



HAL
open science

Mathematics in Engineering: The professors' vision

Gisela Hernandes Gomes, Alejandro S. González-Martín

► To cite this version:

Gisela Hernandes Gomes, Alejandro S. González-Martín. Mathematics in Engineering: The professors' vision. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Charles University in Prague, Faculty of Education; ERME, Feb 2015, Prague, Czech Republic. pp.2110-2116. <hal-01288589>

HAL Id: hal-01288589

<https://hal.science/hal-01288589v1>

Submitted on 15 Mar 2016

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

Mathematics in Engineering: The professors' vision

Gisela Hernandes Gomes and Alejandro S. González-Martín

Université de Montréal, Département de Didactique, Montréal, Canada, gisela.hernandes@umontreal.ca

The debate surrounding the teaching of mathematics in Engineering courses has inspired research for more than a century. However, Engineering students still have difficulties in recognizing the mathematics required to solve problems in non-mathematical contexts. In order to study the vision of mathematics adopted by Engineering teachers and understand its role in shaping teaching practices, we interviewed two Engineering teachers with different backgrounds: one with BSc in Mathematics and MSc and PhD in Engineering, and another with BSc, MSc and PhD in Engineering. Our data reveal differences in the way these teachers approach topics such as mathematical rigor and approximation, and identify challenges faced by their students when working with modelling.

Keywords: Engineering, teachers' background, rigor and approximation.

INTRODUCTION

Mathematics is an important domain in science and technology, and is taught in a variety of university programs such as Administration, Economics, Computation, Engineering, and many other scientific fields of study. In Engineering, the interpretation and solution of certain problems require the direct application of mathematical models. To understand and analyse these mathematical models, it is often necessary to use elements of statistics, linear algebra, or differential and integral calculus. This is one reason why the teaching of mathematics in Engineering courses has been addressed in studies as far back as the early twentieth century (Howson et al., 1986, p. 159). Some authors, such as Murakami (1988), have suggested that mathematics should be taught to Engineering students by mathematicians with the help of engineers, with the latter selecting suitable

exercises, to create a learning environment more in tune with students' professional realities.

Cardella (2006) sought to better understand how Engineering students use mathematics. She also identified the mathematics Engineering students use, and consequently, the brand of mathematics that should be taught in Engineering programs. Her results show that Engineering students use mathematical thinking in various ways to solve problems while designing a project (they apply informal and intuitive knowledge of a given mathematical field; employ facts, definitions, algorithmic processes, etc.). She noted the importance of these various types of mathematical thinking, indicating that professors must carefully address students' mistakes throughout their learning process to ensure they understand what is being taught. Regarding teachers, Maaß and Gurlitt (2009) noted that teachers' knowledge of and beliefs on a given topic influence the way they plan, select, implement, and assess tasks. Later, and Maaß (2011) agreed with Schoenfeld (1992), who states that if we know teachers' resources, goals, and orientations – including their beliefs – we can better explain their actions. In this same vein, the study of Clark and colleagues (2014) provides a framework that divides into four categories those elements that can influence teachers' beliefs: "(a) teachers' professional background and experiences, (b) teacher knowledge, (c) teaching contexts, and (d) students' experiences" (p. 251). The conjecture that different teachers will give different lectures on the same topic on the basis of their knowledge and beliefs was confirmed at the university level by Pinto (2013). Pinto analysed the way two different university instructors implemented an identical course plan (both were capable teachers with a solid background in mathematics). He showed that the two instructors held different beliefs, attitudes and objectives and demonstrated varying degrees of confidence

in different resources. As a result they offered two substantially different lessons.

Regarding the teaching of mathematics through applications and modelling in the context of Engineering education, Cardella (2006) drew on the work of Schoenfeld (1992) and Doer (2007) in recognizing the need for mathematics instructors in Engineering programs to better understand how mathematics and mathematical thinking are used in Engineering. She admits that instructors should be familiar with the mathematical content necessary for engineering, and should consider the importance of problem solving, resources, attitudes, practices and of the learning environment.

In this vein, we are interested in identifying and discussing the attitudes and beliefs of instructors with different academic backgrounds to pinpoint how these beliefs influence their teaching of mathematical elements in Engineering programs. To do so, we follow Schoenfeld's approach to mathematical thinking.

THEORETICAL FRAMEWORK

To analyse the mathematical elements preferred by Engineering teachers with different backgrounds, we draw on Schoenfeld's (1992) proposal of five aspects of mathematical thinking: 1) knowledge base, 2) problem solving strategies, 3) monitoring and control, 4) beliefs and affects, and 5) practices. Of these five elements, we focus on the fourth, because we are interested in analysing teachers' beliefs and whether their different backgrounds (as mathematicians, physicians, or engineers) can influence their attitudes and their teaching practices. At this stage of our research, we do not draw on Schoenfeld's later work (1998), which provides a "theoretical account of how and why teachers do what they do [...] why they are engaged in the act of teaching" (p. 1); instead, we draw on his early proposal (1992), since we base our research on Cardella (2006), who in turn based her own work on this early proposal, which discusses "what it means to think mathematically" (p. 334), including the dimension of beliefs.

Using Schoenfeld's work as a basis, Cardella (2006) refers to beliefs and affects in the following way: "an individual's beliefs about and feelings towards mathematics influence how and when the problem solver uses mathematics. These beliefs and affects may be

cultural beliefs or affects, or they may be particular to the individual problem solver. Additionally, an instructor's beliefs and affects may affect the student's practices." (p. 26). Cardella concluded that one of the most frequent beliefs observed in the execution of Engineering projects was the "Need to be Precise" (p. 119) promoted by teachers. For her, "this belief impacted the way that the team [working on a project] dealt with Uncertainty and also affected their use of Estimation." (p. 119).

According to Schoenfeld (1998), the "definition of thinking mathematically includes having a solid knowledge base, but it also includes knowing a wide range of problem solving strategies, having modelling skills, metacognitive skills, productive beliefs, and more" (p. 79). This vision of *thinking mathematically* is broad and goes beyond mathematical content knowledge and its relationships; other elements are assigned importance as well, and beliefs about mathematics and mathematical activity are also a part of this expanded vision of *thinking mathematically*. For Schoenfeld (1992), teachers' beliefs about mathematics determine the characteristics of their classroom environment. In turn, this environment "shapes students' beliefs about the nature of mathematics" (p. 359). According to Schoenfeld, "whether acknowledged or not, whether conscious or not, beliefs shape mathematical behavior. Beliefs are abstracted from one's experiences and from the culture in which one is embedded. This leads to the consideration of mathematical practice" (p. 360). Certain practices identified by Cardella (2006) in Schoenfeld's work point to elements in the practice of Engineering teachers, such as: "making multiple conjectures, coming to grips with uncertainty, defending claims mathematically, engaging in a science of patterns, extracting tools from the solution of complex problems, having a mathematical point of view, mathematical sense-making, using symbolic representations and manipulating symbols" (pp. 18-19).

Cardella (2006) followed this approach to study the beliefs of Engineering students' that emerge through their work on projects. Her research influences ours, in which we seek to analyse the beliefs of Engineering teachers with different backgrounds and education.

METHODOLOGY

To pinpoint the different beliefs regarding mathematical elements that emerge from distinct teaching practices, we interviewed two teachers (T1 and T2) from the Engineering school in a private university in São Paulo (Brazil). T1 and T2 have different backgrounds: T1 is a female teacher with BSc of Mathematics, MSc of Space Engineering and Technology, and PhD in Mechanical Engineering. T2 is a male teacher with BSc, MSc and PhD in Mechanical Engineering. T1 has taught at the university for seven years, giving courses in Differential and Integral Calculus, Analytic Geometry, and Linear Algebra. T2 has been teaching at the university for six years. His courses include Introduction to Computational Science, Mechanics of Solids, and Resistance of Materials. In addition to the aforementioned courses, both teachers supervise students' capstone projects in the Production Engineering (T1) and the Mechanical Engineering (T2) programs. Because student supervision is common to both teachers, and considering Cardella's work, which focused on capstone design projects and the use of mathematics in a design problem, we centred our research on the teachers' supervision of student projects to identify implicit beliefs concerning the role and use of mathematics in Engineering programs. The interviews addressed the instructors' teaching practices, in particular their approach to the supervision of students completing their capstone projects. The teachers were interviewed together on two occasions to initiate dialogue between them and draw out contrasts between their different visions and beliefs. The first interview centred on the teachers' supervision of capstone projects in general and their approach to this supervision. The second interview focused more on the supervision process for two specific capstone projects that the teachers singled out during the first interview: Artificial Neural Networks Applied to the Prediction of Values in Sao Paulo's Stock Market, the project supervised by T1, and Software of Simulation Applied to Mechanical Engineering, supervised by T2. Students are required to develop their own capstone project over one and a half years (the last three semesters of the Engineering program). During the first six months, students must select a topic and a supervisor, learn the norms of scientific writing, and begin creating a review of literature in a course entitled Scientific Methodology. In the second semester, supervisors begin meeting with students weekly or biweekly to oversee the project's design, data collection, and the

students' first analysis. In the third and final semester, students must finish their project and present it to a panel comprised of two teachers and their supervisor.

The interviews were semi-structured and addressed a variety of topics to extract as much information as possible on the supervision process for capstone projects. In this paper, we focus only on those elements that emerged from the interviews pertaining to the teachers' beliefs, in line with the elements related to beliefs identified by Schoenfeld (1992) and Cardella (2006). These elements illuminate the teachers' approach to supervision, which may influence the way their students use mathematics or how they design their projects. The topics originally proposed for the interviews were: 1) the academic and professional background of each teacher; 2) the supervision of the capstone projects and how this supervision was conducted; 3) the use in these projects of content studied in the Engineering program; 4) difficulties the students encountered while working on the projects; and 5) the resources proposed by the teachers and used by students in completing their projects.

DATA ANALYSIS

After transcribing the two interviews, we created codes based on the teachers' responses and discussions, which allowed us to categorise thematically our data. This codification process was not linear, necessitating multiple readings of the interviews to draft the first codes, which were subsequently divided into subcategories (these subcategories were created because some categories allowed for more than one possible response). The codes were then recombined according to their conceptual similarities or differences, to define the final categories and subcategories (Figure 1).

Both teachers started the interview by providing details of their academic education and their experience in teaching courses and supervising projects in their university's Engineering program. T1, who has a background in Mathematics as well as Engineering, raised the importance of addressing the students' needs:

T1: [...] I came from the Polytechnic School, where I did my PhD working with Artificial Intelligence, neural networks, and genetic algorithms. What could I offer to the student in Production

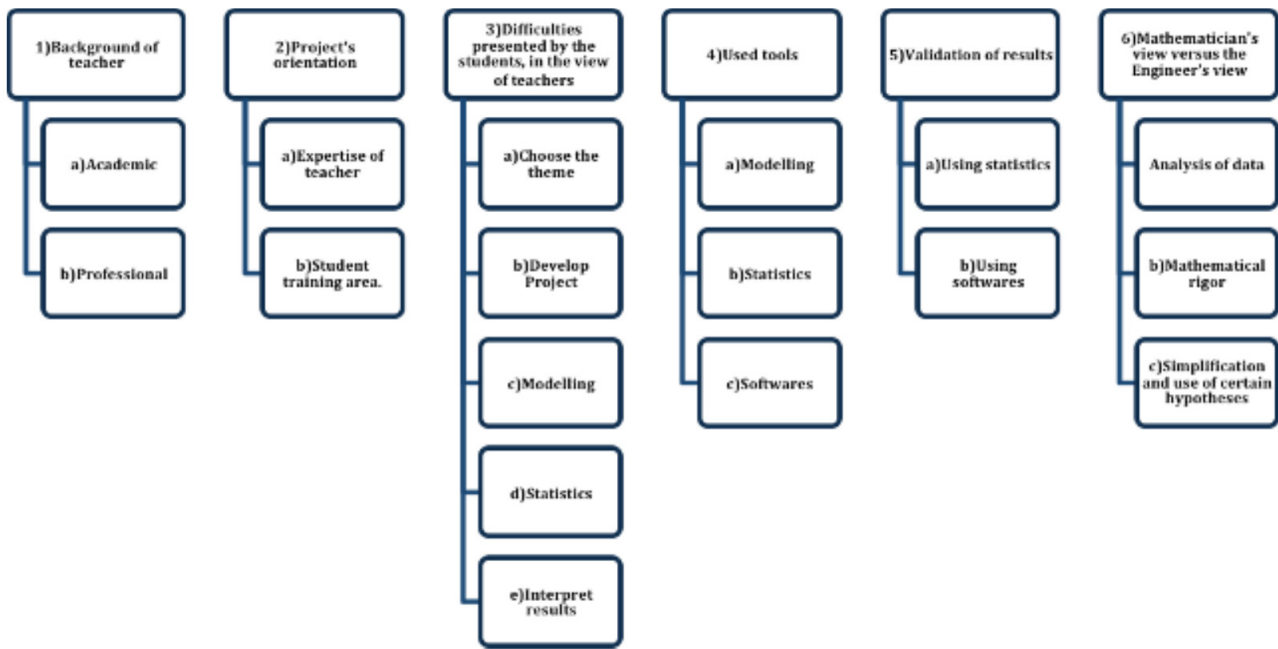


Figure 1: Final categories and subcategories

Engineering? For me to feel reassured, since I was also starting to supervise.

One of the elements mentioned by Schoenfeld (1992), *developing competence with the tools of the trade*, figures prominently in T1's response. In discussing her first experiences supervising students for their capstone projects, she revealed concerns about her ability to adapt her knowledge, which she considered atypical for Production Engineering; we observed that she felt it important to integrate her knowledge in a way that is useful to her students.

Unlike T1, T2's background is in Mechanical Engineering exclusively. However, like T1, his students study topics outside the scope of his education. His courses include Computing and Programming for Materials Engineering, a course in Electric Engineering and another in Mechanical Engineering. On supervision, he noted:

T2: I supervise students in Mechanical Engineering for a simple reason: because I'm a mechanical engineer and within my field of expertise it's more advantageous [...]

We observed a tendency in T2 to supervise students whose capstone projects relate to his field of research, resulting from a belief that it is easier to supervise projects that fall within one's area of expertise, where one masters *the tools of the trade* (Schoenfeld, 1992).

Later in the interview, when discussing a particular Mechanical Engineering student's project involving the measurement of a frame, T2 stated:

T2: [...] with some measurement instruments, you measure the deformations of this frame and obviously it's subject to a static charge, which doesn't happen in reality.[...] And then, as you make a static trial, you affect that result with a coefficient, making the effect bigger. And there you judge, as if it were subject to a real condition, what is in fact an approximation. You can also do it with more sophisticated experimental equipment [...] All of that is experimental. You can also make a mathematical simulation, using computer programs, which are professional, commercial, available in the big firms, in the general market.

With statements such as "which doesn't happen in reality" and "what is in fact an approximation", T2 revealed some of his beliefs about mathematics, as an engineer who deals with approximations of real situations solved using computer simulations. This supports Cardella's findings (2006), who pointed out that engineers, when designing a project, work with approximate dimensions and make comparisons with already existing data to verify whether the ideas they propose fall within acceptable limits. According to Cardella (2006), this aspect of an engineer's way of

thinking could be classified as a mathematical practice used by engineers. Cardella (2006) refers to Gainsburg (2003), who observed a tendency towards conservatism in structural engineers: “to understand a value or procedure as conservative is to attribute to it a broadly general and widely applicable quality, defined informally as ‘safe.’” (p. 9). Cardella observed that engineers adopt this attitude to ensure they meet minimum criteria, and that for more precise solutions, they reduce the margin of conservatism when it is justifiable to do so. She also found that despite the lack of precision inherent in “conservatism,” engineers still use “=” rather than inequalities such as “>” or “<” (Cardella, 2006, p.20).

T2 also discussed software used to model certain structures. However, he stressed that students encounter difficulties using the software and when modelling problems. Many of his responses indicate that software and modelling are typically used by engineers to solve problems – in agreement with Cardella’s (2006) model concerning required knowledge for teachers of mathematics in Engineering – and they revealed T2’s awareness that students have difficulty with both.

T1 discussed the case of a Production Engineering student interested in economy and the stock market. T1 provided the student with data sources to allow the student to build an artificial neural network, and after a few simulations, they achieved satisfactory results: “the network had worked, the results were perfect regarding what we had seen in the site”. However, for T1, it was still necessary to validate the results:

T1: What else can be done for some work, to give it a better basis, to really being able to say: “Ah, this work doesn’t have any problem. Oh, let’s do the statistical analysis of this data”. And then, [the student] performed all the parts regarding regression.

In this excerpt, we can observe elements arising from a belief that only mathematics can be used to prove the validity of a solution found using other tools (in this case, she used statistical analysis), even when the results are satisfactory as a whole. The use of statistics is also identified as a way to validate knowledge obtained through results analysis. Regarding this last point, T1 added:

T1: [...] the analysis performed by a mathematician regarding that data is different from that performed by an engineer.

At this point, T1 was asked to define the difference, and an interesting exchange between T1 and T2 ensued:

T1: The difference I see: within Engineering, we don’t need the exact, exact, exact value, as it is treated with all that...

T2: Rigor.

T1: The rigor with which it’s treated in mathematics.

T2: The engineer... let’s say he makes some hypotheses to simplify.

T1: Exactly. And she [the student] started from those hypotheses.

To further this discussion, the teachers were asked to elaborate their ideas of what constitutes mathematical rigor. T2 provided an example concerning the resistance of materials, called bending displacement:

T2: Among the different methods to determine the bending displacement, there’s [one] called the method of the differential equation of the elastic line of the beam. The elastic line is that configuration the beam acquires when it’s deformed because of the load. The equation from which all the textbooks start off, it’s a second degree, second order differential equation. A very complex equation. And engineers, based on the fact that in a project – in the majority of projects, to speak carefully – we assume that in the angular displacements the slope is very small, the tangent to the angle, when the angle is very small it’s almost the very same angle. And there, a first derivative which makes that differential equation harder for us... and which is to the square, we say it’s so small, so small, that it’s not considered...

T2 identified elements of mathematical practises that engineers use to solve complex problems, and noted their use of certain hypotheses to simplify mathematical calculations. T2 suggested that these practices are not well regarded by mathematicians:

T2: And there, mathematicians get... crazy. They say no, that's not the exact solution. That "we" [the engineers] are lying. And no. I'm just saying that the dislocation, the angle is so, so, so, so, so small, that I'm not gonna worry, and actually in reality it doesn't make a difference. I just simplify...

We see that T2 uses hypotheses to simplify calculations. Cardella (2006) identified this as a solving strategy used by students, which she termed "problem transformation" (p. 88). This strategy seeks both to simplify problems and transform them into familiar problems. Furthermore, we observe in T2 a way of speaking which seems characteristic of an engineer, using a language different from that of a mathematician (Nicol, 2002). Authors as Godfrey (2013) have labelled this as "An engineering way of thinking".

In addressing the notion of rigor, T1 came back to the work of her student in Production Engineering:

T1: In my specific case, when I speak about this mathematical rigor, when you look at a residual graph and you see the dots disperse or not, that interpretation is very subjective. Then, you take a graph [...] and you go validate with others. And there, I think they look alike, she can think they don't, but what was really important for us was to verify whether the system which had been modelled was giving us a prevision as close to reality as possible, which were the data we originally had. And she got to 98.6% of accuracy. [...] Furthermore, that rigor in Engineering is not considered, especially taking into account that the aim was actually to have a prevision of the data, using a computer tool, which is what was done. We wanted to use an artificial intelligence tool to know whether that tool was capable of doing the prevision. That was done, it was shown and with efficacy.

Again, here we can observe her belief that rigor in mathematics and rigor in Engineering are not the same. This aspect of her mathematical thinking (belief) may have a strong impact on others (monitoring

and control, problem solving strategies, practices), and could influence her approach to the teaching of mathematics. Elsewhere in the interview, when discussing her first-year Engineering courses in Differential and Integral Calculus and Linear Algebra, T1 stated:

T1: For me it's extremely important to have that mathematical rigor [...] at the beginning. Mathematics is an extremely important science; [...] even Engineering can't live without those calculations, although in some cases you have: "Ah, it's negligible" or "You don't need a rigor as mathematical as when a mathematician does mathematics". That's one point. Another point is: [the student] needs to know the bond between mathematics, which he has in the first courses, and what he uses in the technical courses. Actually, we try to do that [...], trying to show him that he's gonna use that tool later, the link between all those calculations and the technical courses.

Her response reveals her desire to broaden her students' ability to *think mathematically*, and her beliefs appear to influence her approach to teaching. In the case of T1, we observe a more explicit concern for rigor, and an awareness of the dichotomy between what a mathematician would do and what an engineer would do. According to Schoenfeld (1992, p. 359), these beliefs may determine the way she teaches and, as a consequence, shape her students' beliefs about the nature of mathematics.

FINAL REMARKS

The aspects identified in this study, although taken from interviews with just two teachers, are in keeping with the literature review and the theoretical framework, and they inspire reflection about mathematical practices in Engineering courses.

One issue identified in the teachers' responses concerns mathematical rigor versus approximation. On the one hand, the teachers require students to know (T1) or at least follow (T2) formulas with all the rigor characteristic of mathematics. On the other hand, in applying mathematics to the practice of Engineering and in resolving problems, certain assumptions are

made in order to simplify calculations (T2), which corroborates Cardella's research (2006) on the practices of Engineering professors. When results obtained through this process are close to those anticipated, based on the literature, they are deemed satisfactory by engineers. Regarding modelling, both T1 and T2 identified some difficulties related to 'approximation' and data analysis, which is in accordance with Doerr (2007) with respect to the challenges faced in teaching.

Our analyses have identified certain mathematical elements that could be classified according to the elements of mathematical thinking of Schoenfeld (1992) and in keeping with Cardella's (2006) arguments. However, we believe more research is needed in this area. Specifically, more observation of teaching practices is required, particularly of teachers with different backgrounds. Future research could look at the connections between teachers' beliefs, backgrounds, teaching practices and choices, and identify possible consequences for their students' learning. We intend to explore these aspects further in our future research.

REFERENCES

- Cardella, M.E. (2006). *Engineering mathematics: An investigation of students' mathematical thinking from a cognitive engineering perspective* (Doctoral Dissertation). Seattle, WA: University of Washington.
- Clark, L. M., DePiper, J. N., Frank, T. J., Nishio, M., Campbell, P. F., Smith, T. M., Griffin, M. J., Rust, A. H., Conant, D. L., & Choi, Y. (2014). Teacher characteristics associated with mathematics teachers' beliefs and awareness of their students' mathematical dispositions. *Journal for Research in Mathematics Education*, 45(2), 246–284.
- Doerr, H.M. (2007). What knowledge do teachers need for teaching mathematics through applications and modeling? In W. Blum, P. L. Galbraith, H.-W. Henn, & M. Niss (Eds.), *Modelling and Applications in Mathematics Education: The 14th ICM Study* (pp. 69–78). New York, NY: Springer Science + Business Media, LLC.
- Gainsburg, J. (2003). *Abstraction and concreteness in the everyday mathematics of structural engineers*. Paper presented at the annual meeting of the American Education Research Association, Chicago, IL (retrieved from: <http://files.eric.ed.gov/fulltext/ED479129.pdf>)
- Godfrey, E. (2013). Understanding disciplinary cultures: The first step to cultural change. In A. Johri & B. M. Olds (Eds.), *Cambridge Handbook of Engineering Education Research* (pp. 437–455). New York, NY: Cambridge University Press.
- Howson, A. G., Kahane, J.-P., Kelly, P.J., Lauginie, P., Nemetz, T., Simons, F. H., Taylor, C. A., & de Turckheim, E. (1986). Mathematics as a service subject. *L'enseignement Mathématique*, 32, 159–172.
- Maaß, K. (2011). How can teachers' beliefs affect their professional development? *ZDM*, 43(4), 573–586.
- Maaß, K., & Gurlitt, J. (2009). Designing a teacher questionnaire to evaluate professional development in modelling. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of the Sixth Congress of the European Society for Research in Mathematics Education* (pp. 2056–2065). Lyon, France: INRP.
- Murakami, H. (1988). Mathematical education for engineering students. In A. G. Howson, J. P. Kahane, P. Lauginie, & E. de Turckheim (Eds.), *Mathematics as a service subject* (pp. 63–69). Cambridge, UK: Cambridge University Press.
- Nicol, C. (2002). Where's the Math? Prospective teachers visit the workplace. *Educational Studies in Mathematics*, 50(3), 289–309.
- Pinto, A. (2013). Variability in University Mathematics Teaching: A Tale of Two Instructors. In B. Ubuz, Ç. Haser, & M. A. Mariotti (Eds.), *Proceedings of the Eighth Congress of European Research in Mathematics Education* (pp. 2416–2425). Antalya: Middle East Technical University and ERME.
- Schoenfeld, A.H. (1992). Learning to think mathematically: problem solving, metacognition, and sense-making in mathematics. In D. Grouws (Ed.), *Handbook for research on mathematics teaching and learning* (pp. 334–370). New York, NY: MacMillan.
- Schoenfeld, A. H. (1998). Toward a theory of teaching-in-context. *Issues in Education*, 4(1), 1–94.