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The use of textbooks by pre-university teachers: An example with infinite series of real numbers

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Textbooks are central to the teaching process at all levels, including the tertiary level. However, the phenomenon of textbook use in higher education has not been extensively studied. In this paper, we analyse textbook use in the teaching of infinite series of real numbers at the pre-university level in Quebec. We interviewed five teachers about their textbook use in order to investigate similarities between their personal relationship with series and the institutional relationship with series transmitted through textbooks. Our results show that the teachers' courses generally follow the textbook's presentation, and that their documentation system is reduced to almost a single textbook, to which they adhere almost exclusively. We also compare our results with those of Mesa and Griffiths (2012) at the tertiary level.

Keywords: Textbook use, pre-university, personal relationship, documentation system.

INTRODUCTION AND BACKGROUND

In this paper, we analyse textbook use by pre-university teachers in Quebec (Canada) in the teaching of infinite series of real numbers (*series* hereinafter). Textbooks play a crucial role in the school environment and “have always played a major role in mathematics education” (Sträßer, 2009, p. 70). However, the vision of mathematics transmitted by textbooks can shape what teachers teach and what students learn. Textbooks can also influence students' attitude toward the discipline of mathematics and affect their self-perception as learners (Raman, 2004). Moreover, textbooks can mould students' beliefs about what they can learn and how they can access and use knowledge (Mesa & Griffiths, 2012). For instance, González-Martín, Giraldo and Souto (2013) recently analysed a sample of secondary textbooks to identify how real and irrational numbers are introduced. They showed that the approaches used in textbooks could have ma-

ior consequences for students' learning, influencing their vision of mathematics (e.g., ‘it is possible to show that a statement is true by giving some examples’), and affecting their subsequent learning of other Calculus topics.

In addition, textbooks seem to play an important role in the teaching of tertiary mathematics (Mesa & Griffiths, 2012, p. 85). Although one might expect such textbooks to meet more rigorous academic standards appropriate for university classrooms, research seems to contradict this. For instance, in her study on continuity in Precalculus, Calculus, and Real Analysis courses, Raman (2004) concluded that textbooks send conflicting messages on the status and purpose of mathematical definitions. These results agree with those of Giraldo, González-Martín and Santos (2009), who also found that the presentation of content related to continuity in undergraduate textbooks sometimes conjures erroneous images that can impede the learning of derivatives and integrals. Lithner (2004) analysed the types of reasoning that Calculus textbook exercises can encourage and instil in students, and demonstrated that exercises in which students merely need to reproduce a given example predominate. Regarding the topic of series, González-Martín, Nardi and Biza (2011) also established a preponderance of algorithmic exercises in pre-university and university textbooks, as well as a lack of conceptually-driven tasks; we come back to this last work later. Because most of these studies on the introduction of Calculus reveal problems with the way post-secondary textbooks address mathematical topics, we decided to examine how post-secondary instructors use textbooks in preparing and teaching their courses.

Research on tertiary textbook analysis is rapidly evolving, but the phenomenon of instructors' textbook use in higher education has not been extensive-

ly studied (Mesa & Griffiths, 2012, p. 85). Mesa and Griffiths (2012) addressed this issue and found that tertiary instructors use textbooks for different purposes: generating the syllabus, preparing classes, and designing homework. The various ways textbooks were used included: using the same information contained in the textbook (*offloading*), supplementing it with alternative examples designed by the instructors themselves or culled from other textbooks (*adapting*), or changing the presentation altogether, including using different notation (a form of *improvising*). For all the tertiary instructors in their sample, the textbook appeared to be a crucial artefact in the instruction preparation process. However, the more frequently an instructor gave the same course, the less he or she relied on the textbook due to familiarity with its contents and with what it did and did not offer.

Another important observation was that instructors saw the textbooks they used as written *for* students, and not as a tool from which the instructors themselves could learn (e.g., they did not mine textbooks for new ways of understanding certain topics, nor did they draw inspiration from textbooks to vary their teaching methods). Instructors also did not see textbooks as tools that could help them select problems or decide how to sequence topics in constructing their syllabi. In fact, the textbook features that were perceived to be the most helpful for improving the instructors' teaching were problems and examples.

The research presented in this paper seeks to provide more information on this phenomenon, particularly with regard to the teaching of a very specific topic: series. Series are a key notion in mathematics: already present in early Greek mathematics, they were crucial in the development of Calculus. They have many applications within mathematics (such as the calculation of areas by means of rectangles) as well as outside mathematics (including the modelling of situations such as the distribution of atmospheric pollutants). These factors may partially explain why the study of series is included in introductory Calculus courses in many countries.

This is the case in Canada, where each province develops its own official curricula. In the province of Quebec, compulsory education ends at age 16 and students who wish to attend university must first complete two years of pre-university studies (called *collégial* – other countries, such as Spain, follow sim-

ilar systems). Students pursuing scientific or technical careers are introduced to Calculus during their *collégial* studies, where series first appear. It is in this context that our research, like Mesa's and Griffiths' (2012), seeks to better understand the phenomenon of textbook use. However, while Mesa and Griffiths looked at general textbook use in universities, our work centres on preparatory courses at the *collégial* level and focuses on a topic that is introduced in university in many countries. Moreover, studying the *collégial* experience may help pinpoint gaps and continuities between textbook use at the *collégial* and university levels that could affect students' transition. This could open the door to further research on similarities and differences in practices with regard to a specific topic of study.

Before beginning our investigation of teachers' use of textbooks in the teaching of series, we first developed an analysis of how series are presented in *collégial* textbooks, following an anthropological approach. We also identified some possible consequences of this presentation for students' learning (González-Martín et al., 2011). Our sample consisted of 17 textbooks used in *collégial* studies in Quebec from 1993 to 2008 and our main conclusions can be summarised in four main results:

- R1: Series are usually introduced through organisations that do not lead students to question their application or importance (*raison d'être*).
- R2: Organisations tend to introduce series as a tool to later introduce functional series, but the inherent importance of series is not usually discussed.
- R3: These organisations tend to ignore some of the main difficulties in learning series identified by research.
- R4: The vast majority of tasks concerning series are related to the application of convergence criteria, or to the application of algorithmic procedures.

Having identified how *collégial* textbooks introduce series, the next stage of the research consisted of analysing *collégial* teachers' practices and their use of textbooks (González-Martín, 2010). In this paper,

we discuss how *collégial* teachers use and view their textbooks, specifically in relation to the topic of series.

THEORETICAL FRAMEWORK

We are interested in two main issues: determining how *collégial* teachers use their textbooks, and defining the relationship between teachers and textbooks.

To study how teachers interact with a range of resources, and how these interactions are central to their professional activity, we followed the documentary approach (Gueudet, 2014; Gueudet, Buteau, Mesa, & Misfeldt, 2014). In this approach, a resource is anything that can possibly intervene with the activity of a subject, including artefacts or even a discussion with a colleague. In the case of teachers, they may select, combine, and design their own resources. They may use resources in class, modify them (on the spot or afterwards), or share them. All this constitutes the teacher's *documentation work* (Gueudet et al., p. 142) and, as a consequence, the teacher develops a structured *documentation system*. Teachers, through their use of given resources in pursuit of a teaching objective, develop a *document*: "a mixed entity, associating resources and utilization schemes of these resources" (Gueudet, 2014, p. 2336); this process is called a *documentational genesis*.

The knowledge involved in developing schemes is professional knowledge and can concern a given resource ('this exercise is a good one to trigger an interesting discussion to start this chapter') or the mathematical content to be taught ('I have to work on the idea of slope of functions before introducing derivatives'). The *resource system* is the part of the *documentation system* that refers only to the resources used. This approach therefore considers the professional activity of a teacher in its entirety, both in and out of class.

All the processes studied through the documentary approach are developed in an institutional environment, which establishes (and sometimes imposes) a set of conditions and constraints. Chevallard's anthropological theory provides tools that allow a better understanding of choices made by an institution in organising the teaching of mathematical concepts, as well as the possible consequences of these choices on an individual's practices. A fundamental aspect of this theory is the notion of *institution*. An institution I is defined as a social organisation that allows, and

imposes on its *subjects* (every person x who occupies any of the possible positions p offered by I), the development of *ways of doing and of thinking proper to I* (Chevallard, 2003, p. 82). For instance, a classroom is an *institution* (with two main positions: teacher and student), as is a school (consisting of several more positions: teachers of various disciplines, students in different grades, the principal, course coordinators, etc.), or an educational system.

To analyse how an *institution* approaches notions, further definitions are required. An *object* is any entity, material or immaterial, that exists for at least one individual; in particular, any intentional product of human activity is an *object*. Every *subject* x has a *personal relationship* with any object o , denoted as $R(x, o)$, as a product of all the interactions that x can have with o (using it, manipulating it, speaking of it, etc.). This *personal relationship* is created, or modified, by coming in contact with o as it is presented in different *institutions* I , where x occupies a given position p . From this *personal relationship*, an individual will be endowed with what could be designated as 'knowledge', 'know-how', 'conceptions', 'competencies', 'mastery', 'mental images', 'representations' and 'attitudes' (Chevallard, 1989, p. 227).

This notion of *relationship* is also applicable to *institutions*: given an object o , an *institution* I , and a position p in I , we define the *institutional relationship* with o in position p , $R_i(p, o)$, as the relationship with the object o , which should ideally be that of the subjects in position p within I (Chevallard, 2003, p. 82); this is, 'what is done with o within I ' (Chevallard, 1989, p. 213) for any *subject* in position p . By becoming a *subject* of I in position p , an individual x is subjected to the *institutional relationships* $R_i(p, o)$, which in turn will re-model his or her own *personal relationships*. For our research, we consider as institution the system of mathematics teaching at the *collégial* level (MTCL). The *institutional relationship* of MTCL with series is mainly determined by official programmes and by textbooks that develop the contents required by these programmes.

In the case of teachers, their *personal relationships* will be affected by the *institutional relationships*, which impose constraints on what to teach and how to teach it (for instance, through textbooks). This *personal relationship* can be seen as an element of the schemes developed as a part of a *document*. In this sense, the

document is constructed by taking into account a number of resources, as well as the *personal relationship* of the teacher with the topic being taught, which has a strong influence on his or her view of what should be taught and how this should be done, and guides the teacher in selecting which resources to use.

We can now state the main objectives of the research presented in this paper. We are interested in: 1) analysing *collégial* teachers' *personal relationship* with series and seeing how it relates to the *institutional relationship* promoted by textbooks; 2) analysing *collégial* teachers' *documentation work* concerning textbook use in preparing for the teaching of series.

METHODOLOGY

The research reported here is a part of a larger project aiming at identifying *collégial* teachers' practices regarding series (González-Martín, 2010), guided mainly by our results on the introduction of series in textbooks (González-Martín et al., 2011). To achieve the objectives of this larger project, and guided by the results of our analysis of textbooks, we constructed a protocol for semi-structured interviews. These included questions about the textbook used by the teachers, their opinion on the adequacy of this textbook for the students and for the teacher, the number and type of different representations used, the number and type of examples and applications used to teach series, their opinions on the most important tasks for students to perform during the learning of series, and their awareness of the main difficulties in learning series, among others. For this paper, we focus on questions concerning the textbook and its use as a resource in preparing lessons about series. The interviews were conducted from June to November 2009, with an average length of 45 minutes. They were videotaped and later transcribed for further analysis. Once transcribed, the data was organised into clusters of different topics, with special attention paid to keywords that could serve as indicators of the teachers' *personal relationship* with series and

of their approach to preparing and organising their textbook use.

In order to cover a wider variety of practices for the teaching of series, we selected five teachers from various *collégial* establishments in Montreal, the biggest city in the province of Quebec. We thereby avoided interviewing teachers working in the same establishment, who tend to organise their teaching in similar fashions. These teachers, designated T1, T2, T3, T4, T5, had varying levels of teaching experience at the *collégial* level and varying levels of experience teaching series (Table 1).

It is important to clarify that because we were interested in the use and role of the textbook within the *resource system* of our teachers, we did not collect all the resources of the teachers (see Gueudet, 2014); we had previously analysed these teachers' textbook (which shared the characteristics R1 to R4 described in the Introduction) and inquired about their vision and use of it. The study of the *resource system* as a whole will be the focus of future research. In the following section, we present the main results derived from the interviews.

SOME RESULTS

Coincidentally, at the time of the interview our five teachers were using the same textbook for their courses. This textbook was part of our earlier study's sample, and, as mentioned above, had been analysed by us previously. It displays the general characteristics R1 to R4 of the textbooks within the sample. The teachers' reasons for choosing it were varied (all quotes have been translated from the original French):

- T1: I find that it is better than the others. [...] All textbooks are basically similar, but the order and the way they present content, I think that this one is good.
- T2: Because two teachers from here wrote it [...].

	T1	T2	T3	T4	T5
Experience teaching in <i>collégial</i>	5 years	20 years (mathematics and informatics)	32 years	6 years	7 years
Experience teaching series	5 years	4 years	More than 20 years	4 years	2 years

Table 1: Teachers' experience at the *collégial* level and experience with teaching series

- T3: [...] for many reasons. The simplest reason is that [it] covers the course content so it's a good work tool [for] students [and] the teacher.
- T4: [...] I found that it was better where integration techniques are concerned. Especially the way the exercises are grouped together. [...] At the beginning, the drill exercises are grouped together so that one can associate a concept with many examples [...] So, I find that it is an appropriate learning sequence. [...] I believe a lot in drill exercises. I try to create a balance, but there are many textbooks that I do not like [...].
- T5: Well, it's the one that was used before I started teaching... [...] it is good, it has theory, [...] it has a lot of exercises... it's [...] good enough for the students because [...] it helps the student a lot, [it also helps] the teacher because... it's what we're asked to teach. Meaning that, the parts... I mean, the proofs... [...] here they're not in the textbooks [...] so in that way we know that we don't have to teach those proofs... and [...] it's a well enough organised book... [...] Yes, we use the textbook, but the teachers' syllabi are also useful in order to know how... [...] which is the best way to introduce a concept.

We see here the different purposes of textbook use identified by Mesa and Griffiths (2012): generating the syllabus, preparing classes, and designing homework (when they speak about the exercises). However, the most prevalent use seems to be *offloading*, with the exception of cases where some portions are removed with no alternative information added (which we do not consider to be *adapting*). T2 and T5 seem to be subject to particular restrictions, indicating that they were not entirely free to select the textbook ('two teachers from here wrote it' and 'it's the one that was used before I started teaching'), although T5 adds information concerning his opinion on content and structure. We can observe a phenomenon that was also detected by Mesa and Griffiths: the textbook seems to be perceived as written *for* the student. Moreover, aside from the syllabus, the *resource system* appears, so far, reduced to the use of a single textbook, which suggests that professional knowledge is underdeveloped with regard to the teaching of series in MTCL.

We did not see any of the teachers use more than one textbook in preparing their courses.

We were able to confirm later that the most prevalent way of using the textbook was through *offloading*. When asked whether they follow the order of the textbook when they teach series, T1 and T4 said yes (T4 said 'I think so'), whereas T2, T3 and T5 (transcribed in his previous response) indicated that they '*almost*' do:

- T3: Certain sections are set aside and left out, but on the whole we almost follow the order.
- T4: Yes, but the textbook contains much more than the course content. So you have to make a selection. For example, the convergence criteria in the textbook — almost 10 criteria are presented. In fact, we only have time to get the students to master perhaps half of the criteria.

In each case, it seems that the teacher's *personal relationship* with series does not diverge significantly from what is presented in the textbooks; the only concern seems to be that there is more material in the textbooks than necessary, which results in some sections being set aside. It is possible that this similarity between their *personal relationship* with series and the *institutional relationship* transmitted by the textbooks is at the origin of the teacher's decision not to enrich their *documentation system* with different sources. This could lead the teachers to develop schemes for the teaching of series that privilege the presentation of routine tasks. This became more evident when we asked them whether they thought the textbook they use meets the needs of the students and the instructor:

- T1: Needs of the instructor, yes... I think that it's complete enough when it comes to sequences and series... [...] Meeting the students' needs, well, that's the eternal question of why do we teach that [...].
- T2: Hmm, we are currently re-evaluating what we are doing about sequences and series [...] I would say that for the moment, there are not any textbooks that really correspond to what we would like to do. Especially because we are not certain yet of what we would like to do [...]

- As long as we don't know what we want to do, we will not find the appropriate textbook...
- T3: The one that we use, yes. It's not the only one [...]. But that one, yes.
- T4: The one that I use does, yes.
- T5: Hmm... yes, I would say for the teacher, yes... hmm... for the student [...] well [...] we do not have a lot of time to make the student practise. They are the ones who have to practise. So the textbooks... [...] there are no parts where they can do activities or experiment. Even the teacher does not have a lot of time to experiment with students on the concept of series and organise activities for them. So, time is short for that... and the students... all they have to do is practise with the exercises [...]

These teachers seem to see the textbook as being for the student and they suggest that it meets students' needs in their learning of series. T2 was the only teacher who seems to question how series should be taught (or *what* should be taught), but he ultimately bends to the pressure of the *institutional relationship* and uses the textbook exclusively. On the other hand, T1 also raises the fact that the reason for learning series may not be clear to students, but he does not seem to *adapt* the textbook and provide extra information to clarify this issue. It seems that the weight of the *institutional relationship* with series deters him from taking initiative and using other resources to enrich his *documentation system* and address these issues.

The teachers' adherence to the presentation and organisation of the textbook seems absolute, and they generally insist on the importance of performing the exercises, which was identified as the most helpful textbook feature by Mesa and Griffiths (2012). In fact, the tasks that the teachers consider to be most crucial in learning series were all taken from the textbook (González-Martín, 2010). As in Mesa's and Griffiths's research, no teacher identified the textbook as a tool from which they could learn or that could help them decide how to sequence the topics.

FINAL REMARKS

We cannot present more excerpts from the interviews here, which would allow us to gain better insight into

the use of the textbook by our teachers and paint a clearer picture of their *personal relationship* with series and their apparent compliance with the *institutional relationship*. This will be the subject of future papers.

However, the data presented here allows us to observe a kind of *yielding* to the institution's approach to introducing series (through textbooks). There appears to be a similarity between the teachers' *personal relationship* and the *institutional relationship*, in spite of the weaknesses on the way series are introduced by textbooks (González-Martín et al., 2011). Perhaps the fact that the teachers' *resource system* seems so restricted can be explained largely by this *yielding*: our teachers may not see any reason to seek out complementary resources in preparing their lessons. Our data indicates that the *resource system* of our teachers is by and large reduced to the use of a single textbook that the teachers do not question, and the choice of this single resource as core of the *resource system* appears to be guided by the *personal relationship* of these teachers with series, which appears to be quite close to the *institutional relationship* transmitted in the textbooks.

In general, our results echo those of Mesa and Griffiths (2012). However, textbook use at this school level seems to be more restricted to *offloading* (which in this case includes using material from the textbook while setting some parts aside). *Offloading* is the primary way textbooks were used by our interview subjects. Also, textbook use is quite prevalent, which is common in *collégial* studies in Quebec although it diminishes at the university level. In spite of their varied experience, the instructors relied on the textbook to an equal degree, and in this way they seemed to differ from the tertiary instructors interviewed by Mesa and Griffiths.

The conclusions of the study conducted by Mesa and Griffiths, as well as our own research, reveal a number of important tendencies related to textbook use that may have a strong impact on students' learning at the post-secondary level, especially in light of research results on post-secondary textbook content. We are aware that our sample is quite small, and that we only focus on a specific mathematical topic; however, the similarities between our and Mesa's and Griffiths's results lead us to identify some possible concerns. The need for more research on textbook content and

textbook use at the post-secondary level is therefore crucial.

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REFERENCES

- Chevallard, Y. (1989). *Le concept de rapport au savoir. Rapport personnel, rapport institutionnel, rapport officiel*. Séminaire de didactique des Mathématiques et de l'Informatique. Université J. Fourier, Grenoble, France, 211–235.
- Chevallard, Y. (2003). Approche anthropologique du rapport au savoir et didactique des mathématiques. In S. Maury & M. Caillot (Eds.), *Rapport au savoir et didactiques* (pp. 81–104). Paris, France: Faber.
- Giraldo, V., González-Martín, A. S., & Santos, F. (2009). An analysis of the introduction of the notion of continuity in undergraduate textbooks in Brazil. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (vol. 3, pp. 81–88). Thessaloniki, Greece: PME.
- González-Martín, A. S. (2010). The concept of series: teachers' conceptions and teaching practices. In M. M. F. Pinto & T. F. Kawasaki (Eds.), *Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education* (vol. 3, pp. 33–40). Belo Horizonte, Brazil: PME.
- González-Martín, A. S., Giraldo, V., & Souto, A. (2013). The introduction of real numbers in secondary education: an institutional analysis of textbooks. *Research in Mathematics Education*, 15(3), 230–248.
- González-Martín, A. S., Nardi, E., & Biza, I. (2011). Conceptually-driven and visually-rich tasks in texts and teaching practice: the case of infinite series. *International Journal of Mathematical Education in Science and Technology*, 42(5), 565–589.
- Gueudet, G. (2014). Digital resources and mathematics teachers' professional development at university. In B. Ubuz, C. Haser, & M. A. Mariotti (Eds.), *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education* (pp. 2336–2345). Antalya: Middle East Technical University and ERME.
- Gueudet, G., Buteau, C., Mesa, V., & Misfeldt, M. (2014). Instrumental and documentational approaches: from technology use to documentation systems in university mathematics education. *Research in Mathematics Education*, 16(2), 139–155.
- Lithner, J. (2004). Mathematical reasoning in calculus textbook exercises. *Journal of Mathematical Behavior*, 23(4), 405–427.
- Mesa, V., & Griffiths, B. (2012). Textbook mediation of teaching: an example from tertiary mathematics instructors. *Educational Studies in Mathematics*, 79(1), 85–107.
- Raman, M. (2004). Epistemological messages conveyed by three high-school and college mathematics textbooks. *Journal of Mathematical Behavior*, 23(4), 389–404.
- Sträßer, R. (2009). Instruments for learning and teaching mathematics. An attempt to theorise about the role of textbooks, computers and other artefacts to teach and learn mathematics. In M. Tzekaki, M. Kaldrimidou, & H. Sakonidis (Eds.), *Proceedings of the 33rd Conference of the International Group for the Psychology of Mathematics Education* (vol. 1, pp. 67–81). Thessaloniki, Greece: PME.