagues’ theory includes six areas: Common Content Knowledge, Specialized Content Knowledge, Horizon Knowledge, Knowledge of Content and Students, Knowledge of Content and Teaching, and Knowledge of Content and Curriculum [1]. The last three areas are a further categorization of what Shulman called Pedagogical Content Knowledge. Knowledge of Content and Students, “(KCS), is knowledge that combines knowing about students and knowing about mathematics. Teachers must anticipate what students are likely to think and what they will find confusing” (p. 402). Knowledge of Content and Teaching, (KCT) is knowledge that “combines knowing about teaching and knowing about mathematics” (p. 402), exhibited when teachers decide which tasks or representations to use. Knowledge of Content and Curriculum (KCC) refers to knowledge of the place in the larger curriculum of the various elements of content to teach. Our focus in this paper is on this further elaboration of pedagogical content knowledge as revealed through teach-

ers’ practice with a novel method, IBL, and gleaned through two pieces of information, what instructors find difficult to handle—which signals an area of knowledge needed—and what they propose as a solution for that need, which signals the use of one or various domains of knowledge. Within this theory, we also contend that experience with teaching in general and with IBL in particular informs these domains and that differences in how faculty report concerns and solutions signal the use of those domains.

**METHODS**

Faculty were recruited from the networks associated with the Academies of Inquiry Based Learning and the R. L. Moore conferences and workshops in the United States [2]. Over three years, 74 instructors from 30 different states took part of the study. We collected two types of data, bi-weekly logs and interviews. The bi-weekly logs asked faculty to log into a secure system and document concerns that they experienced over the prior two weeks on 10 different areas (preparing class, designing assessments, using homework, using quizzes/tests/exams, pacing, lecturing, having students do presentations, group work, large group discussion, and mathematics) and the solutions they had found or ideas of what could help them if no solutions were available. In all we collected information on 171 different courses [3] through 943 log entries. In addition to the logs, we selected a purposeful sample of faculty for conducting in-depth interviews. The interviews covered two main areas, instructors’ understanding of IBL and the implementation of IBL in the most recent course they had used to fill the bi-weekly logs [4]. We conducted a total of 30 interviews.

In this paper, we discuss faculty concerns and solutions about two IBL practices, group work and stu-

dent presentations, and two assessment activities, design and use of quizzes, tests, and exams as revealed through the bi-weekly logs. Table 1 shows the number of instructors recruited who logged a concern in each of the areas included in this analysis, separated by their level of familiarity with IBL [5]. Group work and student presentations are two strategies that depart significantly from lecturing and constitute key features of an IBL course. We focus on assessment because our analysis of interviews suggests that this is a major concern for faculty, as they struggle to fit what they understand the role of assessment is with the goals they want to promote by using IBL (Whittemore & Mesa, 2014). In addition, assessment is a practice that teachers, independently of the method, need to manage. We conjectured that the faculty would therefore draw from their prior knowledge more readily when dealing with concerns emanating from these areas than from the other two, therefore providing an important contrast.

We went through several iterations to build a coding system, using constant comparative methods (Corbin & Strauss, 2008), focusing at times on concerns and at times on solutions. Our intention was to create an all-encompassing, parsimonious system for coding concerns and solutions across all areas of teaching. We identified 14 solutions that describe what faculty proposed to solve the challenges they faced over the course of the term when teaching with IBL. The analytical strategy to answer our first question is qualitative in nature, describing the kinds of actions teachers propose, which will be used to connect to the types of knowledge they draw from. To answer our second research question, we model the likelihood that a solution will be chosen to answer concerns in each of these areas (each is coded as 1 or 0 depending on whether there is a concern in that area), controlling for instruc-

	Number of Instructors (L/M/H) <sup>a</sup>	Number of Logs (L/M/H)
IBL Practices		
Group Work	33 (20/12/4) <sup>b</sup>	80 (45/25/10)
Student Presentations	60 (31/24/7)	172 (75/80/17)
Assessment Activities		
Design	38 (21/13/5)	78 (38/28/12)
Use	38 (21/15/5)	79 (45/28/5)

Notes: a. Level of familiarity with IBL, L: low, M: medium, H: High. b. Numbers do not add up because some teachers who participated more than one semester could change their level of familiarity after each semester.

**Table 1:** Number of instructors reporting solutions to selected areas of concerns and number of logs teachers used for reporting concerns by level of familiarity with IBL

tors' familiarity with IBL and type of course. More specifically, we estimated generalized linear logistic regression models with a binary outcome (proposing any given solution or not) for all the areas of concern.

**FINDINGS**

In this paper, we describe the four solutions most frequently coded: Provide Direction, Adjust/Clarify Expectations, Prepare, and No Solution. We first describe the qualitative nature of these solutions and then present the analysis by level of familiarity.

Provide Direction referred to solutions in which the instructor guided students along a desired path while refraining from assuming the role of lecturer. The instructor may provide hints, ask students to break procedures down into smaller steps, or pose questions in general with the intent to incite comprehension/reflection or metacognition:

I try to get the class to realize the problems. Sometimes I jump in, if it involves writing. Sometimes I give the class hints until they get it. I think this is the most challenging part of the experience, and it takes a lot of energy! (Log2.1 I57\_M\_LD) [6]

Adjust/Clarify Expectations referred to solutions in which the instructor reminded students of course expectations, resources available to them, and the motivation and purpose for success within the course. The solution may involve relaxing or tightening classroom procedures, minimizing or adding to assignments/coursework, and adjusting the intensity of adherence to classroom procedures:

For the weak students, I have asked students to come and see me prior to class and work with group members outside of the class. I have asked

students to go to the numerous daily tutorial sessions. (Log6.5\_I58\_L\_LD)

Prepare referred to solutions in which instructors reread course notes and lecture topics, and put together materials for classes in general. This solution describes instructors desiring to remain vigilant and flexible to the evolving nature of the unfolding work in the classes and ideas of actions they intend to take in the next class. It also included tasks that the instructor had found or created, plans to use those in class, or reports that the students had used those tasks:

I have been leaning towards teaching IBL style for the last few semesters, so I am adapting a lot of what I've previously used. I've also found some resources by looking on the Internet and emailing people if I saw something interesting. Since this class is really more of a hybrid IBL and not "pure" IBL and we don't do a lot of theory-related things, some [of] the things I have found (JIBLM) don't quite work. (Log6.3\_I45\_L\_UD)

The No Solution code was assigned when the comments indicated that the instructors were unsure about solutions to their concerns, that they had an idea for a solution but that the solution was not working, or that they were "at a loss" about what would help:

I know I should have the students presenting more proofs in class, but somehow I am not successful this semester at getting proofs presented by the students in class. Most are not even trying the proofs outside of class. Even the examples are not being tried by most of the students. I'm at a loss even to the type of resources I might need. (Log7.4\_I62\_L\_UD)

The frequencies with which these solutions were assigned to the logs by each area of concern are given in Table 2.

	IBL Practices		Assessment	
	Group Work (n = 80)	Student Presentations (n = 172)	Design (n = 78)	Use (n = 79)
Provide Direction	21%	24%	8%	13%
Adj./Clarify Exp.	30%	33%	18%	31%
Prepare	15%	20%	35%	17%
No Solution	24%	7%	13%	17%

**Table 2:** Frequency of Solutions Proposed by Faculty to concerns on IBL Practices and Assessments (N=409 logs)

We run three separate logistic regression models that included all the areas of concern (not only the ones that are the focus of this paper), the course type, and the level of familiarity with IBL as independent variables and the solution as the dependent variable (using it or not, one model per solution: Provide Direction, Adj./Clarify Expectations, and Prepare). The regression coefficient for level of familiarity with IBL was not significant in any of these models; that is, when controlling for the area of concerns and the type of course (upper division, lower division, or future teachers), the probability of using a particular solution was not significantly different for different levels of familiarity. For simplicity in presentation Table 3 shows only the odds ratios estimate, standard errors, and significance of the corresponding coefficient for the three models for the four focal areas of concern.

We highlight four points from Table 3. First, relative to teachers who report that they have concerns with group work, teachers who do not have concerns are 2.5 (1/0.393) times less likely to use prepare as a solution. This suggests, perhaps unsurprisingly, that when solving problems related to organizing group work in the classroom, faculty are likely to expect to spend time in advance, anticipating ways to handle those problems. Second and third, relative to teachers who report that they have concerns with designing assessments, teachers who do not have concerns in this area are 3.2 times less likely to use provide direction as a solution (1/0.309) and 3 times less likely to adjust or clarify expectations as a solution (1/0.332). This suggests, that in solving problems related to designing assessments for IBL work, faculty will resort to modify the assessment or provide better explanations for the requirements or for the work, in a way suggesting that they need to be flexible in order to account for the special nature of assessments in this environment. Naturally, this is a plausible interpretation that would need to be corroborated with a different design in which instructors describe problems

preparing assessments in non-IBL courses. Finally, relative to teachers who report that they have concerns with using assessments, teachers who do not have concerns in this area are 2.3 times less likely to use prepare as a solution. Thus, similar to concerns faculty face with group work, faculty with concerns with the use of assessments for students will spend time thinking through those assessments based on the feedback they gather from how things work during class. When assessments are used they do provide direct information about students' learning, which may trigger realizations for teachers of the need to learn to anticipate issues better.

We obtained an intriguing result when modelling No Solution. The odds ratio coefficient was significant for faculty identified as having low level of familiarity with the method (Odds Ratio, Low Level of Familiarity = 0.296, SE = 0.603,  $p < .05$ ) relative to faculty having high level of familiarity. This means that relative to faculty who had the highest level of familiarity, faculty with low level of familiarity were 3.4 times less likely to report that they had no solution for their concerns.

**DISCUSSION**

Our analysis of the solutions that faculty proposed to the concerns they had with teaching with IBL methods reveals the ways in which faculty use the knowledge they have to manage the situations. Of the three solutions most frequently assigned, Provide Direction, Adjust/Clarify Expectations and Prepare, it seems that the first two need to be deployed by calling upon teachers' appraisal of the instructional situation, namely the way in which students and the material are interacting in the moment and the possible ways in which teachers can provide feedback. In Provide Direction, the teacher refrains from giving a full explanation to the students to answer a particular question. This requires an in-the-moment assessment

Area of Concern	Provide Direction	Adj./Clarify Exp.	Prepare
Group Work <sup>a</sup>	1.137 (0.525)	0.781 (0.458)	0.393 (0.584)†
Student Presentation	1.140 (0.480)	0.835 (0.416)	0.586 (0.598)
Design Assessments	0.309 (0.617)†	0.332 (0.484)*	1.233 (0.617)
Use Assessments	0.595 (0.558)	0.789 (0.454)	0.424 (0.595) †

Notes: a. The reference category for each coefficient is presence of the concern. The odds ratio is the exponential of the coefficient modelled for the cases in which there is no concern reported. The reciprocal of the value gives the relative ratio for the reference category. †:  $p < .10$ ; \*  $p < .05$ .

**Table 3:** Odds ratios and Standard Errors by Solution and Area of Concern

of students' thinking paths vis a vis the mathematics at stake and a decision-making process regarding the mathematical idea or question that may nudge the student in a different direction. We see this as evidence of teachers drawing from their Knowledge of Content and Students. In contrast, when using Adjust/Clarify Expectations, teachers are taking a larger view of the constraints imposed by using the method; the actions taken refer to larger organizational aspects of the course that impinge on the quality of students' work: more resources, tighter or more relaxed requirements. In order to make such decisions, instructors account for the contextual situation in which students are working and how that affects the pacing or other goals of the course; they need to keep in mind that they might need to adjust further the class organization so that the course maintains its coherence. We see this as evidence of drawing on their Knowledge of Content and Teaching. When using Prepare, on the other hand, instructors bring to bear knowledge in both domains. They need to take into account what the students have done in any given class, and decide on a path that will satisfy the goals of course and the demands students have for understanding the content. It also involves the generation of documents informed by such thinking processes.

The level of familiarity did not play a role in the types of solutions proposed, except in the case in which faculty reported having no solution. We take this result as an indication that the method itself makes similar demands on teachers' knowledge independently of the experience they have with the method. This result does not mean that the method does not get easier with time. Recall that we are talking about solutions, that is, about the types of knowledge that could be drawn from to solve problems of practice. Theoretically, there is no reason to believe that instructors of different levels of familiarity with the method will draw from different knowledge sources. This way of teaching, by making students visible, forces faculty to navigate through the spectrum of awareness described by Nardi and colleagues (2005) and puts all the teachers at the same stage in Nyquist and Sprague's (1998) model of teacher development, in which they have to deal with student thinking in every lesson.

This work implies that teachers use their knowledge of teaching when teaching with a new method. While we did see a large amount of no solutions to individual concerns, we see that faculty draw on their experi-

ence and knowledge to deal with the concerns that they face in IBL practice. Because teaching is in the moment, instructors must find a solution to concerns as they emerge in order to manage instruction in the classroom, and working on augmenting their mathematical knowledge for teaching could be a fruitful strategy for sustaining this way of teaching. While there are many outside resources to help teachers using IBL, it seems that for the instructors in this study, given their knowledge, the instructors themselves are their most valuable resource.

## ACKNOWLEDGMENT

We thank Inah Ko, Kyle LaFollette, and Tim Whittemore for assistance in earlier analyses of the data. This work has been supported by the Educational Advancement Foundation and the Undergraduate Research Opportunity Program at the University of Michigan.

## REFERENCES

- Ball, D. L., Thames, M., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389–407.
- Blair, R., Kirkman, E. E., & Maxwell, J. W. (2013). *Statistical abstract of undergraduate programs in the mathematical sciences in the United States. Fall 2010 CBMS Survey*. Washington, D.C.: American Mathematical Society.
- Chazan, D., & Ball, D. L. (1999). Beyond being told not to tell. *For the Learning of Mathematics*, 9, 2–10.
- Cohen, D. K., Raudenbush, S. W., & Ball, D. L. (2003). Resources, instruction, and research. *Educational Evaluation and Policy Analysis*, 25, 119–142.
- Coppin, C. A., Mahavier, W. T., May, E. L., & Parker, G. E. (2009). *The Moore Method: A pathway to learner-centered instruction*. Washington, DC: The Mathematical Association of America.
- Corbin, J., & Strauss, A. (2008). *Basics of qualitative research* (3rd ed.). Los Angeles, CA: Sage.
- Laursen, S., Hassi, M.-L., Kogan, M., & Weston, T. (2014). Benefits for women and men of inquiry-based learning in college mathematics: A multi-institution study. *Journal for Research in Mathematics Education*, 45(4), 406–418.
- Mesa, V., & Cheng, A. (2008). *Inquiry Based Methods Project: Mathematics 175 and 385, Fall 2007*. Ann Arbor, MI: University of Michigan.
- Nardi, E., Jaworski, B., & Hegedus, S. (2005). A spectrum of pedagogical awareness for undergraduate mathematics: From

'tricks' to 'techniques'. *Journal for Research in Mathematics Education*, 36(4), 384–316.

Nyquist, J. D., & Sprague, J. (1998). Thinking developmentally about TAs. In M. Marincovich, J. Protsko, & F. Stout (Eds.), *The professional development of graduate teaching assistants* (pp. 61–88). Bolton, MA: Anker.

Shulman, L. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher*, 15(2), 4–14.

Speer, N., & Hald, O. (2008). How do mathematicians learn to teach? Implications from research on teachers and teaching for graduate student professional development. In M. Carlson & C. L. Rasmussen (Eds.), *Making the connection: Research and teaching in undergraduate mathematics education* (pp. 305–317). Washington, DC: Mathematical Association of America.

Stephan, M., & Rasmussen, C. L. (2002). Classroom mathematical practices in differential equations. *Journal of Mathematical Behavior*, 21, 459–490.

Whittemore, T., & Mesa, V. (2014). *Assessment in IBL mathematics courses*. Paper presented at the R. L. Moore Legacy Conference, Denver, CO.

the amount of mentoring they provide, among other dimensions.

6. The identifier Log2.1\_I55\_L\_LD indicates the semester when the data was collected (second), the number of the log (first in the term), the instructor (#I55), the level of familiarity (Low, Medium, High), and the level of course for which the log was being recorded, (Lower Division, Upper Division, or Future Teachers).

## ENDNOTES

1. Due to space considerations we only describe the domains that are relevant to our study. We refer the reader to Ball and colleagues (2008) for full definitions of the other domains.

2. See <http://legacyrlmoore.org> and <http://www.inquirybasedlearning.org> for details.

3. Lower division courses were intended for students in their first two years of college (e.g., 100–200 level courses: Calculus I, II, Discrete mathematics, Cryptology); Upper division courses were intended for students in their last two years of college (e.g., 300–400 level courses: intro to proof, modern or abstract algebra, topology, real analysis, etc.); Future teacher courses were those intended for future teachers and could be of any level (e.g., geometry for teachers, math for elementary teachers, etc.)

4. The protocols for the logs and the interviews are available from the authors.

5. Familiarity with IBL was divided into three levels, Low, Medium, and High, depending on instructors' described experience teaching with the method (e.g., # of IBL courses), the use of own generated notes, and