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► **To cite this version:**

Dorothea Tubach. ” If she had rolled five, she’d have two more ”: Children focusing on differences between numbers in the context of a playing environment. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Charles University in Prague, Faculty of Education; ERME, Feb 2015, Prague, Czech Republic. pp.2017-2024. hal-01288512

HAL Id: hal-01288512

<https://hal.science/hal-01288512>

Submitted on 15 Mar 2016

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"If she had rolled five, she'd have two more": Children focusing on differences between numbers in the context of a playing environment

Dorothea Tubach

TU Dortmund University, Institute for Development and Research in Mathematics Education (IEM), Dortmund, Germany,
dorothea.tubach@tu-dortmund.de

Mathematically rich games can offer children important informal learning contexts for playful mathematical activities in kindergarten. At the same time, these activities can be used as starting points for formal learning in primary school. In the reported qualitative study, 20 children were observed dealing with three developed playing and learning environments in the last year of kindergarten and the first year of school. This paper concentrates on children's mathematical activities and insights while dealing with a playing environment in kindergarten. On the one hand, the focus is on how children interpret relationships between numbers. On the other hand, two different types of play in the game are outlined. The results will be discussed regarding connecting points for learning processes in primary school.

Keywords: Kindergarten, playing, games, dice, relational understanding.

PLAYING GAMES AND LEARNING MATHEMATICS BEFORE FORMAL SCHOOLING

In recent years, empirical studies have provided evidence that playing mathematically rich games can have a positive impact on children's mathematical learning, including in kindergarten (e.g., Stebler, Vogt, Wolf, Hauser, & Rechsteiner, 2013). Characteristics of mathematical learning situations in games have been worked out and conditions for their development have been investigated, whereby mathematically rich games can provide a meaningful context for mathematical activities and are open for different individual strategies (Stebler et al., 2013). An adaptive guidance through adults can help to unfold the mathematical potential of the game as a learning situation for the child (Schuler, 2011). At the same time,

mathematical activities can themselves – independent from a game – be considered as play. This is the case if the mathematical activity has characteristics of play, e.g. it moves between the poles of rules and freedom (van Oers, 2014).

While current studies emphasize the importance of playing games for mathematics learning in kindergarten, playing as a mathematical activity, as described by van Oers, can be important for both learning locations: kindergarten and primary school [1]. However, there are fewer findings on how to use the idea of 'mathematical play' as a bridge between kindergarten and primary school at present, as well as how mathematical learning in kindergarten is effected in the context of a game (van Oers, 2014). The purpose of this paper is to ascertain the children's mathematical interpretations and characterize the playing activity in the context of a playing environment to outline opportunities for connected learning processes in primary school.

Playing and mathematics learning

As mentioned above, playing games seems to provide an important mathematical learning opportunity. But how can play or playing be characterized? Following van Oers (2014), play is understood in this paper as an activity that is characterized by *high involvement* of the actors, oriented on *rules* and allowing *some degree of freedom*. Accordingly, a characteristic feature of playing activities is that players voluntarily adhere to rules and adopt a specific role, e.g. in a game, they engage themselves in a fictitious competition. The rules of the game can be explicit or implicit, predefined or negotiated in the process of playing. Van Oers (2014) differentiates between four different functions of rules: (1) social rules, namely how to interact and deal

in play; (2) technical rules, concerning how to use play objects properly; (3) conceptual rules, regarding how to act based upon specific concepts; and (4) strategic rules, in terms of how to improve the playing process. Van Oers (2014, p. 62) defines the degree of freedom in a positive way, "as the freedom to change, to resist, to produce extravagant ideas and so on" [original emphasis].

Ginsburg (2006) distinguishes the observable mathematical play of children according to different types: "Mathematics Embedded in Play" and "Play Centering on Mathematics". The former type can arise by playing mathematically rich games. In particular, a game can imply mathematical aspects, although the core is the competition among the players. In contrast, "Play Centering on Mathematics" occurs in operative discovering, exploring and inventing patterns and structures. Children play with mathematical objects by varying them and, thus gaining insights into mathematical relations: They discover the impact of a change of objects and how to react accordingly with another specific change (Steinweg, 2001). Both types of play can offer children a context to verbalize their strategies and interpretations, as well as negotiating mathematical meanings. The playing environment "Who has more?", which is addressed in this paper, links both types of play according to Ginsburg (2006): it enables "Mathematics Embedded in Play" as a game with mathematical aspects. While the choice of structured game materials it is designed to allow play with mathematical objects. Therefore, "Play Centering on Mathematics" can be realized. Here the question is how can the playing activities of the children in the context of the playing environment "Who has more?" be characterized?

Mathematical play and relational understanding

The pivotal activity of mathematically-centered play involves establishing relations between mathematical objects of play. Wittmann and Müller (2009) highlight the ability to identify and use relations between single numbers by counting and calculating as a central objective for learning processes in kindergarten. The epistemological perspective broadens the view for the principally relational nature of numbers. Numbers and relations between numbers are not concrete objects, but rather can only be represented by symbolic or concrete objects. Only by dealing with the concrete and abstract objects and construing differences be-

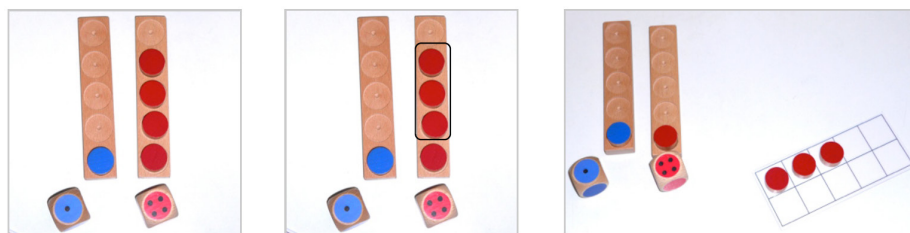
tween them can children acquire the concept of numbers and their relations (Steinbring, 2005).

Due to the relevance of relational understanding, the playing environment "Who has more?" is focused on relations of differences. It is intended to give children the opportunity to identify and use relations of differences in the interactive context of a game and based upon the game material. The main questions addressed in this paper are as follows:

- 1) How do children in kindergarten interpret relations of differences in the interactive context of the playing environment?
- 2) How can the children's 'mathematical play' in the context of the playing environment be characterized?

METHODS

The research method is oriented by Cobb and colleagues' method of "design experiments" (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003). Accordingly, three complementary playing and learning environments for kindergarten and primary school were designed, focusing on the exploration of relations between numbers. In spring and summer, children in the last year of kindergarten were involved in playing environments. At the beginning of the first school year (in autumn and winter), they were involved again, albeit now in the context of learning environments. Overall, about 20 children were observed by video over two cycles dealing with the playing and learning environments in kindergarten and primary school. The sequences presented in this paper are from observation in kindergarten. To reconstruct interactive processes of understanding, a qualitative approach is chosen, oriented in the interpretative classroom research (Krummheuer, 2000). Essential for the mathematical analyses is the epistemological triangle, as described by Steinbring (2005), which enables identifying specific reference contexts that children use by interpreting differences. The epistemological analysis particularly focuses on the reconstructions of the interactive process of constructing knowledge based upon actions and interactions.



According to the rolled number, counters are put in the block of 5

Comparison: There are three red counters more.

The red player is allowed to take the three counters and put them on his or her ten-frame.

Figure 1: Rolling and comparing

Construction of the playing environment "Who has more?"

In the following, one of three designed playing environments is exemplified: "Who has more?" The learning environment building on this playing environment is not considered here (for further details, see Tubach & Nührenbörger, 2014). "Who has more?" (WHM) is a game for two children, whereby each child receives a wooden block of five, a ten-frame and a dice (with the numbers 0 to 5) in the color blue or red. The game material is completed by small round gaming pieces, called counters (with a red and an alternate blue side).

Rule of the game: Both players roll their dice and put the appropriate number of counters in their respective blocks of five, according to the number rolled. The player with the higher number of counters is allowed to take the difference of counters (the ones that he or she has more) and puts them on the ten-frame (see Figure 1). Afterwards, the blocks are cleared and the dice are rolled again. The first player to fully fill the ten-frame wins the game.

At the heart of the playing environment is the comparison of two numbers and the determination of their difference. Children have the opportunity to gain insights into relations of differences. Meanwhile, children collect, structure and determine the number of counters on their ten-frame and gain experiences in composing and decomposing of numbers.

UNDERSTANDING OF MATHEMATICAL RELATIONS BY PLAYING "WHO HAS MORE?"

Maya (5,7) and Leon (6,4) play the game WHM with their guiding adult in kindergarten, a few months before entering primary school. Maya is already acquainted with WHM from previous game experience,

whereas Leon is playing for the first time. From this game, two sequences are selected in which the children are engaged with the comparison of two numbers. One focus of the following analysis is on the children's interpretation of differences. The leading questions are: How do children interpret differences based upon the game material? Which reference contexts do the children use for the interpretation of differences in the gaming process? The second focus is on the characterization of the playing activities in these sequences. This will help to gain deeper insights into the mathematics activity in the gaming process, as well as understanding and clarifying the role of this game for mathematical and playing activities.

Focus 1: Interpretation of differences – Comparison of 3 and 2

Maya (M) and Leon (L) start the game WHM. Maya has chosen the red color and rolls number 3, while Leon rolls number 2 with the blue dice. Both children put the corresponding number of counters in their blocks of five (see Figure 2). They agree that Maya has one counter more. The following scene begins as the guiding adult (GA) asks once more.

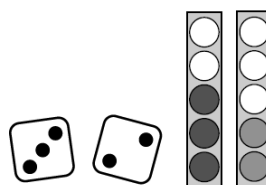
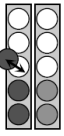


Figure 2: Initial situation

- 1 GA Ok. But how do you see that it's one more?
- 2 Maya 'Cause, 'cause three (takes her dice with the rolled number 3 and covers one dot with her finger)



- 3 GA Why aren't there two more?
- 4 Maya If, if you (.) because I didn't roll two. Leon rolled two and I have three one more (*lifting her third red counter and putting it back again*) 
- 5 Leon # If she had rolled five, she'd have two more.
- 6 Maya # then I can take it (*takes the counter and puts it on her ten-frame*) [...]
- 7 Leon Ah I know, I know (*points at the blocks*) Look here, one, two (*tipping the blue counters*) and if it was there (*tipping the third now empty field of M's block*), it's always the one above, which is one more. And if she had five and I had two, then would be one, two, three, four, five (*tipping the fields from bottom up in M's block*) So one, two, three (*tipping the empty fields of M's block*) more. Three (.) more. Three pieces more (.) If she had rolled five.

In reply to the GA's question (l. 1), Maya turns to her dice and covers one dot. Therefore, instead of three, there are only two dots visible. By showing with the dice pattern how to change the number 3 to achieve 2, Maya relates the two rolled numbers 3 and 2: if you cover one dot of three, both dice show the same number of dots (l. 2). As the GA asks once more why there were not two more, Maya emphasizes that she rolled another number than Leon: she demonstrates the difference between the rolled numbers with the help of the blocks of five and lifts her third counter. Hence, she creates an *equality* of the red and blue numbers of counters. She emphasizes the difference 'one' by putting her counter back on the block (l. 3). Therefore, she interprets the difference as the change needed to establish an equality of two numbers; in this case, by removing or covering the difference. Leon, however, constructs an example of a number pair (5 and 2, l. 7), which he regards will have the difference 'two' (l. 4): if Maya had rolled number 5, she would have had two more. Accordingly, Leon increases the minuend to achieve a greater difference. In the following, he gains a new insight into how to compare two numbers ("I know"): he divides the counters in the blocks into 'lower' and 'upper', stating that the counters above are the additional ones (l. 7). Consequently, he matches the equal number of red and blue counters and highlights that only those counters that cannot be matched have to be considered. These counters can subsequently be counted one-by-one. By separating the numbers

of counters into 'lower' and 'upper' and focusing on the 'upper', Leon creates a new initial situation for comparing: instead of 3 and 2, he compares 1 and 0. Here, he unconsciously uses the mathematical relation that the difference remains the same if minuend and subtrahend are increased or decreased by the same amount. He illustrates this insight with an own number pair of 5 and 2 and, thus, probably picks up on his assumption in line 5 to check it. He determines the difference by first identifying the fields where red counters would be located. The counting procedure of the fields seems to represent or replace the concrete action of putting counters in the block. He then counts the upper three empty fields in Maya's block where counters would be located, thus concluding that the difference is three. He succeeds in matching the equal number of blue and red counters, which are no longer important for determining the difference, whereby he can compare 3 and 0 rather than 5 and 2.

Conclusion: The children's interpretation can be characterized by their different reference contexts. Maya's actions indicate that she uses a reference context for interpreting the difference, which can be called "equalizing". She answers the question of how many counters have to be removed to establish an equality between the two numbers of counters. Expressed algebraic: $a-d=b$; $b+d=a$. She represents this change dynamically in the material. In contrast, Leon interprets the difference in a spatial-static way, which can be described as "matching", whereby he matches the same number of red and blue counters and determines as difference the number of counters that cannot be matched. Accordingly, he changes minuend and subtrahend by the same number to create a 'to zero' comparison: $a-b=(a-c)-(b-c)=d-0$, with $c=b$. By interpreting the arrangement of counters, however,

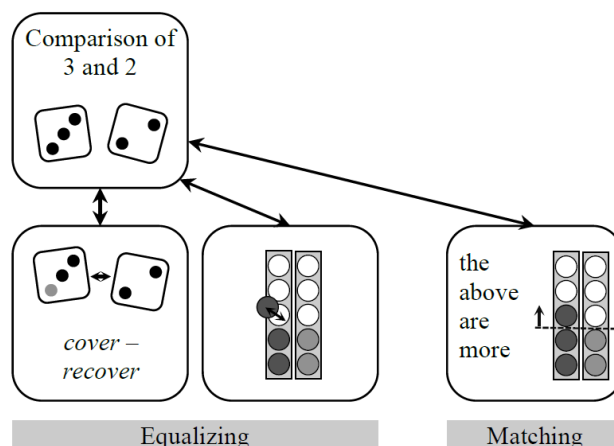


Figure 3: Epistemological triangle: 3 and 2

it is not necessary to identify the specific number of the equal change as subtrahend ($c=b$); rather, it is sufficient to remove or separate two unspecific yet equal amounts. This view on the arrangement of counters in the blocks of five helps Leon to determine the difference of his own example of a number pair.

The interpretation of "matching" builds upon the equality of two numbers to focus on the 'inequal'. "Equalizing", however, concentrates on inequality to determine the necessary change to establish equality between the two sets of counters. For both interpretations, it is not necessary to quantify the number of the equal counters; rather, a qualitative estimation 'equal' is sufficient.

Comparison of 5 and 2: What does Maya have to roll?

In the following, Maya and Leon win further counters. Now Maya has nine and Leon has seven counters collected on the ten-frame (see Figure 4). Leon rolls number 5 and is happy. Maya protests that number 5 would not be possible, since Leon would win too many counters. Hence, the question arises of what Maya has to roll for Leon to win.

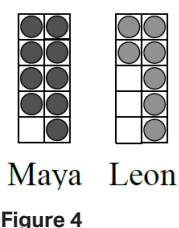


Figure 4

- | | | |
|----|------|--|
| 8 | GA | What does Maya have to roll so that you've won <u>immediately</u> ? |
| 9 | Leon | Mhh, one or zero or two. |
| 10 | GA | Then you could win? |
| 11 | Leon | 'Cause, 'cause wait (<i>takes his dice with the rolled number five and covers two dots with two fingers</i>), she's got to roll two. |
| 12 | GA | # Why? |
| 13 | Maya | # Heh? |
| 14 | Leon | Because then I have three more (<i>shows his dice with the two covered dots</i>) |



Leon answers the GA's question by giving three number pairs (5,1; 5,0; 5,2) with a difference greater than or equal to three (l. 9). Apparently, he assumes that he can win with every difference that is not less than three. In response to the question of whether he would win then (l. 10), Leon takes his dice with the rolled number 5 and covers two dots with two fingers so that only three dots are visible (l. 11). He concludes that Maya has to roll number 2 and then he would have three more. This approach leads to the following in-

terpretation: Leon covers as many dots from the dice pattern of 5 so that three dots are left. The number of covered dots represent the number that Maya has to roll. In opposite to Maya, who covered the difference, Leon covers the subtrahend and the three dots left represent the difference.

It is remarkable that, on the one hand, Leon uses the dice pattern of 5 to *construe the difference* of 5 and 2. On the other hand, he uses it to *construct differences* due to the context of the game required to find a number pair with 5 for the given difference 3.

Conclusion: Leon's interpretation of the difference shows an expansion of his previous idea, whereby he decomposes the minuend 5 in two parts: 2 and 3. This enables both the statement that Maya has to roll number 2 to win three counters and the determination of the difference 3, if Maya had rolled number 2. Therefore, he compares 5 and 2 by "subtracting" 2 from 5. Expressed algebraic: $a-b=d$. Here, the activity of covering dots of the dice pattern has a double function: the construction of differences and the constructing of differences.

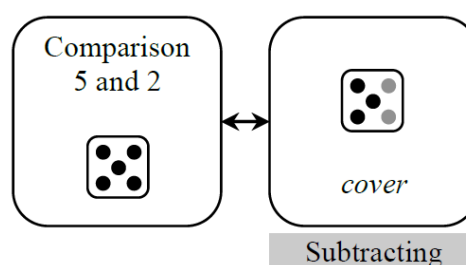


Figure 5: Comparing 5 and 2

Focus 2: The playing activity – Mathematically-centered play while playing a game

The playing activity of Maya and Leon in the context of "Who has more?" can primarily be described as "Mathematics Embedded in Play" (Ginsburg, 2006). Mathematical aspects are relevant, although the main issue is to win counters to fill the ten-frame the first. In addition and more surprisingly, "Play Centering on Mathematics" (ibid.) can be identified as a mathematical activity focusing on differences (see Figure 6). In this section, both mathematical playing activities are analyzed by the following characteristics: *involvement*, *rules* and *degree of freedom* (van Oers, 2014).

Children seem to be highly *involved* by playing WHM. They engage themselves in the fictitious competition and are pleased with a 'good' rolled number, for ex-

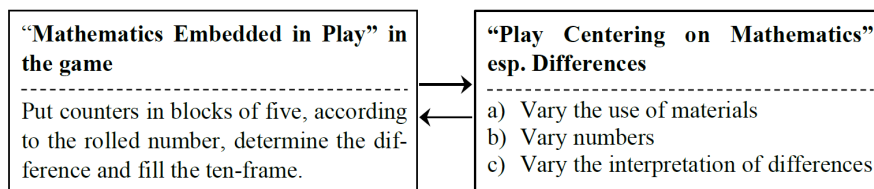


Figure 6: Mathematically-centered play in the context of playing a game

ample. The game is framed by different explicit technical and conceptual *rules*, while further rules are negotiated in the gaming process (e.g. that the ten-frame has to be filled exactly). The children are *free* to choose whereupon they put their attention: social or mathematical aspects. Another analysis of WHM (Tubach & Nührenbörger, 2014) showed exemplarily that children keep the rule that 'the ten-frame has to be filled', aside from which they try to interpret the rules in their favor, e.g. manipulate the dice to get a full ten-frame quicker. Therefore, playing the game WHM embeds mathematics.

Nevertheless, this deviation from the intended playing course allows another type of play, namely "Play Centering on Mathematics". This playing activity can be recognized in the variation of using the game material, the variation of numbers and the interpretation of differences in the game material. In the following, these characteristics of mathematically-centered play are illustrated based upon the two sequences:

Variation of using materials: The reinterpretation of the game material is the key aspect for mathematically-centered play: Maya uses the dice pattern, which originally randomly determines the number of counters to put in the block of five, to represent the difference. In the second sequence, Leon also uses the rolled number as a representation to construct a number pair for a given difference. Furthermore, Leon regards the blocks of five 'as if' counters would lie there. Therefore, it is possible for him to cope with fictitious number pairs, not only to determine differences but also to test assumptions (5 and 2 have the difference 2).

Variation of numbers: Maya decreases the rolled number 3 by one and establishes a relationship to Leon's rolled number 2. Leon tries to increase the difference and increases the minuend of a number pair (5 and 2 instead of 3 and 2).

Variation of interpretations of numbers and differences: The analyzed different reference contexts show that children vary their interpretation of differences and

also shorten the process of comparing two numbers in their blocks of five.

Children are obviously *involved* in these varying activities, as they are based upon their own ideas. They play together rather than against each other and inspire themselves towards new ideas, pick them up and develop them further. The *rules* are not explicit and predefined, but rather implicit and arise in the interaction in the playing process. In the context of the mathematically-centered play, transparency in own ideas seems important (social rule). At the same time, the children's ideas and interpretations are oriented on the properties and functions of the game material, e.g. they choose realistic numbers (technical rules). Their mathematical considerations are oriented on the game, which means that the game builds the conceptual framework for the mathematical activity (conceptual rule). The *degree of freedom* is apparent in the conceivable variations of how children (re-)interpret materials and their view on differences, as well as how they change numbers.

By means of the selected scenes, two types of play can be reconstructed, namely playing the game WHM and therein realizing mathematically-centered play with numbers, materials and interpretations.

CONCLUSION

The selected sequences of Maya and Leon indicate that the playing environment "Who has more?" offers rich possibilities to discuss interpretations of relations of differences. The epistemological analysis shows that children use individual reference contexts to compare two structured numbers of counters. However, these reference contexts are not used constantly, but rather are varied and become more sophisticated. This is reflected by the representations used and the ways of interpreting differences. As representation for differences, children use:

- a) dice patterns, covering dots to represent a second or third number

- b) blocks of five, to compare two linear structured (real or imagined) numbers

In addition, three different interpretations of differences could be worked out:

- a) Equalizing: The difference (d) is the change needed to achieve equal numbers of counters: $b=a-d$ or $a=b+d$.
- b) Matching: Equal amounts are matched to determine the number of counters that cannot be matched.
- c) Subtracting: The two amounts are related so that the smaller number is interpreted as a part of the minuend, which can be removed: $a-b=d$.

It becomes clear that the recurring new situations for comparing and discussing in the game provide occasions to use not only individual different strategies but also make situational different and new interpretations, which become more differentiated (Nührenbörger & Steinbring, 2009; Stebler et al., 2013).

The analysis of the playing activities in the game also provided evidence that "Who has more?" not only enables "Mathematics Embedded in Play" but also allows "Play Centering on Mathematics", even while playing the game (Ginsburg, 2006). This mathematically-centered play can be distinguished from the social play where mathematics is embedded, in that it involves another intention (instead of winning, the focus is on varying numbers, materials and interpretation), it follows other rules and children are involved in other roles (explorer instead of competitor) (van Oers, 2014). Essential for mathematical play is the reinterpretation of the game materials as representations of mathematical relationships. Mathematically-centered play gains space in the process of playing the game. Thus, the space for mathematically-centered play is always limited by the game. To maintain the game process, there are only short periods for mathematically-centered play; for example, a new rolling of the dice interrupts the play with mathematical objects. If children's play stays focused on mathematical relations, e.g. they try to find further number pairs with a given difference, the game fades into the background or disappears.

Hence, connecting points for the arrangement of learning processes in primary school can be deduced: here, the mathematically-centered play can achieve more space in the context of a learning environment in primary school. Accordingly, children's experiences to construe and construct differences can be picked up, to play with the same material but independent from game, to construct differences, namely finding number pairs with a given difference.

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ENDNOTE

1. German kindergartens (depending on the federal state) normally have recommendations but not an obligatory curriculum for mathematical education. Compulsory education begins with school entry for children aged 5 to 6.