



HAL
open science

Use of mathematics in engineering contexts: An empirical study on problem solving competencies

Malte Lehmann, Bettina Roesken-Winter, Sven Schueler

► **To cite this version:**

Malte Lehmann, Bettina Roesken-Winter, Sven Schueler. Use of mathematics in engineering contexts: An empirical study on problem solving competencies. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Charles University in Prague, Faculty of Education; ERME, Feb 2015, Prague, Czech Republic. pp.2173-2179. hal-01288607

HAL Id: hal-01288607

<https://hal.science/hal-01288607>

Submitted on 15 Mar 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Use of mathematics in engineering contexts: An empirical study on problem solving competencies

Malte Lehmann, Bettina Roesken-Winter and Sven Schueler

Humboldt-Universität zu Berlin, Berlin, Germany, malte.lehmann@hu-berlin.de

The research field of problem solving in mathematics is highly relevant in mathematics education. There are manifold approaches to understand the process of problem solving like Polya's phase model or Bruder and Collet's heuristic methods. In two studies, we investigate how engineering students' performances in higher mathematics and technical mechanics are connected (study 1, n=37) and what role their problem solving competencies play (study 2, n=8). In the first study, we ascertained that mathematical competencies and beliefs about physics are substantial for success in technical mechanics. In the second study, students had to complete sequences of tasks and their usage of heuristics was investigated. The results show that successful students' heuristic tools and strategies are more elaborated.

Keywords: Problem solving, engineering education, technical mechanics, qualitative content analysis.

INTRODUCTION

Mathematics is an important subject in engineering education. The first courses of study are characterized by a high usage of mathematics, be it in mathematics, in physics or further engineering lectures. Besides continuously improving their declarative and procedural knowledge in the respective fields, it is important for students to develop their problem solving competencies as this is one of eight competencies that engineering students need to learn according to the SEFI¹ (2013) framework: *thinking mathematically, reasoning mathematically, posing and solving mathematical problems, modelling mathematically, representing mathematical entities, handling mathematical symbols and formalism, communicating in, with and*

about mathematics and making use of aids and tools. In accordance with findings in mathematics education (cf. Törner, Schoenfeld, & Reiss, 2007), in SEFI (2013) problem solving as competency for engineering students is characterized as follows:

This competency [mathematical problem solving] includes on the one hand the ability to identify and specify mathematical problems [...] and on the other hand the ability to solve mathematical problems (including knowledge of the adequate algorithms). What really constitutes a problem is not well defined and it depends on personal capabilities whether or not a question is considered as a problem. (p. 13)

The formation of these competencies is, however, often hampered by an asynchronicity of mathematical and engineering education. The overarching aim of the project KoM@ING² is to measure mathematical competencies of engineering students and the relations between the different lectures by combining a quantitative and a qualitative perspective. In the quantitative approach (project partners from IPN³ Kiel and University Stuttgart), IRT-based measures for higher mathematics and technical mechanics are developed to capture students' development in their first year of study (Nickolaus, Behrendt, Dammann, Ștefănică, & Heinze, 2013). Thus, individual competencies are measured reliably and validly, but no insight is provided into the students' actual problem solving processes. This paper reports on the work of the qualitative project that scrutinizes these processes.

1 European Society for Engineering Education

2 German acronym for "Modelling Competences of Engineering Students", funded by BMBF (Ministry of Education and Research)

3 Leibniz Institute for Science and Mathematics Education

THEORETICAL FRAMEWORK

Mathematical problem solving is an important field in mathematics even at school and at university (cf. Halmos, 1980). There are many different approaches for research on this topic (cf. Schoenfeld, 1985; Chinnappan & Lawson, 1996; Rott, 2013). One perspective focuses on the *inner structure* of problem solving processes considering heuristics and beliefs, another perspective elaborates on the *outer structure* in terms of timing and organizing of processes. In our study, we focus on both aspects by investigating phases of the problem solving process (cf. Polya, 1945) and the use of heuristics (cf. Bruder & Collet, 2011). In his seminal work, Polya (1945) differentiates *understanding the problem*, *devising a plan*, *carrying out the plan*, and *looking back* as essential phases of any problem solving cycle. Schoenfeld (1985) extends the model of Polya by adding an *exploration* phase which interacts closely with the *planning* phase, but also allows a throwback to the first phase. Likewise, Chinnappan and Lawson (1996) stress the importance of the first two phases: “[...] the planning process forced the solver to make optimum use of information that was identified and information that was generated” (p. 13). In addition, Chinnappan and Lawson (1996) could show that not only the single use of heuristics is important, but also a training in management strategies that allows for effectively coordinating different phases. However, Rott (2013) could show that completely linear models are not always suitable to describe problem-solving processes and provides empirical evidence for applying a more flexible model. Bruder and Collet’s (2011) work on heuristics places the methods in the center and explains the process of problem solving by heuristic tools (e.g., *informative figure*), heuristic strategies (e.g., *using analogies*) and heuristic principles (e.g., *symmetry principle*). In addition, the problem solving process is influenced by the interplay of mathematics and physics as described in the framework by Tuminaro and Redish (2007) that elaborates on so-called *epistemic games*. These *epistemic games* like, for instance, *mapping mathematics to meaning*, allow the description of how students make the transition from novices to experts, and contribute lenses to elaborate on either *individual-related* or *task-related* characteristics. In order to solve problems with physical contexts, students need adequate beliefs about the physical concepts involved. From beliefs research we know that pre-service teachers “who approach learning physics with a more favourable belief

structure are more likely to achieve higher learning gains” (Mistades, 2007, p. 185). An instrument to reveal students’ beliefs about the force concept is provided by Hestenes, Wells, and Swackhamer (1992) which proved to be useful in several studies. Based on these theoretical foundations, we investigate how students’ performances in higher mathematics and technical mechanics are connected and what role their problem solving skills play:

Research question 1: Do students’ performances at school, their mathematical skills, and their beliefs about physics concepts (related to the force concept) predict their achievement in technical mechanics?

Research question 2: Can task difficulties in higher mathematics and technical mechanics be described through analysing occurrences of Polya’s phases and students’ use of different heuristics?

Research question 3: Does working on task variations in technical mechanics that imply increasing difficulty requires using more and different heuristics?

The first and second research question will be answered using the results from study 1. Research question 3 is answered by referring to the results of study 2.

METHODOLOGY

The research comprises two different, but related surveys. The first one is taken from our larger study to investigate the development of students’ problem solving competencies in their first year at university. That is, students work on the IRT-scaled test for higher mathematics (HM) and technical mechanics (TM) from the quantitative project. The other survey is task-related and investigates four sequences of tasks about statics and the different usage of problem solving competencies. Here, students are observed while working on and discussing tasks of increasing difficulty.

Our project partners developed *TM/HM pre-tests* delivered at the beginning of students’ studies. The pre-tests consist of 28 (TM) and 36 (HM) items with a combination of closed (multiple choice, multiple true-false) and (semi-) open questions. The TM pre-test covers items on statics, kinematics, kinetics, energy and momentum, oscillations and basic concepts, all on a higher school level. For example, one task is:

Time	Content
09/2013	TM/HM pre-test, IQ-test, FCI; Video: easy/difficult tasks (pre-test)
04/2014	Video: easy/difficult tasks (pre-test)
09/2014	TM/HM post-test, FCI; Video: easy/average/difficult tasks (post-test)

Table 1: Overview study 1

Draw a possible amplitude curve of a damped oscillation in the diagram.

The HM pre-test, based on work by Hauck (2012), asks questions on mathematical basics, calculus and geometry. One example is:

Given is a plane $E: 6x_1 + 6x_2 - 3x_3 = -12$ and the point $A (3/6/4)$

a) Determine the distance d of the point from the plane.

b) Calculate the coordinates of A' (A mirrored at the plane).

These tests are IRT-scaled (pilot study with $N=1069$ students) and thus enable us to categorize our participants in terms of their mathematical, physical and technical competencies at the beginning of their university course. Additionally, the scaling affords us selecting the five easiest and most difficult tasks.

The Force Concept Inventory (FCI; Hestenes, Wells, & Swackhamer, 1992) investigates students' beliefs about the concept of force which is an important part of competences in mechanics. It consists of 30 items, each of them describing a physical situation. One must choose among five possible answers, one relating to the correct Newtonian concept and four relating to alternative concepts. Opportunities to analyse the scores vary. Either the raw score of correct answers can be used or special attention can be paid to how the wrong answers relate to misconceptions. We decided to use the raw score to investigate students' changes during their first year (cf. Hestenes & Halloun, 1995),

because we have been interested in analysing the changes in students' beliefs during the first year.

Design of study 1: Development of problem solving competencies

In the first study data was collected at three points (Table 1). In September 2013, the students ($n=37$; male=27, female=10) worked on four tests: the TM pre-test, the HM pre-test, an intelligence test (CFT-3, Culture Fair Intelligence Test) and the FCI. In this paper, we only focus on this first measurement point, but provide an overview on the whole study in Table 1. The longitudinal design was chosen to capture students' development of their mathematical and physical skills, and problem solving competencies. These results will be presented elsewhere.

In addition, the students worked in groups of two or three on the five easiest and five most difficult tasks of the two pre-tests (Table 2).

The group work was recorded on video for later analysis. The *thinking aloud*-method (Ericsson & Simon, 1984) rendered thoughts and ideas observable. In order to analyse the video data and the students' work, we developed a category system based on the theoretical framework described above (Table 3).

The category system was carefully tested and optimized in a pilot study. As a result we arrived at 38 categories as an assessable selection with reasonable frequencies.

HM-Topic	Difficulty	TM-Topic	Difficulty
Equation	-3.71	Kinetic energy	-1.98
Number line	-3.39	Deflection curve	-1.89
Reflection at a plane (see above)	2.80	Trajectory	1.80
Exponential function - Extremum	3.22	Load	3.17

Table 2: Overview on topics of the chosen tasks (in excerpts: 2 easy/2 difficult tasks)

Structure	Category	Sub-categories
Inner Structure	Heuristic tools	Informative figure, Table, Equation
Outer Structure	Polya's phases	Understanding the problem, devising a plan, Carrying out the plan, Looking back

Table 3: Category system (in excerpts)

Design of study 2: Task-related problem solving competencies

The second study investigates characteristics of tasks by special sets of TM tasks at two measurement points (cf. Table 4). The set consists of two to four tasks with increasing difficulty for the four topics of gravity, reaction forces, displacement and deformation, free body diagrams and force systems (cf. Figure 1). Four students handled every sequence. These tasks are taken from the piloting phase of the TM post-tests where they showed inadequate model fit and therefore were rejected in the IRT-model. In our study we investigate how slightly changing task features influence students' problem solving behaviour.

Time	Content
05/2014	Video: easy tasks
06/2014	Video: more difficult tasks; FCI

Table 4: Overview study 2

To control for students' competences we considered their first term grades (calculus, linear algebra and mechanics) as indicators. The participants (n=8, male=6, female=2) worked alone on the tasks and received in advance a detailed introduction into the *thinking aloud*-method (Ericsson & Simon, 1984), including a preceding practice task. After that, students chose a topic, were given two to four tasks accordingly

(cf. Figure 1), and processed the tasks in ascending difficulty. Finally, all students were interviewed and asked to reflect their courses of action. The video data was analysed by using the category system (see study 1). While students' work was analysed as in study 1, the interview data was also checked for task characteristics and the conscious use of problem solving strategies.

RESULTS

Study 1

As mentioned earlier, only the findings from the first measurement point can be reported at the moment, as the data from the second and third survey have not yet been completely analysed. Our participants are comparable with the students from the larger pilot study conducted by the quantitative project. In detail, they received the following mean personal ability estimates and results: $M_{HM} = -0.11$ ($SD = 1.38$), $M_{TM} = -0.86$ (1.01), FCI=11.35 (5.78) out of 30 points. To clarify which factors affect students' performance in the processing of tasks due to physical contexts, several multiple linear regressions and the corresponding correlation matrices were calculated. For the correlations of pre-test results and prior knowledge, represented

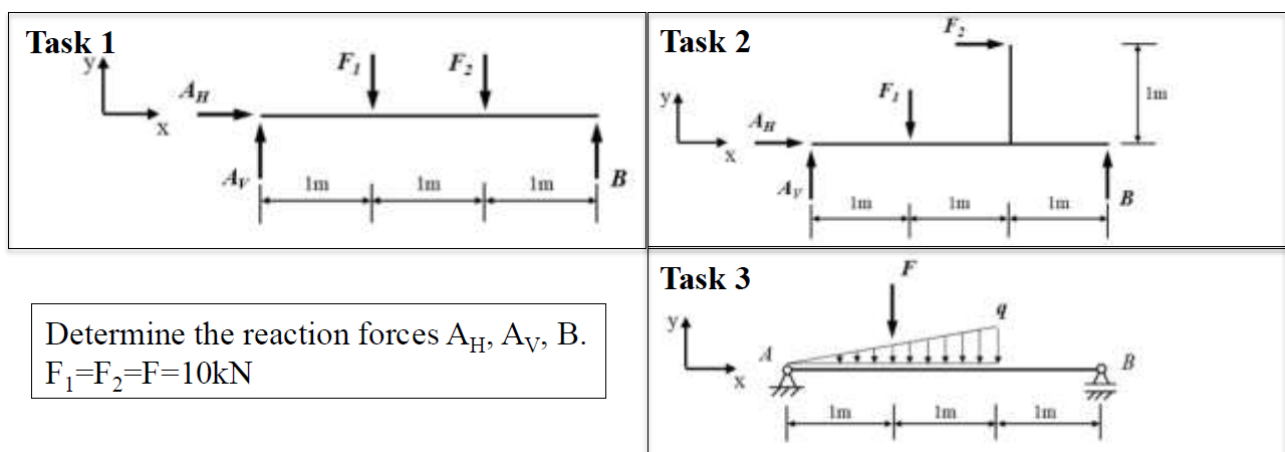


Figure 1: Task sequence: "reaction forces"

	Results TM pre-test	Results HM pre-test	Results FCI	Grade School Certificate
Results TM pre-test	1			
Results HM pre-test	.676**	1		
Results FCI	.744**	.696**	1	
Grade School Certificate	-.382*	-.612*	-.372*	1

** $p < .01$, * $p < .05$

Table 5: Correlation matrix (first measurement point)

Predictor	b_i	β_i	Sig.
(Constant)	-1.883		$p = .00$
Results HM pre-test	.236	.306	$p = .05$
Results FCI	.093	.531	$p < .01$
$R^2=0.602$			

Table 6: Multiple linear regression

	Easy tasks	Difficult tasks
Heuristic Tools (HM/TM pre-test)	31%	69%
Heuristic Strategies (HM pre-test)	37%	63%
Heuristic Principles (HM/TM pre-test)	0%	100%

Table 7: Distribution of heuristics

by the final grade in school certificate, the results are provided in Table 5.

Collinearity analysis detected that the factor *Grade School Certificate* contains the same information as the factor *result HM pre-test*, it is therefore not important for further calculations (cf. Table 6).

The model explains 60% of the variance in the TM pre-test results. In particular, beliefs about physics have a great influence on these results. In contrast, only 20% of all observed uses of heuristic strategies occurred when students worked on physics problems. Thus, the mathematical problem solving approaches, such as problem solving strategies and Polya's phase model, are only partly suitable for describing processes in solving physics problems. Here analyses by guide of the epistemic games framework, which are reviewed in a forthcoming analysing step, will probably deliver further explanations. However, when just looking at the use of heuristic tools and heuristic principles, we can conclude that these are used in a similar way when solving both mathematical problems and physics tasks. Table 7 shows the distribution of heuristics for the processing of easy and difficult tasks.

The results show that the students are increasingly using both tools and principles for difficult tasks. The situation is similar for mathematical tasks when using heuristic strategies. Overall, the students used heuristics rather rarely. Only in slightly more than half of the task processing of the difficult tasks, the use of heuristics could be observed (tools: 54%, strategies: 55%, principles: 27% of all difficult tasks). When solving the easy tasks, heuristics were less observed (tools: 30%, strategies: 45%, principles: 0% of all easy tasks).

Applying Polya's phase model to analyse the mathematics tasks, it becomes clear that for difficult tasks, students worked much more in cycles (for example, between *carrying out a plan* and *devising a plan*) in their solution process than for easy tasks, where they tend to go through the problem solving process linearly.

Study 2

For the sake of brevity we limit ourselves to presenting only the results of the sequence of tasks to reaction forces (see Figure 1). All students successfully solved the first task, using one specific algorithm. At the end of their solving process, only two students recognized

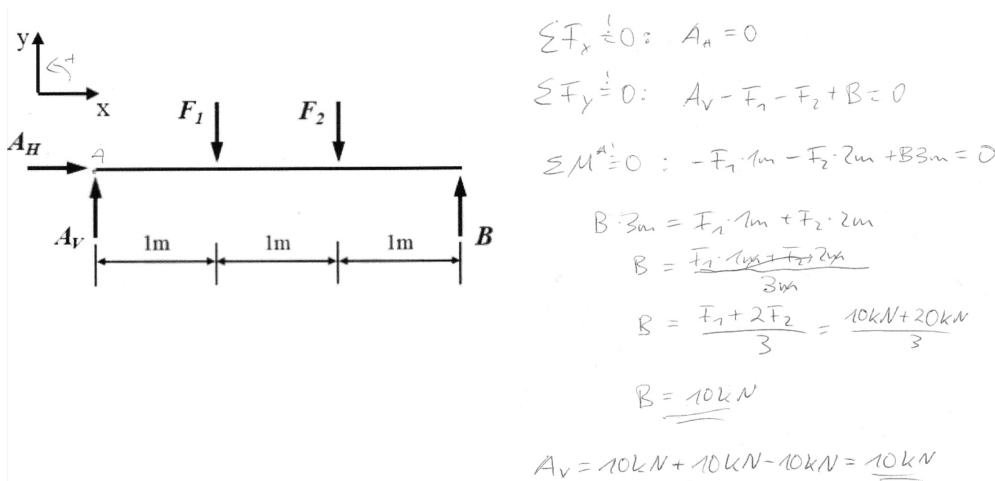


Figure 2: Student D's work on task 1

the symmetry aspect of the task. Students' work differed only slightly concerning the amount of the heuristic tools and strategies used. It is remarkable that the students who had mastered the more difficult tasks of the sequence, switched more frequently between the heuristic tools used when solving the simple task (here *informative figure* and *equations*). Table 8 summarizes the results that the four students received. As it is shown in Figure 1 and Figure 2, students were asked to compute the three forces A_H , A_V and B in each task; yielding the correct result was coded 1, otherwise 0 (see Table 8).

The following transcript of Denise's work shows the alternating use of the two heuristic tools mentioned above. She was able to determine the first two equations for calculating the x- and y- directions, and thus A_H , and started to calculate the third equation for the momentum.

From her think aloud protocol we gain the following explanations:

So, for example, the sum of all momentums, what will be best? So you have to have any equation with only one unknown that you are looking for, yes. Then you could take, for example, the point (showing on the point, A_V and A_H point to), because the two go through that, then they do not

have to be calculated. [The] Sum of all momentums in point A which is here (draws a point to the place to which A_V and A_H point and labels it with A) must be 0. So the positive direction of rotation is this (draws a curved arrow). F_1 (redraws arrow F_1) rotates negatively with the lever arm 1m, meaning minus F_1 times 1m, F_2 negative with 2m, and B (indicates B) is positive with 3m equals 0. F_1 and F_2 on the other side mean B times 3m is equal to F_1 times 1m minus F_2 plus F_2 times 2m. Then, you need to convey the 3m here (indicates the side of the equation with F_1 and F_2), by dividing, and then we have B alone in the end.

This procedure of combining heuristics is also seen in the processing of the two other tasks. The two other students (Alice, Bob) do not connect the *informative figure* and the *equations* in a similar way. They fail to solve the third task, since they apparently cannot transfer all the information from the figure correctly into their equations.

DISCUSSION

We were able to provide evidence for the expected influence of mathematical skills and beliefs about physics on the achievements in technical mechanics even in our small sample. In view of the above-mentioned asynchronicity of engineering education, this

	Alice ¹	Bob	Chris	Denise
Task 1 ($A_H/A_V/B$)	1/1/1	1/1/1	1/1/1	1/1/1
Task 2 ($A_H/A_V/B$)	1/0/0	1/1/1	1/1/1	1/0/0
Task 3 ($A_H/A_V/B$)	1/0/0	1/0/0	1/1/1	1/0/1

Table 8: Results of the task sequence "reaction force"

means that students first need to master the mathematical basics to be able to successfully work on physics problems and not, as is often the case, learn them at the same time or after encountering the physics contents. The results about problem solving in our first study suggest two conclusions. On average, more difficult tasks require a greater use of heuristic tools, strategies and principles. In addition, we noticed that in some tasks which are assigned a high difficulty estimate by the IRT scaling, the students' use of heuristics is comparable to their processing of the easy tasks. Overall, the students rarely use heuristics in solving the tasks, both in the easy and in the difficult tasks. We note that only in just over half of the difficult tasks the students use tools and strategies when working on them. The cause may lie in a low consideration of problem-solving tasks in mathematics education at school.

The results of the second study clearly show that it is not enough to simply master heuristics, although the use of individual heuristics helps solving tasks, as can be seen in the first item of the task sequence "reaction forces". Only when these heuristics are meaningfully connected, in a way that they support and enhance each other, students are able to solve challenging tasks successfully. An exemplary situation is the following one: first, a student used the diagram to understand the situation. The situation requires a certain algorithm which the student formulated in a general form. Then the components were adjusted using the diagram in the given situation. After that the result was checked for plausibility referring to the diagram. Therefore learning heuristics should attend to two competencies: Students must be able to use some heuristics deliberately, and they need to be able to effectively combine tools, strategies and principles.

REFERENCES

- Bruder, R., & Collet, C. (2011). *Problemlösen lernen im Mathematikunterricht*. Berlin, Germany: Cornelsen Verlag Scriptor.
- Chinnappan, M., & Lawson, M. J. (1996). The effects of training in the use of Executive Strategies in Geometry Problem Solving. *Learning and Instruction, 6*, 1–17.
- Ericsson, K. A., & Simon, H. A. (1984). *Protocol Analysis. Verbal Reports as Data*. Cambridge, MA: The MIT Press.
- Halmos, P. (1980). The heart of mathematics. *American Mathematical Monthly, 87*, 519–524.
- Hauck, C. (2012). *Vergleich der mathematischen Kompetenzen von Gymnasiasten an allgemeinbildenden und technischen Gymnasien in Baden-Württemberg*. University of Stuttgart, Germany: Unpublished Master's thesis.
- Hestenes, D., & Halloun, I. (1995). Interpreting the force concept inventory – A response to Huffman and Heller. *The Physics Teacher, 33*, 502–506.
- Hestenes, D., Wells, M., & Swackhamer, G. (1992). Force Concept Inventory. *The Physics Teacher, 30*, 141–158.
- Mistades, V. (2007). Physics teachers' beliefs and their performance in an in-service training program. *Science Education International, 18*(3), 173–187.
- Nickolaus, R., Behrendt, S., Dammann, E., Ștefănică, F., & Heinze, A. (2013). Theoretische Modellierung ausgewählter ingenieurwissenschaftlicher Kompetenzen. In O. Zlatkin-Troitschanskaia, R. Nickolaus, & K. Beck (Eds.), *Kompetenzmodellierung und Kompetenzmessung bei Studierenden der Wirtschaftswissenschaften und der Ingenieurwissenschaften* (Lehrerbildung auf dem Prüfstand, Sonderheft, pp. 150–176). Landau, Germany: Verlag Empirische Pädagogik.
- Polya, G. (1945). *How To Solve It*. Princeton: Paperback Printing.
- Rott, B. (2013). *Mathematisches Problemlösen. Ergebnisse einer empirischen Studie*. Münster, Germany: WTM-Verlag.
- Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press.
- SEFI (2013). *A framework for mathematics curricula in engineering education*. Brussels, Belgium: European Society for Engineering Education (SEFI).
- Törner, G., Reiss, K., & Schoenfeld, A. H. (2007). Problem solving around the world: summing up the state of the art. *ZDM – Mathematics Education, 39*(5–6), 353.
- Tuminaro, J., & Redish, E. (2007). Elements of a cognitive model of physics problem solving: Epistemic games. *Physics Education Research, 3*, 0201011–02010122.

ENDNOTE

1. All students' names are changed to preserve anonymity.