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# Preschool teachers' understanding of playing as a mathematical activity

Christina Svensson

Malmö University, Malmö, Sweden, [christina.svensson@mah.se](mailto:christina.svensson@mah.se)

*In this paper, preschool teachers' documentation of their learning about playing as a mathematical activity are examined using Wartofsky's three levels of artefacts, primary, secondary and tertiary. Playing is one of Bishop's six mathematical activities and was a new consideration for most teachers and in contrast to how play is generally conceived as a tool for learning. Wartofsky's three levels of analysis provided insights into how the teachers were able to visualize and understand playing as a mathematical activity.*

**Keywords:** Artefact, mathematical activity, preschool, preschool class.

## BACKGROUND

In this paper, I analyse preschool teachers' reflections on their learning about Bishop's (1988a) mathematical activity Playing using Wartofsky's (1979) three levels of artefacts. This investigation is a part of a wider study investigating preschool teachers' mathematical learning in a professional development project designed around the introduction of a revised curriculum (Skolverket, 2010). The project involved teachers completing written documentation about their reflections and this paper explores how this documentation was analysed. According to the Swedish curriculum (Skolverket, 2010), the preschool teachers are responsible for that activities in groups of children are performed in a way to stimulate and challenge children in their mathematics development. Skolverket (The Swedish National Agency for Education) (2008) highlighted how, ten years after instigating a curriculum for preschools, mathematics still had an inconspicuous position in preschools.

Previous research has suggested that preschool teachers perceive mathematics only to be about counting and measuring (Clements & Sarama, 2007;

Ginsburg, Lee, & Boyd, 2008). Consequently, there have been suggestions that mathematics in preschool should include patterns (Björklund, 2008), and geometry (Clement & Sarama, 2007; Ginsburg et al., 2008). More broadly there is an argument made in favour of encouraging preschool children to think and make many mental relationships rather than to teach them specific subject content (Kamii, Miyakawa, & Kato, 2009). Counting, measuring, patterns and geometry are mathematical content and do not necessary include the expectation of mathematical thinking, such as Playing. This is in contrast to curricula such as the Swedish preschool curriculum (Skolverket, 2010) which emphasis mathematical thinking. In a background document to the curriculum (Utbildningsdepartementet, 2010), mathematics in preschool is discussed on the basis of Bishop's (1988a) six mathematical activities Counting, Measuring, Locating, Designing, Playing and Explaining, which perceive mathematics as a cultural activity, developed in all cultures. Bishop (1988a) considered the activities to be processes that lead to the development of mathematics. These six fundamental activities, he claimed are universal for two reasons. Firstly, because they seem to have been performed by each culture group that has ever been studied, and secondly because they are both necessary and sufficient for the development of mathematical knowledge.

Bergen (2009) asserted that Play can be defined as the medium for learning process for all ages because many qualities of play enhance learning process. She continues with saying that play is valuable for children primarily because it is a medium for development and learning, and it is important to understand that playing and playfulness is a quality which is valued by mathematicians, engineers and scientists. Helenius, Johansson, Lange, Meaney, Riesbeck, and Wernberg (2014b) also discuss that Play is an important means

for learning and continues by saying that; with for mathematicians Play describes as a necessary component of their creativity in problem solving. Thus, enhanced playful learning at every level of education that prepares professionals for scientific, mathematical and engineering fields is warranted (Bergen, 2009). Playing is the mathematical activity, which deals with aspects of mathematical thinking. Bishop (1988b) considered that Playing involves designing and participating in games and pastimes, which have more or less formalised rules which all players must follow. Bishop (1988b) included games for adults and claimed that Playing is an important activity for developing mathematical thinking for all ages. He considered Playing as characterized by thinking hypothetically (imagining a potential action to take in the game and is the beginning to think abstractly), modelling (abstracting something from reality) and abstracting (identifying the relevant features to focus on within a situation), guessing, estimating, assuming or adopting.

The role of play in education is a major concern of early childhood educators (King, 1979) and so even in Sweden. Play has a long history in preschool curriculum in Sweden, which could be the means of that preschool teacher are unlikely to naturally connect it with mathematical thinking. Wernberg, Larsson and Riesbeck (2010) claimed that the early learning of mathematics needs to be problematized so that it not only consists of numbers and calculations. They argued that the learning of mathematics in preschool should be based on playing. They considered Playing to be the most important means for learning in preschool, because it promoted interactions between teachers and children or between children and children. But the concept of learning by Playing needs to be more nuances to be categorized as a mathematical activity Playing (Bishop, 1988b). Helenius and colleagues (2014b) mean that, for play to be considered mathematical, it must include all or most of the following three parts. The first is that the participants must abide the implicit or explicit rules of the play. The second is that if the rules change there needs to be negotiation by participants. The third is that negotiating the rules contributes to forming the boundaries of the play situation and thus what aspects of reality can be suspended and what aspects are modelled in what ways. "However, young children are unlikely to know the rules of mathematics so for play to count as mathematical, there must be abiding by group nego-

tiated rules, but these may not necessarily be about mathematical content knowledge per se" (Helenius et al., 2014, p. 7).

As a result, it is possible that Swedish preschool teachers have difficulties understanding Playing as a mathematical activity. In research about the impact of a professional development project based on Bishop's (1988a) six mathematical activities, Helenius, Johansson, Lange, Meaney, Riesbeck & Wernberg (2014a) noticed that preschool teachers focused more on counting and measuring. Very few teachers labelled the activities that they described as Playing. They argued that changes in teachers' understanding take time and "a more explicit discussion of Bishop's six activities could be beneficial for future professional development programs" (p. 10).

Previous research on professional development in Sweden shows that, teachers claimed that they wanted to acquire extended knowledge and understanding of representations, and ideas in many different areas in mathematics (Doverborg, 2006). Furthermore, she found that continuous reflection with colleagues about videos, photographs, observations and interviews contributed to teachers' extending their knowledge and views on mathematics and pedagogical discussions. Alnervik (2013) focused on sharing and discussing pedagogical documentation between colleagues as a tool for visualising and developing their teaching practices. Pedagogical documentation is mentioned in the foreword of the curriculum as a way of making preschool practices visible and subject to discussions and evaluations of the quality and quantity of preschools (Skolverket, 1998). However, it is only when documentation is used for reflection that it becomes pedagogical, which has also been emphasized in published literature to provide guidelines for the use of documentation (Taguchi, 1997).

This research investigates teachers' documentation of their reflections on their learning about Playing as a mathematical activity. In the next section, Wartofsky's (1979) three levels of artefacts are discussed, as Alnervik (2013) used this in analysing the pedagogical documentation of preschool teachers in her research. The professional development was a project for developing preschool teachers' understanding of mathematics, including Playing as a mathematical activity. The project provided data for my research when it was necessary to determine

whether teachers faced a conflict between Playing as a mathematical activity and other conceptions of play in order to improve the possibilities for learning opportunities within the professional development. Thus, the research question is: *How can Wartofsky (1979) three levels of artefacts in the analyses inform how preschool teachers developing understanding of Playing as a mathematical activity?*

### WARTOFSKY'S THREE LEVELS OF ARTEFACTS

Wartofsky's (1979) three levels of artefacts have been used as tools for understanding the learning that occurs in mathematics classrooms (Hemmi, 2010; McDonald, Le, Higgins, & Podmore, 2005; Radford, 2008). There are several ways to classify artefacts (Säljö, 2005; Wartofsky, 1979). Säljö (2005) defined artefacts as tools that mediate between the individual and the social practice. He divided artefacts into two groups: *intellectual tools* like discourses and systems of ideas and *physicals tools* like texts, maps and computers. He also classified them into *primary tools* like a hammer and *symbolic tools* like those used for communicating ideas. Radford (2008) argued that we think with and through cultural artefacts, like the wooden ruler, the number line, and the mathematical signs on a piece of paper. A similar argument can be made for pedagogical documentation. Alnervik (2013) used Wartofsky's (1979) three levels of artefacts, primary, secondary and tertiary, to analyse teachers' pedagogical documentation to identify different perspectives. Artefacts, such as a die, a photograph or a film about a mathematical activity are unlikely to support changes in teaching practices without a task, a reflection and a conversation between colleagues and a re-reflection for reconstructing or developing the educational work focusing on the activities (Alnervik, 2013).

Following the Soviet tradition, Wartofsky (1979) focused on historic-cultural objects as artefacts. His interest was to define the ways humans worked to create cultural artefacts. He claimed that perceptions are culturally conditioned and represent a person's internal mental model which, through different representations, indicates that possible changes have occurred. Wartofsky (1979) connected the function of tools for use in internal mental models of human understanding by going from the practice and use of the tool to the theoretical and imaginative understandings of the tool. He provided a strategy

to do this by separating artefacts into three levels, primary, secondary and tertiary, although he also acknowledged the importance of linking the three levels. The artefact or tool itself does not determine whether it is primary, secondary and tertiary, but rather how it is used.

*Primary artefacts:* These are tools, which are being used, in a special context. To utilise the tools means possessing the knowledge of how the tools operate, including the skills needed to operate them.

*Secondary artefacts:* These tools act as a model or pattern in which the artefact is used to describe how people are supposed to do something i.e. seeing something that was not apprehended earlier.

*Tertiary artefacts:* These are tools or aids for thinking, where the artefacts help people to see the environment in a special "new" way. These artefacts can be considered as a way to "create, understand and analyse the world" (Säljö, 2005, p. 98). Hence, new ways of viewing the world can be discovered and these can contribute to altering and understanding a practice in a new way (Alnervik, 2013).

### METHOD

This paper reports on a part of a larger study aiming at investigating how preschool teachers use their understandings of the six mathematical activities (Bishop, 1988a) in their teaching practices during a professional development programme. This programme was part of Matematiklyftet (Skolverket, 2013) and used materials developed specifically for teachers in preschool and preschool class. Preschool class is a "bridging" year between preschool and school and school starts in the year when children turn seven, preschool is for one to five year olds. The material is based on collaborative learning (Timperley, Wilson, Barrar, & Fung, 2007) and Bishop's (1988a) six mathematical activities. Ninety preschool teachers and preschool class teachers, divided into four groups, actively participated in this project during a period of eighteen months, 2013–2014. The mathematical activity Playing (Bishop, 1988a) was the focus of three meetings with the teachers. The teachers were given mathematical tasks and questions to discuss, in order to develop their understanding of Playing. Before the first meeting the participants were supposed to have read the prescribed texts and looked at video(s), from



the professional development material. During the first two meetings, the texts and videos were discussed and between the meetings the teachers implemented tasks in their preschools that could be linked to *modelling*, *abstraction* and *hypothetical thinking*.

Following Helenius and colleagues' (2014) advice about the need for more explicit discussions of Bishop's six activities, teachers got the opportunity to discuss mathematics in preschool and preschool class with their colleagues at the professional development. To base the discussions on teachers' own practices, the teachers brought with them documentation in the form of photographs or videos. Every third meeting was a reflection meeting where the teachers discussed their documentation. During these discussions the teachers used a reflection protocol as support for their discussions so that they could further develop the mathematical activities. As one of the facilitators of the professional development, I kept my own notes of the meetings and these were also analysed.

In this paper, the purpose is to look at how Wartofsky's (1979) three levels of artefacts were used to identify what understanding of Playing as a mathematical activity teachers displayed in their documentations during the professional development. Bishop's (1988b) description of modelling, abstraction and hypothetical thinking as key features of Playing was used in identifying how teachers visualises this mathematical activity in their work in preschools. Wartofsky's (1979) three levels of artefacts were chosen as a way of seeing whether a conflict between understandings of Playing and other conceptions of play in the Swedish preschool was evident in the teachers' pedagogical documentation. In order to operationalise Wartofsky's (1979) three levels of artefacts so that they could be used in analysing the teachers' documentation, descriptions of how they could be used in the data analysis were developed:

*Primary artefact:* Tools as a photograph, video recording or a note from the teacher to visualise Playing as a mathematical activity.

*Secondary artefact:* Tools in the documentation from the teacher, where the notes described what occurred in photographs or video recordings in form of the three concepts; modelling, abstraction and hypothetical thinking.

*Tertiary artefact:* Tools that aid teachers' thinking to see the environment in a special "new" way of Playing as a mathematical activity.

## ANALYSIS

In the following sections, I provide a description of a teacher's documentation and how it was analysed using Wartofsky's (1979) three levels of artefacts.

### **Pedagogical documentation as a tertiary artefact**

A teacher in a preschool class chose to video record a play situation where three children, six years old, played "doctor". This is a common play situation in preschools and preschool classes and models children's perceptions of a real hospital. Consequently it was considered to be an example of the *modelling* component of Playing as a mathematical activity. In the video, one child is a patient, one child was a doctor, and one child was a nurse and secretary. The teacher talked to the children and asked them questions during the episode captured on the video.

The teacher discussed with the children different aspects of Playing by asking questions. From this, she then added comments to the documentation.

The teacher asked the children question about objects they were using as part of their hospital game but which did not necessarily look like they did in reality. Such a situation occurred when the child, who acted as a secretary, sat down at the table and pressed the buttons of a calculator. The child answered that the calculator was a typewriter on which he wrote prescriptions. This was analysed by the teacher as illustrating the *abstraction* component of Playing because the child had only focused on specific aspects of a typewriter, the buttons, as being important for what he was doing. After that, the teacher asked the doctor about the patient's condition. The doctor informed the teacher the patient had a very high heart rate, so he had to stay at the hospital for a few days. When the patient heard that his condition was very serious, he lay down on the sofa again. The child who was acting as the patient imagined what it was like to have such a bad condition so he followed the rules of the game situation in order to ensure that he received further treatment from the doctor. This was considered to be an example of *hypothetical thinking* because he had to work out what would be expected

of him, if he really was a patient with a bad heart condition the rules changes and there was negotiation by participants.

In her documentation, the teacher showed how her choice of implementing this situation and her conversation with the children, illustrated how she had perceived and worked with the three components of Playing as a mathematical activity. At the reflection meeting, my notes suggested that she had observed this play before, but had not seen it as a mathematical activity. During the reflection meeting, she also, of raised how she felt about her own presence during the children's "free play", and how this affected her further development and planning of this situation.

Her documentation of Playing was analysed as a tertiary artefact, as the teacher showed that she had gained new understanding, that might contribute to her changing how she saw this situation in the future Playing by providing her with new ways of understanding what she did.

### **Pedagogical documentation as a secondary artefact**

The example of a secondary artefact comes from the documentation of two teachers who decided to work together. They took turns in participating in the children's free play and simultaneously photographing it. The teachers provided texts to accompany their photographs about what the children said and did.

However, they did not mention modelling, abstracting and hypothetical thinking (Bishop, 1988b) in their documentation. The children did discuss how chairs should be placed in order to resemble a ferry and in so doing organized the chairs from a model they had been aware of earlier and the rules contributes to forming the boundaries of the play situation. This situation could have been identified by the teachers as *modelling*. The children also put toys in a swimming pool to represent a pool on the ferry that could give massages. This second situation could have been identified by the teachers as *abstraction* like aspects from the reality. Later, the children informed the teachers that a chair was missing and it meant that not all the children could get a massage at the same time. This last situation could have been identified by the teachers as *hypothetical thinking* because of

how the children pretended that the game could not continue if not all of the children participated.

This documentation was categorised as a secondary artefact because an individual reflection from the teachers on their roles, which was present in the first pedagogical documentation, was left out. The notes that they made allowed them to explain what the children had done but not reflect on it, thus making it a secondary artefact. Without the reflection component, it seems that teachers can remain unaware of how the components of Playing were present in the situation. It is interesting to note that when the teachers presented their documentation to their colleagues at the reflection meeting, they then could discuss the different aspects of Playing in the situation but this was not part of their documentation.

### **Pedagogical documentation as a primary artefact**

In this documentation, the teacher video recorded a situation where a four-year-old child was supposed to sort material, containing teddy bears in different colours and sizes. The teacher asked questions during the documentation, which were related to the numbers and the sizes of the teddy bears, such as "how many teddy bears do you have?" and "do you have more orange teddy bears than red?" However, the child played with the material and responded with statements such as "this is a ring [of teddy bears], dancing around a Christmas tree". However, this situation could be an example of *modelling*, as the child presents a possible model of a real-world situation. The child continued with "I get to do this instead when I throw out the Christmas tree" which could visualized be a *hypothetical thinking* as she presents a possibility of something occurring. The teacher used the video camera, as a direct tool to produce a description of a mathematical situation or a play situation with math material. In the documentation, the situation was neither explained nor reflected upon. Hence, it was categorised as a primary artefact, a tool as a video used in a specific context to visualize mathematic in an organised situation.

## **COMPILATION OF THE RESULTS**

A total of thirty-seven sets of documentation were collected from the teachers. Several teachers chose to do the documentation together, others chose to resist. About a quarter of the documentation were

categorised as primary artefact (Wartofsky, 1979). In many of these, it emerged that the teachers perceived the situations as playful environments which they connected to mathematical learning situations. In other documentation categorised as primary artefacts, the teachers described the situation as a representation of Playing (Bishop, 1988), similar to sorting the teddy bears, but which contained no explanation or reflection. The rest of the collected data was analysed as secondary and tertiary artefacts.

During the two meetings before the reflection meeting, when Playing (Bishop, 1988a) was discussed in different ways, the teachers expressed that they had difficulties to seeing the children engaged in modelling and the abstraction. The facilitators therefore chose to focus the meetings on these two parts, by showing many different representations of modelling and abstraction. However, in the results it is hypothetical thinking that many educators had difficulties with seeing. Consequently many of their sets of documentation could not be classified as indicating a tertiary artefact, because this lack of awareness hindered the teachers from changing their understanding about their practices.

Many of the teachers found that it was in the collegial learning, when they discussed their documentation of their own activity that contributed to opening up their view of Playing as a mathematical activity. In addition, the teachers identified the importance of the facilitators being involved in the discussion of their documentations in the reflection meeting in order for them to see Playing as a mathematical activity.

## CONCLUSION

Analysing how teachers in preschool and preschool class were able to visualize Playing as a mathematical activity is very complex. By using Wartofsky's (1979) three levels of artefacts primary, secondary and tertiary it became possible to detect how preschool teachers developed their understanding of Playing as a mathematical activity. Helenius and colleagues (2014a) showed that teachers struggled to catch the sight of the mathematical activity Playing in their practice. The same phenomenon appeared in the documentation from this professional development. This suggests that teachers' conceptions of play were connected to the curriculum understanding that learning occurs through play or that mathematics maybe is only about

counting and measuring. Wartofsky's (1979) three levels of artefacts could inform analysis of preschool teachers developing understanding of play as a mathematical activity. More research in this area is needed on how the mathematical activity Playing (Bishop, 1988b) is made visible and teachers understanding of the activity to enhancing playful learning in early childhood education for prepare professionals in the scientific, mathematical and engineering fields.

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