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Teacher management of learning calculus: The case of sequences in the first year of university mathematics studies

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In this paper, I present my methodological tool for analyzing regular mathematics courses on calculus and an application of this tool in the transition from secondary school to university. The tool is based upon the Theory of Didactic Situations (TDS), and especially on the constructs of the “didactic contract” and the “milieu”. The data are taken from the transcription of a regular lesson on sequence convergence which took place during a first year university course. The aim is to investigate in what ways university calculus teachers attend to students’ prior knowledge in their teaching. The results are not surprising but the used tool does suggest a method of analyzing university teaching and its affordances or limitations for bridging the gap between secondary school and university.

Keywords: Transition, sequence convergence, university teaching and learning, didactic contract, milieu.

AIM OF THE PAPER

It is widely acknowledged that the transition from secondary school to university on calculus requires students to move from a problem-solving orientation to a formal orientation. A previous study (Bloch & Ghedamsi, 2004), focussed on crucial differences between the secondary school mathematics contents and the university one on calculus, led to the categorization and the formalization of many important changes that should occur in the way students are required to work at the first year of their university studies. We deploy the TDS construct of didactical variables (Brousseau, 1996), which are defined as parameters that influence the mathematics students’ work, to characterize these changes. Here are three main relevant didactical variables:

- The use of proof setting: At the first year of the university, the mathematics organisation on calculus is based on Bourbaki’s rules; as a result students have to deal with proof by using formal definitions, theorems, logical operations such as negation of quantified predicates, reductio ad absurdum, etc. At the end of secondary school, calculus contents focus on graphic or numerical proofs and indeed on algebraic proofs. At most, students use general statements concerning the operations on the limits of convergent or divergent sequences to “calculate” a limit of a sequence given by its general term.
- The use of technical methods: At the first year of the university to solve calculus tasks an amalgam of technical methods is introduced, whereas, at upper secondary school, a few methods are very well identified and many exercises allow a work on each of them. Students then are never surprised by the work they have to achieve. Yet, at the university they have the responsibility of choosing the adequate technical methods. For instance, to check the convergence or the divergence of a sequence at the university, one can identify adjacent sequences, use sub-sequences, use Cauchy theorem, use De l’Hospital’s rule, etc.
- The use of conversion between semiotic settings: At the end of secondary school, the tasks emphasize a fruitful conversion between the setting of algebraic semiotic representatives and the graphic one. These tasks become rather common and helpful for students’ work. However, students have almost no possibility to take the initiative of using a graph in a heuristic way since these conversions are generally explicitly enunciated. At the first year of the university, there are no more

graphs. Students have the responsibility to draw a diagram or a graph and exploit by themselves their potentialities as a heuristic support during a phase of a control or an exploration.

The modifications of the values of didactical variables in the transition from secondary school to university suppose changes in the didactic contract, which is “the implicit set of expectations that teacher and students have of each other regarding mathematical knowledge and regarding the distribution of responsibilities during the teaching and learning processes.” (González-Martin et al., 2014, p. 119). In the case of the study above, the values given to the didactical variables are mutually exclusive which may lead to an alteration of major rules of the didactic contract.

According to these results, it is important to investigate the reality of the work during regular mathematics courses, especially the role of the teacher to manage such crucial changes into the students’ work at the university. The research questions of this paper therefore are: To what extent does university teaching support the students’ shift to formal calculus? How does university teaching help students learn through adjusting previous knowledge? Finally – and this is the main question explored here – how can we model teaching and learning processes in order to allow the assessment of both the students’ actual work and the teacher’s management of these processes?

For this, I demonstrate my methodological tool for analyzing teaching and learning processes in a regular lesson on calculus, with a particular focus on the transition from secondary school to university. Then, I apply the tool to analyze a regular lesson which took place at the first year of the university, on sequence convergence.

METHODOLOGICAL TOOL – ASPECTS OF MAIN TDS CONSTRUCTS

The central object of TDS is the notion of Situation which is “defined as the ideal model of the system of relationships between students, a teacher, and a mathematical milieu.” (González-Martin et al., 2014, p. 117). The learning process is highlighted through the interactions taking place within such system.

In the Situation, the students’ work is modelled at several levels with a main focus: on the action “knowing

appears as means for action through models that can remain implicit” (p. 119); on the formulation “knowing develops through the building of an appropriate language” (p. 119) and on the validation “knowing becomes part of a fully coherent body of knowledge” (p. 119). The students’ work grows up within a milieu “namely the set of material objects, knowledge available, and interactions with others” (p. 119) including the interactions with the teacher.

The foundations of TDS constructs focus on the optimization of interactions taking place within the system mentioned above, “in ways that maximize the students’ responsibility for producing knowledge”. The use of TDS at the university level compels researchers to reconceptualize the “maximal responsibility” (p. 121) given to the students and leads to an adjustment of the role of the teacher, especially in helping students overcome the new requirements at the first year of the university.

In this sense, in the transition between secondary school and university in calculus, teacher’s interventions should not be neglected since he/she has the responsibility to manage students’ evolution from problem-solving skills to formal calculus. These interventions should enrich the students’ work and its evolution within and against a mathematical milieu during the phases of action, formulation and validation. González-Martin and colleagues (2014) illustrate the potency of TDS to design and to experiment Situations at university level, and demonstrate its application in three recent studies related to calculus and proof.

In regular (non-experimental) mathematics courses, the interactions taking place within the system formed by the teacher, the students and the milieu are governed by the actual didactic contract and evolve according to its nature. As a result, the quest for optimizing the interactions taking place within such system, as stressed in the TDS constructs, has to be the essence of methodological tool that will be used to analyze a regular lesson and that will allow the assessment of the students’ actual work and of the teacher management. The emphasis on the phases of action, formulation and validation in the students’ work materialize this quest. This should be done with taking into account teacher’s interventions to manage these interactions.

Taking these considerations, the methodological tool for analyzing a regular lesson introduced in this paper focuses on two categories of students' utterances that deal with the phases of action, formulation and validation in the students' work; and three categories of teacher utterances, one related to managing interactions and two related to managing phases that support learning. The teacher categories are divided into subcategories that cater for the particularities of paradigmatic examples of didactic contracts (Brousseau, 1996). The definition of these subcategories is helped by the use of Robert's studies (2003; 2007) concerning teacher practices in order to achieve better meaning of transition phenomena. In particular, Robert (2003) attaches great importance to the organization of knowledge as a condition of learning and argues for the "comparison of several methods and the simultaneous operations of several properties at once, including old and new." (p. 70). The subcategories referring to students' work are outlined according to the structuring of the milieu. In the following, I set out the methodological tool with more explanations for each subcategory.

Teacher management

1) Management of interactions

MI1: Initiate discussion by asking questions about specific knowledge in relation with the aimed one.

MI2: Leave openings that help students to make a choice, to ask questions and to organize knowledge (Robert 2003; 2007).

MI3: Abbreviate students' work, including questions.

MI4: Splitting tasks into elementary subtasks, or specify technical methods to use. In this case, it imports to clarify whether the teacher limits the students' work to an application of juxtaposed knowledge (Robert, 2007).

MI5: Guide students to take distance from what is happening and to work at the meta-cognitive level (Robert, 2007).

2) Management of action and formulation

MAF1: Treat examples and counterexamples.

MAF2: Support students' formulations by providing them with opportunities "to make conjectures, to experiment with heuristic solution, and search for adequate means of reasoning." (González-Martin et al., 2014, p. 122).

MAF3: Foster the changing of knowledge context by developing an operational status of the notions, if any, by emphasizing relationships among notions, by changing the setting of semiotic representations, etc.

3) Management of validation

MV1: Enunciate statements about knowledge

MV2: Argue by using formal proof.

MV3: Argue by using formulations, explanations, and by changing semiotic settings in a relevant way.

MV4: Exemplify general statements and discuss the implications of these statements on a certain class of notions (functions, sequences, sets of real numbers, etc.).

MV5: Make assessments of knowledge (local or global syntheses, including those relating to the use of formal rules of calculation).

Among some of these subcategories, the teacher's interventions do not enable students to undertake efficient interactions within the milieu and to progress in the learning process; this is the case of MI3, MI4, MV1 and MV2. The remaining subcategories emphasize the role that the teacher can play to enrich students' work especially in the case of MI2, MI5, MAF3, MV4 and MV5.

Students' work

1) Action and formulation

WAF1: Formulate questions concerning specific knowledge in relation with the aimed one.

WAF2: Express spontaneously knowledge by changing semiotic setting, by making examples and counterexamples, by linking several notions, etc.

WAF3: Formulate views on knowledge.

2) Validation

WV1: Indicate technical methods.

WV2: Perform on validation.

WV3: Discuss validation patterns proposed by peers or by the teacher.

1) What is the nature* of the sequence if $a = 0, 1$ and -1 ?

2) We suppose $a \in]-1, 1[$ et $a \neq 0$. Prove that the sequence converges to 0.

3) We suppose $a \notin [-1, 1]$. Prove that the sequence diverges.

* Across the text 'what is the nature of a sequence' is meant as 'Study this sequence in terms of its convergence or divergence'

When students demonstrated evidence of maladjusted knowledge, it is important to stress this in the analysis of the given lesson.

EXPERIMENTAL SETTING

Data collection

In Tunisia, mathematics courses at university level are organized into lectures and tutorials. The tutorial constitutes a setting to apply definitions and theorems already studied in the lecture. The lesson I focus on in this study functions as a tutorial, and concerns the applications of the main theorems of sequence convergence studied at the first year of the university. The sequence convergence is a concept met by the students from the third year of secondary school (scientific direction), which lasts four years in Tunisia, and which makes it possible to build a very rich and diverse corpus of knowledge.

This paper draws on the tasks planned by the teacher and the transcription of the whole lesson translated verbatim from French. The lesson lasted 2 hours in which approximately 30 students participated.

Mathematical tasks

Three mathematical tasks were planned by the teacher. The first one is related to the convergence of geometric sequences, the second one is related to the study of several sequences given by their general terms and the third one focuses on the use of Cauchy's theorem. In the following, I present succinct, *a priori* analyses of tasks in order to identify the mathematical milieu, namely the targeted knowledge, the students' previous knowledge and the elements that may optimize learning.

Task 1: General statements for geometric sequences

Let $(u_n)_n$ a sequence defined by $u_n = a^n$, $a \in \mathbb{R}$

At the end of secondary school, the theorem on the limit of geometric sequence is stated only for the case of $] -1, 1[$. The validation is based on the use of both the graphic semiotic setting and the numerical one. This is done by plotting and discussing geometric sequences graphs, or by computing terms of larger orders. Both cases show how the sequence tends to a specific value.

For this task, students have to pick out, among several university technical methods used to prove convergence or divergence, the relevant one in the case of such sequences. Some of the most useful methods to prove divergence at the first year of the university are: use formal definition; use Cauchy's theorem; prove that the sequence is not bounded; find two subsequences which don't behave the same way; etc.

Task 2: Sequences defined by general terms

What's the nature of these sequences?

$$u_n = (-1)^{\frac{n(n+1)}{2}} \left(1 + \frac{1}{n}\right); v_n = \frac{\cos(2n^2+1)}{n+1}; w_n = \sqrt{n^2+1} - n;$$

$$t_n = \sin\left(\frac{n\pi}{2}\right); s_n = \frac{c^n}{n^p}, p \text{ a natural number } \neq 0.$$

The study of $(v_n)_n$ and $(w_n)_n$ requires routine methods from the secondary school which correspond to the use of cosines properties and algebraic operations. The sequences $(t_n)_n$ and $(u_n)_n$ diverge, numerical calculation of some terms permit to identify subsequences to prove the non-convergence. The general term of $(s_n)_n$ is not familiar but its algebraic transformation permits to deduce that the sequence converges to $\frac{1}{p!}$.

Task 3: Convergence and Cauchy's theorem

1) Let $u_n = 1 + \frac{1}{2} + \dots + \frac{1}{n}$, $n \geq 1$.

Prove that $u_{2n} - u_n \geq \frac{1}{2}$. Deduce the nature of this sequence.

- 2) Let $v_n = 1 - \frac{1}{2} + \dots + \frac{(-1)^{n-1}}{n}$, $n \geq 1$.
 Prove that $v_n - v_m \leq \frac{1}{n+1}$, $0 < n < m$. Deduce the nature of this sequence.

This task requires more formal methods referring to logical operations such as negation of quantified predicates – which is nevertheless rather implicit – necessary condition, and sufficient condition. The intermediate subtasks allow students to use Cauchy theorem in order to conclude. Studying such sequences refers in reality to the study of $\sum_n \frac{1}{n}$ and $\sum_n \frac{(-1)^{n-1}}{n}$.

Under these conditions, the task is widely different from what is common at the end of secondary school. As said by González-Martin and colleagues (2014), “The role of the teacher becomes essential in helping students overcome their difficulties and fully grasp the subtleties they are confronted with.” (p. 130).

Overview of transcript data according to each task

A global description of transcripts shows few interactions between teacher and students. Teacher utterances are generally isolated from those of the students. The students’ utterances are short and the interactions among peers are non-existent. This compelled me to organize the analysis of the teacher and students utterances separately, and to do it in tandem when possible.

The lesson proceeded as follows in Table 1.

DATA ANALYSIS

Teacher management

The details of teacher utterances are as follows in Table 2.

About 26% of teacher utterances concern the argumentation by using classical formal proofs (MV2), but no interactions were observed with the students. Likewise, the utterances relating to the enunciation of statements about knowledge (MV1) aren’t correlated to students’ work and refer to the definition of convergence, theorems on the convergence, the definition of Cauchy sequence and its negation, etc. For instance, this is the case of the utterances (35 and 53) below:

- 35 Teacher: If $(a_n)_n$ tends to 0 and $(b_n)_n$ is bounded then $(a_n b_n)_n$ tends to 0.

The second utterance is the only one (from MV1) that is preceded by a student utterance formulated as a question:

- 23 Student: What’s the negation of a Cauchy sequence?
 53 Teacher: $(u_n)_n$ isn’t a Cauchy one $\Leftrightarrow \exists \epsilon > 0, \forall n_0 \in \mathbb{N}, \exists n \geq n_0, \exists m \geq n_0; |u_m - u_n| \geq \epsilon$.

The teacher intervened only five times to argue by using formulation and explanation (MV3). Among these interventions (20, 26, 31, 34 and 39), one of them carries some students’ knowledge:

- 6 Student: [talking about $\forall n \in \mathbb{N}, n \leq \frac{\log |M|}{\log |a|}$] This statement isn’t true because \mathbb{N} isn’t bounded!
 20 Teacher: Yes, it’s absurd... this means that the set of integers is finished!

The rest of these utterances are isolated from students’ work as shown below:

- 26 Teacher: [talking about $u_n = (-1)^{\frac{n(n+1)}{2}}(1 + \frac{1}{n})$] There’s no problem for $\frac{1}{n}$ which tends to 0, then $1 + \frac{1}{n}$ tends to 1.

Task	Teacher utterances	Student utterances	Duration (approximately)
Task 1	1 – 24	1 – 7	35 min.
Task 2	25 – 45	8 – 22	50 min.
Task 3	46 – 57	23 – 24	20 min.

Table 1: Overview of transcript data

	MI ₁	MI ₂	MI ₃	MI ₄	MI ₅	MAF ₁	MAF ₂	MAF ₃	MV ₁	MV ₂	MV ₃	MV ₄	MV ₅
Occurrence	7	0	4	10	0	3	3	0	10	15	5	0	0

Table 2: Details of teacher utterances

- 34 Teacher: [talking about $v_n = \frac{\cos(2n^2+1)}{n+1}$] Cosine of any number is between -1 and 1.
- 39 Teacher: [talking about $0 < w_n \leq \frac{1}{n}$ and $\frac{1}{n} \xrightarrow{n \rightarrow +\infty} 0$] This statement isn't available for 0... there is no problem as we search limit on $+\infty$.

However, more than 60% of the total interventions concerning interactions (MI) don't contribute to lead students to make links between knowledge and especially their previous ones (MI3 and MI4). More precisely, in the utterances concerning (MI4), the teacher specifies university technical methods to use; this way of doing limits students' work to an application of juxtaposed knowledge, such as the use of sub-sequences, Cauchy theorem and the theorem on bounded sequence. This is the case of utterances (9, 14, 16, 36 and 40) highly linked to students' work:

- 2 Student: $|u_n| = |a^n| < \varepsilon \Leftrightarrow |a|^n < \varepsilon$
- 9 Teacher: We can apply logarithm for the two members of the equality! [to prove that geometric sequence is convergent given $a \in]-1, 1[$ and $a \neq 0$]

In the utterances 14 and 16, teacher seems to limit the choices of the students by imposing over and over again his technical methods:

- 4 Student: We can use the subsequences! [to prove that geometric sequence is divergent given $|a| > 1$]
- 14 Teacher: Perhaps, we also can use a *reductio ad absurdum*.
- 15 Teacher: What's the main property of a convergent sequence seen in the lecture?
- 5 Student: A convergent sequence is a Cauchy one.
- 16 Teacher: Otherwise, it's bounded and then we suppose it and we conclude that it is absurd.

The teacher reacted in the same way for the utterance (36) that follows:

16 Student: $w_n = \sqrt{n^2+1} - n = \frac{1}{\sqrt{n^2+1}+n}$.

- 36 Teacher: We can put $\frac{1}{\sqrt{n^2+1}+n}$ between two members that converge to zero.

The teacher kept on imposing his technical method for the utterance (40):

- 20 Student: $t_n = \sin(n\frac{\pi}{2})$, $t_0 = 0$, $t_1 = 1$, $t_2 = 0$, $t_3 = -1$, $t_4 = 0$.
- 40 Teacher: It's clear that the sequence diverges. You can easily find subsequences which converge to different values.

Among teacher utterances which explicitly abbreviate students' work (MI3), the following one prevents student to use his/her own method to prove that the sequence converges (for the others, 37, 38 and 56 see students' work section):

- 7 Student: In this case we can use subsequences to prove that the sequence diverges.
- 22 Teacher: This is not fast. [meaning the method]

Finally, only 10% of the teacher utterances could help students' work in the phases of action and formulation (MAF). These interventions focus on the potentialities of the graphic setting or the numeric one to make conjectures, as well as, permit to investigate some examples and counterexamples related to aimed knowledge. Nevertheless, these interventions are initiated by the teacher and isolated from students' work.

Students' work

The details of students' utterances are anonymized and are as follows in Table 3.

Among the few questions asked by students (WAF1), three of them (17, 19 and 24) were shortened by the teacher (students who intervened in the exchange below aren't the same):

- 17 Student: Can't we directly calculate its limit? [talking about the sequence $w_n = \frac{1}{\sqrt{n^2+1}+n}$]
- 37 Teacher: Well! You're used to do this at the secondary school. Now I apply the theorem on sequence bounded.

	WAF ₁	WAF ₂	WAF ₃	WV ₁	WV ₂	WV ₃
Occurrence	4	7	0	3	10	0

Table 3: Details of students' work

- 18 Student: $0 < w_n \leq \frac{1}{n}$ and $\frac{1}{n} \underset{n \rightarrow +\infty}{\rightarrow} 0$ then $\lim_{n \rightarrow +\infty} w_n = 0$.
- 19 Student: What's the theorem on sequence bounded?
- 38 Teacher: You saw it in the lecture.

A little further:

- 24 Student: Why n is an integer different of zero? It isn't mentioned in the definition of a sequence?
- 56 Teacher: It depends on the sequence.

However, the few students' interventions which express spontaneous knowledge (WAF2) emerged in response to questions posed by the teacher (MI1). Nevertheless, both the teacher questions and the students' responses are basic and are not significant of real requirements on convergence.

Finally, students' work in the phase of validation (WV) is rather thin. In this phase, students employed technical methods widely used at the secondary school; some of their interventions are expressed in tandem with the teacher ones.

RESULTS

In this paper, I present my methodological tool, based on TDS constructs, for analyzing a regular lesson on calculus. The application of the tool for analyzing a regular lesson on convergence sequence at the first year of the university allows a global illustration of teacher management and its implications for the learning process, as well as a more local description of effective learning about the convergence of sequences, if any. In this situation, the teacher seems not to care much about students' work as he doesn't intervene to enrich this work by emphasizing relationships among notions, by changing the setting of semiotic representations, by allowing openings to organize knowledge, by making assessments of knowledge, etc. The interventions of the teacher failed to enable students to undertake efficient interactions with the mathematical milieu. For instance, he limits the choices of the students by imposing over and over again university technical methods, as well as, by abbreviating efficient students' interventions including questions that may contribute to make links between knowledge. Furthermore, the few teacher interventions that focus on the potentialities of secondary

semiotic tools are initiated by him and isolated from students' work. The teacher's use of the formal semiotic tool is done in an "ostensive way" (Brousseau, 1996, p. 45), which is defined as the act of the teacher who "shows" a mathematical object under the illusion that the students see the object that this "showing" aims to achieve. As a result, students' work is not consistent. The students express basic knowledge with little appropriate knowledge on convergence at the university level. During the whole lesson, the students use methods from secondary school and do not succeed to shift to the use of methods expected at university level.

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