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Contributions and limits of a specific course on manipulation of formal statements for fresh university students

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In this paper, we present preliminary results from an on-going project about an innovative course for first year students in Computer Sciences aiming to improve students' abilities in manipulation of formal statements and writing proofs. The data analysed show that although the course seems to improve syntactic abilities, the semantic control of formal statements remains problematic for most students.

Keywords: Didactics of mathematics at university level, logical analysis, proof, syntax, semantics.

INTRODUCTION

It is well known that the manipulation of formal statements is a main difficulty for many students starting university. In particular the widespread use of symbolic register to characterize the notions minimizes the use of natural language and faces students with the obstacle of formalism (Dorier, 1995). As a consequence, an important question is to find mathematical organisations [1] that allow students to both grasp the meaning of these notions and become familiar enough with formalism in order to be able to engage in proof and proving (Durand-Guerrier et al., 2012). In this paper, we describe such an attempt carried out at the University of Mons (Belgium), and we present elements of an explorative research aiming to investigate the effectiveness of such an innovation.

BACKGROUND

In the teaching of mathematics at university, definitions play a crucial role, in particular in the dialectic between proof and definition. From an axiomatic perspective, the role of definitions is to introduce new concepts relying on concepts already known, and avoiding contradiction. Ouvrier-Buffet (2006) showed the relevance of the activity of construction of definition for the process of mathematical conceptualization. However, in university courses, in France as in Belgium and in many other countries, the most common method of teaching is the axiomatic one (definitions, examples, theorems, proofs) in particular regarding formalising, unifying and generalizing (FUG) concepts such as the concept of vector space (Dorier, 1995). FUG concepts unify and generalize objects, tools or methods that exists previously in the students' background in various forms by using a new formalism. These concepts are difficult to introduce because the distance between previous and new knowledge is very big. Bridoux (2011) highlighted the nature of formalising, unifying and generalizing of the main notions of Topology, whose definitions require an intensive use of mathematical and logical formalism.

Durand-Guerrier (2013) stresses the importance of taking into account the logical complexity of the statement at stake in the tasks given to students in order to allow students to recognize the productiveness of operational formalism (the syntax) and to articulate it with the meaning of the mathematical objects involved in the tasks (the semantics). At the University of Mons (Belgium), in order to take into account this perspective, an innovative course was implemented in 2012 to replace the traditional mathematical courses for students in Computer Sciences.

Although these students are *a priori* well educated in mathematics, we hypothesize that they will face the classical difficulties identified in the literature when using mathematical language and notations, produc-

ing examples or engaging in proofs (e.g., Thomas et al., 2012). In this paper, we present the first stage of a project aiming to explore the following research question: How students involved in the innovative course deal with formal statements, in particular how do they take in consideration the relationship between syntactic and semantic aspects?

METHODOLOGY

The data was collected during a written examination. Due to the limited place, we choose two items for which the manipulation of formal statements was the key. The methodology that we use is a qualitative one relying on the consideration of the distance between *a priori* analysis, including a mathematical and a logical analysis of the tasks, and *a posteriori* analysis. The *a priori* analysis opens the possible answers that could be expected from students and the foreseeable difficulties. This *a priori* analysis has been done afterwards, for the purpose of the research, and reveals an underestimated logical complexity of the statements and the tasks considered. We first describe the main characteristics of the innovative course, and then present the data and the participants. In the next section, we provide the *a priori* analysis of the two items chosen for this paper, on which *our a posteriori analysis* relies. The last section of this paper is devoted to our conclusion.

DESCRIPTION OF THE DIDACTICAL INNOVATION INTRODUCED IN MONS UNIVERSITY

The teachers of the department of mathematics at UMONS took the opportunity of the introduction of the LMD [2] reform in Belgium to reconsider the content to be taught to first year students, and also the mathematical abilities that students should have developed at the end of their studies. The general objective for the first year at university is the deepening of concepts studied in upper secondary school in classical domains such as: Calculus, Algebra and Linear Algebra (e.g., Convergence of numerical sequences – Groups – Linear mapping – etc.) and at the same time helping students to master the following points: giving meaning to concepts, without neglecting technical and operational aspects; being able to use relevant knowledge even when they are not explicitly required; developing flexibility between the various representations, frames and registers of mathematical objects as suggested by Douady (1987); making explicit reasoning and providing justifications. All these aspects involve the manipulation of formal definitions and the properties studied during the courses. According to these aims, there are many tasks devoted to the manipulation of definitions during the tutorial classes and the assessment of students.

The course named « Mathématiques pour l'informatique 1 » takes place in the first semester. It comprises 20 hours of lectures and 40 hours of tutorials and deals with logic, set theory, methods for proof and proving (mathematical induction, proof by contradiction, proof by contraposition, etc.), and some topics in arithmetic. A characteristic of this course is to attempt working with students on the syntax and the semantics of quantified statements, involving more or less familiar or intuitive concepts. The quantified statements we work with involve simple inequalities, classical functions or classical sets and subsets such as intervals. At this stage, convergence of numerical sequences or infinite sequences of sets had not been introduced [3]. Here is an example of tasks proposed to students in this course.

Let X and Y be subsets of $\mathbb{R}$ *and f:* $X \to Y$ *be a function.*

a) Define « f is injective ».

b) Define « f is surjective ».

c) Is the function g: $\mathbb{R} \to \mathbb{R}$ *defined by* $g(x) = x^4 + x^2$ *injective ? Surjective? Justify your answer using the definitions provided in the previous points.*

This kind of task is generally not given in secondary school. As a consequence, the teacher has to use didactical means such as explanations on the methods, orally as well as in writing. During the tutorial classes, the students are asked to present their work to their peers in order to facilitate exchanges between students and to allow collective discussions on reasoning and other difficulties.

PRESENTATION OF THE DATA AND OF THE PARTICIPANTS

In accordance with the aim of the course, exercises involving reasoning and manipulation of definitions were included in the assessment. We present below two items, the first one requires to prove by contrapo-

sition the statement that characterises non positive real numbers; the second one requires the definitions of injective and surjective function and their use for studying two given functions. In the first exercise, we will mainly observe the mastery of the syntax, while in the second one, we will focus more on the articulation between syntax and semantics (Weber & Alcock, 2004, Durand-Guerrier, 2011).

52 students were assessed in January 2014 and the assessment lasted 4 hours. Around 80% of the participants attended the whole course. Our data consists in the assessment papers submitted by the students. The seven questions given to students challenged them to manipulate quantified statements. It is not possible to say how much time the students spent on these two particular questions. The assessment was a closed book exam.

A PRIORI ANALYSIS OF THE TWO EXERCISES

In this paragraph, we present the *a priori* analysis of the two chosen exercises taking into account their logical structure.

First exercise: A proof by contraposition [4]

Let us consider the proposition: $\forall x \in \mathbb{R}, (\forall \varepsilon > 0\ x \leq \frac{\varepsilon}{2}) \Rightarrow \Rightarrow x \leq 0$ *(1)*

The goal is to prove this statement by contraposition

— *Provide the proposition to prove in order to achieve this proof*

— *Write down the proof*

In the first item, it is required to provide the statement that will be proved, that is providing the contrapositive of statement (1). During the course, students were encouraged to perform this step prior to engage in the proof in order to avoid confusion between negation, proof by contraposition and proof by contradiction. The statement (1) is a universally quantified conditional statement whose antecedent is a universally quantified statement.

Global structure of the contrapositive

Let us denote $A(x)$: $(\forall \varepsilon > 0\ x \leq \frac{\varepsilon}{2})$ and $B(x)$: $x \leq 0$ respectively the antecedent and the consequent. Statement (1) can be formalized as: $\forall x[A(x) \Rightarrow B(x)]$. We hypothesize that students will recognize this structure. The global structure of the contrapositive is hence: $\forall x[\neg B(x) \Rightarrow \neg A(x)]$. It is possible that some students could apply incorrectly the following rule: *while negating a statement, change all the quantifiers* (this rule is valid only for quantifiers in prenex position) providing: $\exists x[\neg B(x) \Rightarrow \neg A(x)]$. Also some students may confuse negation and contraposition (or proof by contraposition and proof by contradiction) and propose: $\exists x[\neg B(x) \wedge \neg A(x)]$.

The next step is applying the negation to $A(x)$ and $B(x)$.

Negation of A(x) and B(x)

$B(x)$ is an atomic formula; taking its negation is elementary: $\neg B(x)$: $x > 0$.

$A(x)$ is a complex formula comprising a variable in the scope of a bounded universal quantifier: $(\forall \varepsilon > 0\ x \leq \frac{\varepsilon}{2})$. Taking the negation provides a statement $(\neg A(x))$ with a bounded existential quantifier: $(\exists \varepsilon > 0\ x > \frac{\varepsilon}{2})$.

Two rules are involved:

a) While negating a universally quantified statement, change the quantifier to the existential one.

b) Negating "≤" provides ">" and vice-versa, that applies correctly to "$x \leq \frac{\varepsilon}{2}$", but not to "$\forall \varepsilon > 0$".

Although the treatment of bounded quantifiers had been discussed during the course, some students may apply this rule incorrectly, providing "$\exists \varepsilon \leq 0$,".

The method to prove the contrapositive consists of providing a number strictly superior to a given number. It had been widely discussed during the course, and the mathematical knowledge required is elementary. So in this exercise we aimed to test the reasoning structure and the consideration of the logical structure (mainly the manipulation of syntactic rules). Although this exercise might appear rather easy, this brief *a priori* analysis shows the complexity of the first item, whose understanding should help students to successfully write down the proof.

Second exercise: Manipulation of definitions

First, students are asked to provide the definition of an injective function and the definition of a surjective function. The formal definitions are expected, this should be clear for students due to the focus of the

course. Indeed, during the course, the two classical definitions of an injective function were given, both in natural language and in formalized language:

$$\forall x \in D_f \forall y \in D_f (x \neq y \Rightarrow f(x) \neq f(y)) \text{ (1)}$$

$$\forall x \in D_f \forall y \in D_f (f(x) = f(y) \Rightarrow x = y) \text{ (2)}$$

Various examples of functions had been studied and the necessity of identifying the logical structure and the mathematical needed to prove that a given function is injective (surjective) or not had been considered. This is required in the two following items, for which the students should indicate if the given function is or not injective, resp. is or not surjective, and then prove their claims.

Type of proofs expected from students

It should be obvious that to prove that a given function is not injective it is enough to simply provide a counterexample. To prove that a given function is injective students need to provide a proof by generic element using definition (1) or (2). Likewise, to prove that a given function is not surjective, students need either to determine the set of all the images, or identify an element b with no antecedent. To prove that a given function is surjective, students have to consider a generic element b of the outputs set and find an antecedent. This leads to prove that for any b in codomain, the equation $f(x) = b$ has at least one solution.

In the course expectations, critical review of the reasoning produced by students is emphasised. A strong requirement of the course is also making explicit the logical structure of formal statement that should appear in the students' work. Another requirement concerns justifications: students have to explicitly indicate the results of the course.

Mathematical analysis

a) $g: \mathbb{R} \to \mathbb{R}: x \to g(x) = x^4 - 1$

Injection: Here $D_g = \mathbb{R}$. is an even function; hence it is not injective. It is also possible to prove this by providing a counterexample, i.e. an element satisfying the negation: any couple of element $(-a, a)$ with $a \neq 0$ is suitable. Note that attempting to prove that function g is injective leads to the class of counterexamples $(a, -a)$ with a in $\mathbb{R}^*$ (nevertheless, some students could provide an erroneous proof).

Surjection: As $Im\ g = [-1, +\infty[$, while the set of outputs is $\mathbb{R}$, function g is not surjective. It is also possible to prove directly that any given element of $]-\infty, -1[$ is not reached, i.e. that equation $x^4 - 1 = b$ with b an element of $]-\infty, -1[$ has no solution.

So in this question only elementary knowledge of functions, equations and real numbers previously studied at secondary school is involved. As indicated in the course description, similar functions had been studied during the course.

b) $h: 2^{\mathbb{N}} \times 2^{\mathbb{N}} \to 2^{\mathbb{N}}$

The function h associates to each couple of subsets of the set of natural integers the intersection of the two subsets. No similar function has been studied during the course, but the set theory knowledge that is necessary to prove the function is surjective had been studied, and numerous examples had been treated in the corresponding chapter, so that *a priori* the repertoire of students is sufficient to provide a counterexample for injection, in order to prove that function is not injective.

Injection: It is possible to find different couples of elements with the same image; example: $h(\{1\}, \{1\}) = h(\{1, 2\}, \{1\}) = \{1\}$. Hence, h is not injective.

Surjection: $\forall X \in 2^{\mathbb{N}}\ X = h(X, X)$; as a consequence h is a surjection.

Nevertheless, as considering couples of sets as inputs for a function is a rather abstract process, it is likely that many students will not be able to provide a correct proof. In particular, considering a pair of pairs of subsets in order to engage in a general proof of injection (in case non injection is not recognized) is difficult in terms of formal manipulation as well as conceptually. We hypothesize that such a proof will not appear in the students' work.

This brief *a priori* analysis of this second exercise emphasizes the fact that here both syntactic and semantic aspects of proof are closely and dialectically intertwined (Durand-Guerrier & Arsac, 2005). So although syntactic forms are clue aspects of mathematical reasoning, a semantic control on objects is needed in order to elaborate the expected proofs.

$\forall x \in \mathbb{R}, x > 0 \Rightarrow \exists \varepsilon > 0\ x > \frac{\varepsilon}{2}$	21	$\forall x \in \mathbb{R}, x > 0 \Rightarrow \exists \varepsilon \leq 0\ x > \frac{\varepsilon}{2}$	16
$\exists x \in \mathbb{R}, x \leq 0 \wedge \exists \varepsilon > 0\ x > \frac{\varepsilon}{2}$	2	$\exists x \in \mathbb{R}, x \leq 0 \vee \exists \varepsilon \leq 0\ x > \frac{\varepsilon}{2}$	2
$\exists x \in \mathbb{R}, x > 0 \Rightarrow \exists \varepsilon > 0\ x > \frac{\varepsilon}{2}$	2	$x > 0 \Rightarrow \exists \varepsilon > 0\ x > \frac{\varepsilon}{2}$	2
$\forall x \in \mathbb{R}, x > 0 \Rightarrow \forall \varepsilon \leq 0\ x > \frac{\varepsilon}{2}$	1	No answer	6

Table 1: Students' answers for the contrapositive

A POSTERIORI ANALYSIS OF THE TWO SELECTED QUESTIONS

First exercise: a proof by contraposition

Statements provided by the students are given in Table 1.

For this item, 42 students provide a conditional statement, among them 21 provide a correct answer; 6 change the universal quantifier in prenex position in an existential one, and 19 negate "$\forall \varepsilon > 0$" in "$\forall \varepsilon \leq 0$". Two students provide a conjunction, which is not the negation, and two students provide a disjunction not equivalent to the contrapositive.

We observed that the statement provided by students for the contrapositive had an impact on the way they shaped their proof, as in the following example, where the student fulfils the expectation of the teachers by writing down the proof in detail:

Given $x \in \mathbb{R}$. Let us consider $x > 0$. Let us show then: $\exists \varepsilon \leq 0, x > \frac{\varepsilon}{2}$. *Let us take $\varepsilon = -1$. Let us show $x > \frac{\varepsilon}{2}$, i.e. $x > -\frac{1}{2}$;* *true as x>0 by hypothesis.*

Table 2: An example of an incorrect proof in the first exercise

We interpret this as an indicator that this student remains in a syntactic point of view, neglecting the possible semantic control that could have lead him to come back to (and may be modify) the statement provided for the contrapositive. This example illustrates the importance of the relations between syntax and semantics in proof production, in line with consideration by Weber & Alcock (2004).

Second Exercise: manipulation of definitions

Providing the definition of an injective (resp. surjective) function

Most students (45 out of 52) were able to provide the definition of an injective function. Among them 38 choose the definition considering two distinct elements in the antecedent of the implicative statement. For the definition of a surjective function, we have 38 correct answers; all students, except one who used the image set, provided the definition with quantifiers. Wrong answers are either incorrect statements in natural language like "*f* is surjective if every real number has at least one image" or incorrect quantified statements like "*f* is injective if $\forall x_1 \in C\ \forall x_2 \in D\ x_1 \neq x_2 \Rightarrow f(x_1) \neq f(x_2)$".

Manipulation of definitions

a) The first function considered in the *a priori* analysis is $g(x) = x^4 - 1$.

As already said this function is not injective. 36 students gave the correct answer, first writing down the negation of the statement (which is a teacher's expectation) and then giving a counterexample taking two opposite numbers. 5 students did not write down the negation, but provided a counterexample. This confirms that the rule "to prove that a universal statement is false, provide a counterexample" is available at that level for most students. What is often problematic is the relationship to the negation of a given statement (Njomgang & Durand-Guerrier, 2011) – here it is difficult to know if those who provide the negation make the links, or just fulfil the teacher's expectation.

Function *g* is not surjective. We obtained 27 out of 52 correct answers. Half of these 27 students began to write down the negation of the definition with a correct logical structure. Then, they provided a counterexample: mainly saying that −2 has no antecedent, sometimes without mathematical justification; 3 students provided *Im g* without mathematical justification. These students seem to master the mathematical knowledge involved. It is not the case with the 8 students who assumed the function is surjective, providing an antecedent for each given real number, without controlling its effective existence. In the following example, the student does not take in account the fact that $\sqrt[4]{y+1}$ is defined only on the interval $[-1, +\infty[$.

Function g is surjective, i.e. $\forall y \in \mathbb{R}, \exists x \in \mathbb{R}, g(x)=y$. *Given* $y \in \mathbb{R}$. *Let us take* $x = \sqrt[4]{y+1}$. *Let us show* $g(x) = y$ *i.e.* $x^4 - 1 = y$. $(\sqrt[4]{y+1})^4 - 1 = y$, *i.e.* $(\sqrt[4]{y+1})^{\frac{4}{4}} - 1 = y$, $y + 1 - 1 = y$, $y = y$.

Table 3: An example of an incorrect proof of the false claim that *g* is surjective

The second function considered in the *a priori* analysis is $h: 2^{\mathbb{N}} \times 2^{\mathbb{N}} \rightarrow 2^{\mathbb{N}}$ that associates to each couple of subset of the set of natural integers, the intersection of the two subsets. Function *h* is not injective. We obtained 20 correct answers; 18 students did not answer this question. As in the previous exercise, most of the students began to write the negation of the definition. It appears that those who were able to identify that the elements of the inputs set were couple of subsets succeeded in providing the proof. Recognizing the nature of these elements was a difficulty for many students: some identified them as a subset of $\mathbb{N}$, or as a pair of numbers. In the following example, the logical structure of the proof is more or less adequate [5], but the proof is invalid due to the involvement of inappropriate objects.

h is not injective, thus $\exists x_1, x_2 \in 2^{\mathbb{N}}, f(x_1) = f(x_2) \Rightarrow x_1 = x_2$. *Let us take* $x_1 = (0,1)(1,2)$ *and* $x_2 = (1,2)(0,1)$. $f(x_1) = f(x_2) = 1$.

Table 4: An example of incorrect proof

Function *h* is surjective. We obtained 13 correct answers; 20 students did not provide an answer. Here the nature of the mathematical objects at stake is problematic for many students.

Concerning the results for both questions, we are able to say that students successful in the first question are, except one, successful in the second one. One perspective is to study whether the difficulties are the same in each question, for example for the sense given to the objects or for the manipulation of formal statements.

CONCLUSION

In this paper, we have focused on the elaboration of an innovative course for students attending the first year university in Computer Sciences in Belgium (Mons University). The aim of this course is to work with students on formal statements and proof and proving in mathematics. A main issue of this course is to emphasise some logical aspects that are often not elaborated upon by teachers, such as explicitly teaching and insist on *how to negate* or *how to take the contrapositive of* a conditional statement. In addition, specific attention is paid to the production of examples and counterexamples.

As a first exploration of the impact of this course on the students' logical and proving abilities, we have analysed two of the exercises focusing on manipulation of formal statements including definitions that were given in the assessment at the end of the course. A first result is that a rather large number of students attending the course were able to provide the required definitions, to recognize the logical structure of statements and to correctly negate a statement involving quantifiers. Referring to the existing literature that attest of strong difficulties with negation for university students (e.g., Njomgang Ngansop & Durand-Guerrier, 2011), we hypothesize that this could be a benefit of the course. Nevertheless, difficulties remain: we have observed that for some students, the manipulation of formal statements remains at a syntactic level. They seem to use routines without any semantic controls. In the case where familiar mathematical objects (such as real numbers) are involved, the related difficulties are limited, while these difficulties seriously increased when knowledge of less familiar objects is required. A second result concerns the logical complexity of the tasks that the students had to achieve, complexity that had been partly underestimated when designing the assessment. We mention in particular those difficulties related to the negation of statements involving bounded quantifiers, so that difficulties related with the dialectic between syntax and semantics, in particular in cases unfamiliar mathematical objects are being considered. The next step of this on-going study is to conduct interviews in order to better understand the way students deal with this type of tasks, for which mathematical and logical aspects are closely intertwined.

REFERENCES

Barbé, J., Bosch, M., Espinoza, L., & Gascon, J. (2005). Didactic Restrictions on the Teacher's Practice: The Case of Limits of Functions in Spanish High Schools. *Educational Studies in Mathematics, 59*(1–3), 235–268.

Bridoux, S. (2011). *Enseignement des premières notions de topologie à l'université. Une* étude *de cas.* Thèse de doctorat. Université Paris Diderot (Paris 7, France).

Dorier, J.L. (1995). Meta level in the teaching of unifying and generalizing concepts in mathematics. *Educational Studies in Mathematics, 29*(2), 175–197.

Douady, R. (1987). Jeux de cadres et dialectique outil/objet. *Recherches en Didactique des Mathématiques, 7*(2), 5–31.

Durand-Guerrier, V. (2013). Quelques apports de l'analyse logique du langage pour les recherches en didactique des mathématiques. In *Actes de la XVIème* école *d'été de didactique des mathématiques* (pp. 233–265).

Durand-Guerrier (2011) Semantic perspective in mathematics education. A model theoretic point of view. In *ICME electronic proceedings, materials from ICME 11*. Monterrey, Mexico, July 2008. http://www.mathunion.org/icmi/publications/icme-proceedings/materials-from-icme-11-mexico/regular-lectures/

Durand-Guerrier, & V. Arsac, G. (2005). An epistemological and didactic study of a specific calculus reasoning rule. *Educational Studies in Mathematics, 60*(2), 149–172.

Durand-Guerrier, V., Boero, P., Douek, N., Epp, S. S., & Tanguay, D. (2012). Examining the role of logic in teaching proof. In G. Hanna & M. DeVilliers (Ed.), *Proof and Proving in Mathematics Education* (pp. 369–389). New York, NY: Springer.

Njomgang-Ngansop J. & Durand-Guerrier V. (2011). Negation of mathematical statements in French in multilingual contexts – an example in Cameroon. In *Proceedings of the ICMI Study 21 – Mathematics and Language Diversity* (pp. 268–275). Sao Paulo, Brazil: ICMI.

Ouvrier-Buffet, C. (2006). Exploring Mathematical Definition Construction Processes. *Educational Studies in Mathematics, 63*(3), 259–282.

Thomas, M., Freitas Druck, I., Huillet, D., Ju, M., Nardi, E., Rasmussen, C., & Xie, J. (In press). Survey team 4: Key mathematical concepts in the transition from secondary to university. In *ICME 12 Proceedings*, Seoul, South Korea, 8–15 July 2012.

Weber, K., & Alcock, L. (2004). Semantic and syntactic proof productions. *Educational Studies in Mathematics, 56*(2–3), 209–234.

ENDNOTES

1. A mathematical organisation of a subject to be taught consists of teachers' or institutionnal's choices among « a set of mathematical elements » (types of problems, techniques, notions, properties, results, etc.) (for developments in English see, for example, Barbé et al. 2005).

2. LMD is the acronym for the three degrees of Higher Education in the Francophone area: Licence (Bachelor's degree), Master (Master's degree), Doctorat (Doctorate).

3. These concepts are taught in the course named « Mathématiques pour l'informatique 2 », organised during semester 2.

4. In the course, the coma is used to indicate that the scope of the universal quantifier is the conditional statement, so that the statement at stake is closed. In predicate calculus, we would write: $\forall x \in \mathbb{R}\ [(\forall \varepsilon > 0\ x \leq \frac{\varepsilon}{2}) \Rightarrow x \leq 0]$.

5. We notice an incorrect use of *thus* (in French *donc*), while what is expected here is *That is to say* (*c'est-à-dire* in French). Moreover, the statement provided by this students to formalise « *f* is not injective » is not correct.