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# Characterising university mathematics teaching

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*This paper reports early findings from university mathematics teaching in the tutorial setting. The study distils characteristics of two tutors' mathematics teaching and through interviews, their underlying considerations. Analysis of a teaching episode from each tutor illuminates their different teaching approaches and suggests ways in which approaches are linked to students' meaning making.*

**Keywords:** University mathematics teaching, small group tutorials, meaning making.

## INTRODUCTION

University mathematics teaching is of major significance in mathematics education. Some of the numerous benefits from research in this area of study are the production of professional development resources for novice and experienced university teachers as well as support to those teachers to create rich learning opportunities (Speer, Smith, & Horvath, 2010). However, the benefits from research go beyond university mathematics teaching and learning. Speer and Wagner (2009, p. 537) stressed that by studying the practice of teachers with strong mathematical knowledge, teacher educators “are better able to detect directions for growth in other areas of knowledge” such as “knowledge of typical ways students think (correctly and incorrectly) about the task or content in question” (p. 558), which teachers at all levels may need.

This report is based on the analyses from my ongoing doctoral project that examines university mathematics teaching through an exploration of tutors' teaching practice with first year undergraduate mathematics small-group tutorials. The analysis suggests that teaching practice can be construed in terms of three elements *tools*, *strategies* and *characteristics* of teaching (Mali, 2014). I focus here only on *characteristics* of teaching, which are patterns in the ways that

tutors teach in the tutorials. From observations of tutors' teaching and interviews with tutors about their teaching, the aim of the doctoral project is to identify aspects of teaching practice and knowledge and connect them with students' meaning making in mathematics. Tutorials are studied since they provide opportunities for teacher-student dialogue and interaction through which meaning making can be discerned.

The focus of this report is on two tutors' teaching approaches. It is significant to juxtapose *characteristics* which look similar but are used in different ways by the different tutors to promote student meanings. So, the three research questions are: What are the *characteristics* of the two tutors' teaching? In what ways do two different tutors implement similar *characteristics* of teaching? What are the tutors' actions that encourage students to make meaning (promoting meaning making.) and what do tutors do to find out what meanings have been made (discerning meaning making)?

## THEORETICAL BACKGROUND AND LITERATURE REVIEW

University mathematics teaching is an area of interest where research is still rather limited. Speer, Smith and Horvath (2010), after conducting a systematic literature review in university mathematics teaching, reported that there is no systematic data collection and analysis focusing on teachers and teaching. As to the small group tutorial setting, the number of studies is far fewer. Certain studies (including this one) focus on the nature of tutor's teaching and the knowledge that frames it. For example, Jaworski and Didis (2014) introduced the *questioning approach* to teaching and suggested tutor's awareness about her teaching as the basis of *knowledge in practice* which informs future action. Mali, Biza and Jaworski (2014) focused on *characteristics* of university mathematics teaching, such as the use of generic examples, and suggest that the research practices of the tutor (math-

ematician) influence her teaching practices; an influence which accords with findings in the format of lectures (Petropoulou, Potari, & Zachariades, 2011). Jaworski (2002) distinguished tutors' exposition patterns (*tutor explanation, tutor as expert* and forms of *tutor questioning*) as the main teaching aspect in the context of tutorials. She also stressed that teaching/learning was idiosyncratic to the tutor and to some extent to the particular students. These studies give insight into elements of tutors' teaching practices and their reflective thinking. Gaining access to students' meaning in relation to teaching provides significant information about how tutors encourage students to make meaning; for example, Jaworski and Didis (2014) relate students' meaning making to the *why questions* of the tutor. In our study reported here, we investigate, through a sociocultural perspective, teaching practices of selected tutors that encourage students to make meaning.

The socio-cultural paradigm, rooted in Vygotskian psychology, considers the overall social and cultural context, which frames mathematics teaching and learning in its complexity. Concepts and meanings are experienced and understood in the social and cultural small group tutorial practices (e.g., engagement, participation and interaction). Meaning, thinking and reasoning are products of social activity and take place first on the social plane. Tutors' teaching mediates students' mathematical meaning making by using material (e.g., textbooks, problem sheets) or intellectual tools (e.g., exposition).

In this paper, I embed tutors' *characteristics* of teaching in the socio-cultural tutorial practices of tutor-student interaction and participation relating them to students' meaning making. For the purposes of the analysis, I draw on the literature which considers meaning making in terms of making connections within mathematics through different representations, such as symbols, diagrams, pictures (Haylock, 1982); and between mathematics and "other aspects of the world" (Ormell, 1974, p. 13), such as real world situations. I interpret this collective mathematical meaning making through observing and analysing tutors' and students' actions in the classroom. The tutor's actions relate to the nature of teaching and the approach, and accord with what the tutor says (I can read in transcripts); does (gestures, body language) and intentions (I can ask in interviews or hear in the

classroom). The students' actions are what they say and do during the tutorials.

## THE CONTEXT OF THE STUDY

The study is being conducted at an English University, where students are in their first year of a straight or a joint programme in Mathematics. They are expected to attend lectures (in analysis modules and linear algebra) and a small group tutorial of 5 to 8 students. Tutorials are 50 minutes weekly sessions and work is on the material of the lectures (lecture notes, problem sheets, coursework and exams). Students are expected to work on the material of the lectures beforehand and bring their questions to the tutorial. The tutors are lecturers in modules offered by the mathematics department and conduct research in mathematics or mathematics education. Phanes and Alex are experienced lecturers as well as researchers. Phanes holds a doctorate in mathematics and Alex holds a doctorate in mathematics education.

## THE METHOD OF THE STUDY

This study is part of a doctoral project, which has analysed data from one tutorial from each of 26 tutors, as a basis for conceptualisation of teaching. This has been followed by a systematic study of the teaching of three of the 26 tutors for more than one semester. Phanes and Alex are two of the three tutors. Data consists of observation notes and transcriptions of their audio-recorded tutorials and follow up interviews. The interviews are discussions with them about their thinking behind the teaching actions in these tutorials. A grounded analytical approach is taken to the data in which aspects of tutors' actions that seemed to be informed by their teaching knowledge are coded. Analysis is based on the identification and grounded study of teaching episodes; there are several cycles of interpretation: from initial ones using open coding to more advanced ones creating categories. *Characteristics* of teaching have been identified repeatedly throughout the analysis of each tutor's teaching, emerging from this as a category in the nature of teaching. Examples are provided in the accounts below.

## RESULTS

Both Phanes and Alex work on a number of questions/mathematical issues in their tutorials, which is a fre-

|    | Grounded characteristics of teaching  | Phanes | Alex |
|----|---|--------|------|
| 1  | Use of graphs, diagrams or gestures to provide a visual intuition for formal representations            | ✓      | ✓    |
| 2  | Know-how exposition about procedures or techniques for the work on mathematics                          | ✓      | ×    |
| 3  | Use of problem-solving techniques   | ✓      | ✓    |
| 4  | Use of kind(s) of/multiple examples   | ✓      | ✓    |
| 5  | Use of different mathematical representations/notation  | ✓      | ✓    |
| 6  | Explanation/revision of theory from special to general cases devising less to more complicated examples | ✓      | ×    |
| 7  | Provision of time to students to work on their scripts while tutor is circulating and supporting        | ✓      | ✓    |
| 8  | Request to students to devise an example  | ✓      | ✓    |
| 9  | Tutor's intuitive or formal explanation of concepts   | ✓      | ✓    |
| 10 | Use of funneling  | ✓      | ✓    |
| 11 | Invitation to specific students to answer   | ×      | ✓    |
| 12 | Request to students to find definitions in lecture notes  | ×      | ✓    |

**Table 1:** Characteristics of the two tutors' teaching

quent general practice at tutorials. As preparation for the tutorial they look at the lecture materials, including problem sheets, a few minutes before the tutorial. In the following Table, I present *characteristics* of teaching, identified so far, from both tutors. These characteristics emerged in the process of data analysis after the tutorial observations.

In order to scrutinise the different ways tutors implement common *characteristics* and the different issues that are raised, I offer a teaching episode from each tutor. These episodes are paradigmatic cases of the tutors' teaching in terms of manifesting a number of *characteristics* of their teaching.

**Phanes' approach**

This episode is situated in the second tutorial of the year and concerns work on an exercise from the first problem sheet in analysis:

“Rewrite  $||x|-1|$  without modulus signs, using several cases where necessary.”

Reading the exercise, Phanes suggests: “we can just sketch the graph of the function”. He uses *exposition about know-how* to get rid of the modulus sign (*characteristic 2/*Table 1): “You see, to get rid of the modulus sign of  $|x|$ , you need to know that  $x$  is positive or negative. You have to consider cases. But there is another outer modulus. It's external. Again, to get rid of it, you need to either consider the case whether

the expression inside it is positive or not.” He offers a *less complicated example to reveal the work on modulus signs* (*characteristic 3/*Table 1); he constructs on the board the graph of  $|x^3|$  reflecting the negative part of the graph of  $x^3$  about the  $x$ -axis. Then he requests students to work on their scripts for  $||x|-1|$  (*characteristics 7/*Table 1), after which the following episode occurs:

Phanes: So, how do I solve this problem? I'll show you. I saw correct pictures; all of you had correct pictures. So, what am I going to do? I will do it step-by-step. First, I will construct  $|x|$ , right?  $|x|$  is this. [Phanes sketches the graph of Figure 1.] Ok? Then, we do  $|x|-1$ .  $|x|-1$  means that you take  $|x|$  and you shift it down by 1. This means  $-1$ , right? So, it gives you this [g in Figure 1]. These points are 1 and  $-1$ . And this point is  $-1$ . This is the expression under the modulus sign. And then, you take the modulus of this function and it means that you reflect this negative bit about the  $x$  axis, right? And you get this function. Ok? This is the graph of the function. Now, we have to write down the equations for this. You can see that it's given by different functions on different intervals. For instance, this expression is what [f in Figure 1]? This was  $y=x$  [e in Figure 1], and then, you shift it

by 1, so this is  $x-1$  [f in Figure 1]. Is this clear? Please stop me if something is unclear. So, this is  $x-1$  [f in Figure 1]. So, what is this [c in Figure 1]? What is this – this bit [c in Figure 1]? It has the same slope as  $x-1$  but it's shifted it up.

St2: It's  $x+1$ .

Phanes: It's  $x+1$ . So, this bit is  $x+1$  [c in Figure 1]. Now, what is this [a in Figure 1]? This graph is  $y=-x$  [b in Figure 1], and we shift it down, so it's  $-x-1$ . So, this thing is  $-x-1$ . And what is this [d in Figure 1]?

St4:  $-x+1$ .

Phanes:  $-x+1$ .  $-x+1$ . So, what can we now say about this function  $||x|-1|$ ? It equals. Now, it depends on where  $x$  is, right? So, we know for this function that on this [Phanes points to interval  $[1, +\infty)$ ], it's  $x-1$  if  $x$  is greater than or equal to 1. Agree? It is  $-x+1$ ,  $-x+1$ , if  $x$  belongs to  $(0, 1)$ . It is  $x+1$ ,  $x+1$ , if  $x$  belongs to  $(-1, 0)$ . And finally, it's  $-x-1$  if  $x$  belongs to  $(-\infty, -1)$ .

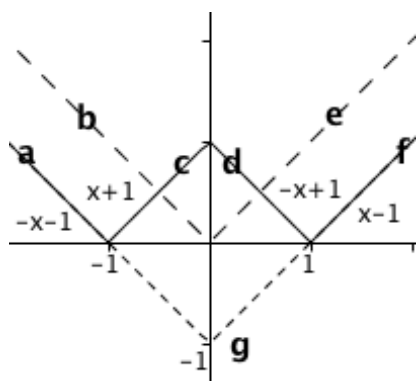


Figure 1: Graph on board, episode 1

In the above episode, Phanes uses the *graph of  $||x|-1|$  to provide a visual intuition for rewriting the algebraic expression  $||x|-1|$  without modulus signs (characteristic 1/ Table 1)*. He first constructs the graph “step-by-step” and then the equations; in this way, he *divides the mathematical task into steps (characteristic 3/ Table 1) and uses geometric and algebraic representations (characteristic 5/ Table 1)*. For the construction of the graph, he offers *know-how exposition for the work on modulus signs (characteristic 2/ Table 1)*. Furthermore, for the construction of the equations, he shows how to find  $x-1$  and  $-x-1$  asking students for  $x+1$  and  $-x+1$  respectively (characteristics 4/ Table 1).

The above characteristics (1–5/ Table 1) are within Phanes’ thinking on the mathematics. Phanes uses

the graph of  $||x|-1|$  as a tool to think on the mathematics; he adjusts basic graphs (i.e.  $|x|$ ,  $x$ ,  $-x$ ) to construct  $||x|-1|$  and from that, he extracts the essential information (i.e. equations and intervals) for the solution of the specific exercise. He uses the graphical representation  $||x|-1|$  and visual intuition of the equations as a problem solving technique (*characteristic 3/ Table 1*), thereby negotiating different contexts (geometric and algebraic) of the concept of modulus sign. Connecting the two contexts/representations, he promotes students’ meaning making of the modulus sign. To this end, he also uses know-how exposition, problem-solving techniques, multiple examples of expressing equations and provision of time to students for individual work (*characteristics 2–4, 7/ Table 1*).

In the beginning of the episode, Phanes comments that “I saw correct pictures; all of you had correct pictures.” This suggests that while circulating and supporting students (*characteristic 7/ Table 1*), he also made some judgements about their meaning making of the modulus sign. These judgements arise from his assessment of the students’ scripts and indicate that he used *characteristic 7* to discern their meaning making; not only to promote it. Phanes can also use the multiple examples of equations (*characteristic 4/ Table 1*) to discern students’ meaning making of the graph by assessing their correct answers for  $x+1$  and  $-x+1$ .

The use of visual intuition of the equations on the graph does not provide enough insight into the intervals. After the episode, Phanes stresses to students that the function is continuous, so “it doesn’t matter” if the endpoint is included in the interval; he says to them “strictly speaking, you should include it”. In our discussion and in response why he chose a geometric solution when some mathematicians avoid choosing them, Phanes connected his choice with mathematicians’ research practices.

It depends on your research area. If you are a geometer [Phanes is a geometer], you are happy with geometric solutions; it depends on your background I think. [...] You see to me it is easier to see the graph. [...] For instance if you are a programmer writing computer programs, then it is more convenient to you to give an algorithm.

Phanes approaches mathematics teaching putting emphasis on the mathematics and geometric thinking, whereby he relates geometric solutions to his

research area. From this thinking on mathematics, he draws out his teaching practice which I recognise through his actions (*characteristics* 1–5, 7/ Table 1) to promote and/or discern students' meaning making. In this episode, Phanes presents the ways he is working through the graphs and symbols dissecting the mathematical task to make its aspects more visible to students. He thus works within his thinking about the graphs in *characteristics* 1–5. *Characteristic* 7 is different in nature from the others since it can be used in the teaching of other subjects as well as mathematics.

### Alex's approach

In his third tutorial of the year, Alex used Venn diagrams to explain the definition of injectivity (*characteristic* 1/ Table 1) as well as examples and non-examples of the concept (*characteristic* 4/ Table 1). In discussion after the third tutorial, he reflected:

By the reaction I got when I asked for the definition [of injectivity] the student couldn't even say what the symbols were there. So, I had to repeat it for him. There was not so much meaning making there. So, that's why I decided to use examples, use the Venn diagrams for the sets and what exactly it means to be injective and surjective. [...] If the students get it, I am not sure about that, because after that they still have the face of 'what are you talking about?' So, at that point you say 'Mmm if I carry on with more examples, eventually they will get it', because I don't have any other didactical instrument to make it even clearer for them. Ah in fact when I was preparing my module for another lecture, I thought of a very good example of the function. When you go to the supermarket and I am going to say to them next time [...] to explain what an injective and a surjective function is. [...] And I think that's more near the experience of the students, so that they can say "ah yes, I get it now".

Alex implemented the "good example of the function" in the fourth tutorial. He said to the students: "A function is a relationship between a set of inputs, in this case the products in the supermarket, a loaf of bread, and the set of permissible outputs, in this case the prices. So it relates each product to the one, the only one price, it cannot be related to two." In the following episode, we see Alex's implementation of the example for the concept of injectivity. Before the start of the episode, Alex asked the students to express injectivity

in the context of his example. As a response to their inability to do so, he asked them to find the definition of injectivity [ $\forall x, y \in \text{Dom}(f), f(x)=f(y) \Rightarrow x=y$ ] in their lecture notes (*characteristic* 12/ Table 1). He then wrote the definition of injectivity on the board.

Alex: How would you read that [the definition of injectivity] in the supermarket example? Which are the  $x$ 's? What's the domain of the function? St3, what would the  $x$ 's be in this example in Tesco [i.e. supermarket]?

St3: Products. Products..

Alex: The products, exactly. So for all the products in Tesco

St3: They should be  $x$  and  $y$ .

Alex: So why would it be  $x$  and  $y$ ?

St3: Because it's product and price;  $x$  is product,  $y$  is price.

Alex:  $x$  could be read,  $y$  could be milk, mm? So what would this mean, this then? [Alex points to  $f(x)=f(y)$ .]

St2: The same price. [St2's voice is almost inaudible.]

Alex: What would that [Alex points to  $f(x)=f(y)$ ] mean in the example? I want you to contextualise a very abstract formal definition so we do an everyday job that you can understand; that you give some meaning to those things. Try to think on the example of the supermarket, what would  $f(x)$  equal, what would  $f(y)$  equal, what would  $x$  be, what would  $y$  be?

St5:  $f(x)$  would be prices.

Alex: Yes, the prices, OK. So it says if the prices are equal, let's say 99p, what has to happen to  $x$  and  $y$ ? [Alex sketches Figure 2 on the board.] Let's say  $x$  is bread and  $y$  is milk, OK? And I notice that the price of the bread and the price of the milk are the same, they are both 99p. Yes? If this function was injective, then the bread would have to be milk, well that's impossible isn't it? [Alex deletes milk on Figure 2.] In other words, I cannot have the price of 99p that belongs to two products, two different products, mm, in the abstract definition, there is no way that 99p comes from bread and milk. Does that make sense or not? Say no if ... well your faces say no.

St1: No, I'd say no.  
 Alex: OK. Can you think of another example?  
 [...] Do you play a sport?

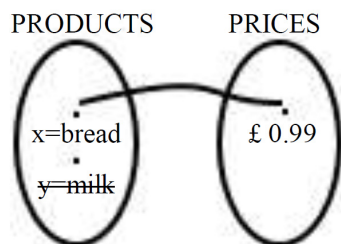


Figure 2: Diagram on board, episode 2

St1 mentioned hockey and Alex devised another example regarding a function that relates hockey players with their scores (*characteristic 4/*Table 1). Despite the real world context of the example of function in the supermarket (a product cannot be related simultaneously to two final prices), a function that relates products/players with their prices/scores is not injective in real life since, there, two different products/players can have the same price/score.

In the above episode, Alex devises a real word example regarding a supermarket (*characteristic 4/*Table 1) to promote students' meaning making of the concept of injectivity: "I want you to contextualise a very abstract formal definition so we do an everyday job that you can understand; that you give some meaning to those things." He uses funnelling (*characteristic 10/*Table 1) by asking what the  $x$ ,  $y$ ,  $f(x)$  and  $f(y)$  are and invites st3 to answer (*characteristic 11/*Table 1); then, he uses the Venn diagram of Figure 2 to provide a visual intuition for the definition of injectivity in the context of the supermarket example (*characteristics 1, 5/*Table 1). In the interview excerpt, Alex stresses that in order to promote meaning making he uses multiple examples (*characteristic 4/*Table 1) and Venn diagrams (*characteristic 1/*Table 1). In discussion with Alex about the use of real world examples, he connects it with research in mathematics education.

By making it [the example] nearer to the students' experience; that comes from mathematics education. [...] Because you need to make connections in order to make meaning. To understand something you need to make the appropriate connections from your own experiences.

Alex discerns students' meaning making from their faces: "Does that make sense or not? Say no if... well

your faces say no" (episode excerpt) and "If the students get it, I am not sure about that, because after that they still have the face of 'what are you talking about?'" (interview excerpt1). In the episode, when st1 answers he doesn't make sense of the example, Alex asks him to devise an example close to his interests (*characteristics 8, 4/*Table 1). After the fifth tutorial, Alex reflects:

I thought it went a bit better last time when I asked st1: "What do you do in your life?" I play hockey he said. And it went well I thought; at least they said: "Oh yeah I understand now what you mean." That's the design at least to connect with what they do outside.

He also discerns meaning making by the reaction he gets from students.

Alex approaches mathematics teaching bringing in awareness from research in mathematics education; he connects mathematics with students' everyday experiences for meaning making (e.g., Ormell, 1974). A number of his actions (*characteristics 8, 10, 11, 12/*Table 1) to promote and/or discern students' meaning making relates to students' participation, and can be used in the teaching of other subjects as well as mathematics. In this episode, Alex steps out of mathematics, goes into the context of the students and chooses examples there that he can use to parallel injectivity (*characteristic 4/*Table 1). So, Alex roots the abstract mathematics in examples of an everyday nature; starts in the abstract mode through symbols; discerns that students do not make meaning of them; and then brings in a diagram as an alternative way of representing injectivity. He uses this diagram as a tool to explain the mathematics to students; it constitutes another representation of the formal definition which he enriches with explanatory exposition.

## CONCLUSIONS

In this paper, I presented two different approaches to teaching, where both tutors put a considerable effort so that students make meaning of the mathematics of the lectures. I related this effort to their actions to promote and/or discern students' meaning making coded in *characteristics* of teaching. I thus looked at the tutor's perspective for students' meaning making, acknowledging that there is no right or better approach. Zooming in each tutor's approach to teach-

ing, they deploy different ways to implement common characteristics; for instance the *characteristic use of graphical representations to provide visual intuition for formal representations*. Phanes uses the graph to make fundamentally mathematical ways of thinking transparent to students, whereas Alex uses it as an alternative to explain the mathematics. In order to promote students' meaning making of how a process works for the modulus function, Phanes uses exposition, problem-solving techniques, explanation, examples and representations all within his thinking of the mathematics. By his exemplification of the process, students can potentially gain mathematical expertise (i.e., use of graphs and visual intuition) to apply in other problems thereby being enculturated into mathematical practices. Alex's real world example is localised around the concept of function and its properties (e.g., injectivity), which is fundamental in mathematics. In order to promote students' meaning making of the concept of injectivity, he uses examples, representations, funneling and invitations as well as requests to students. Evaluation of students' scripts, responses and facial expressions are ways tutors discern students' meaning making. However, promoting and discerning meaning making are two processes that cannot be separated in some cases; for instance, while the tutor *provides time to students to work on their scripts*, he both supports and evaluates them. In future studies, I will analyse data from the other tutors and juxtapose their characteristics of teaching in order to identify aspects of teaching practice and knowledge and ultimately connect these aspects with students' mathematical meaning making.

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