Learning interventions supporting numerosity in three year old children

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This paper reports from a preliminary qualitative case study and accompanying theoretical considerations in preparation for a study of interventions designed to support the learning of numerosity in three year old children in a Norwegian kindergarten. Teaching interventions are analysed and discussed both from the perspective of children's learning and the teacher's mediation. Luis Radford's theory of objectification is used as theoretical framework. This is a new approach, as most research papers in the field use quantitative methods and a cognitive approach. Our study indicates that preschool teachers face challenges in taking the perspective of the children they teach, and that knowledge of children's learning phases and assessment of children is needed to improve teaching.

Keywords: Numerosity, objectification, give-N task, parallel individuation system.

INTRODUCTION

Already at the age of about two children begin to learn the sequence of counting words, but further development takes time. More than a year is often needed from when children can count small sets of objects properly until children respond correctly to questions involving the use of numbers (see Sarnecka & Carey, 2008, p. 664). We hypothesize that this time can be reduced by planned learning interventions conducted by preschool teachers or other competent persons. In a preliminary qualitative case study over a period of three months we followed learning interventions in a Norwegian kindergarten conducted by a preschool teacher who was supervised by two researchers. Twelve children took part more or less throughout the study. We emphasised activities which use and give meaning to counting and number, but also the learning of counting skills. The interventions in the preliminary study started up with low quality. During the study we observed improved instruction and many signs of learning. Interventions of mixed quality proved much better than no interventions at all. A follow up study is planned and will build on the findings from the preliminary study.

Norwegian children can attend kindergarten from the age of one until they start school the year they turn six. As much as 90 % of Norwegian children between one and five years old, attended kindergarten in 2013. As remarked by Hundeland, Erfjord and Carlsen (2013), "the Norwegian kindergarten is regarded by OECD (2006) as situated in a social pedagogy tradition, i.e. an educational institution where core enterprises are upbringing, care, play and learning". Norwegian preschool teachers usually do not support children's learning of counting and number through planned and systematic interventions. Now and then they initiate simple counting activities, and sometimes they initiate conversations about numbers with children based on free play situations. The preschool teacher in our study does not have mathematics as a subject in his education. However, since 1995 mathematics has been compulsory in preschool teacher education in Norway.

From the adult expert perspective, numerosity is the same as the cardinality of sets, but we use the former term to underline the very different perspective of the child and the educated adult. The learning of numerosity means to catch the meaning and cultural use of number. This is complex and diverse, starting early in children's lives. In the research literature, the understanding of the numerosity N has been operationalized by the so called give-N test (Wynn, 1990; 1992; Sarnecka & Carey, 2008). In give-4 the child is asked to retrieve four objects, for instance plates, from a heap or a larger set. Research has shown the give-N tasks to be far from obvious for the child. Wynn (1992) found a progressive development in children from

first managing give-1, then give-2 and so on up to give-5 or give-6. This has been confirmed by several others (see Sarnecka & Carey, 2008). When give-5 or give-6 is mastered, children seem to respond correctly to give-N, for N as large as the child is able to count. We will analyse excerpts of interaction of such challenges both for the children and for the preschool teacher trying to support them. Finally, possible improvements in the design of interventions will be discussed.

RADFORD'S THEORY OF KNOWLEDGE OBJECTIFICATION

The research literature on young children's acquisition of number has been dominated by psychologists, cognitive scientists and neuroscientists. Their perspective is cognitive with emphasis on understanding, mental models and what happens in the brain, and they use quantitative methods. The give-N activity has been seen as a test of "understanding the cardinal principle" or being a cardinal principle knower, Sarnecka and Negen (2012). The cardinal principle was introduced by Gelman and Gallistel (1978), and states that the last counting word in a correct applied counting procedure, is the cardinality of the set. The latter refers to a concept as possessed by mature adults. We claim that the child does not experience concepts, but activities which gradually become mastered and turned into procedures. To possess numerosity as a concept comes much later in the child's development. In place of speaking about understanding, the noticing of regularities in and connections among the activities is emphasized. This approach is based on Luis Radford's variant of cultural historical activity theory. In his theoretical framework, mathematical objects exist in the culture among us. According to Radford (2008, p. 222), "[...] mathematical objects are fixed patterns of reflexive human activity incrusted in the ever-changing world of social practice mediated by artifacts". Both counting and give-N activities are examples of cultural activities. Objectification is the crucial moments of the child's learning when the child notices important aspects hidden in cultural activities.

> The term objectification has its ancestor in the word *object*, whose origin derives from the Latin verb *obiectare*, meaning" to throw something in the way, to throw before". The suffix – *tification* comes from the verb *facere* meaning "to do" or "to make", so that in its etymology, objectification becomes related to those actions aimed at bringing or throwing something in front of somebody or at making something apparent – e.g. a certain aspect of a concrete object, like its colour, its size or a general mathematical property. (Radford, 2006, p. 6)

The child does not notice an abstract property, but a pattern or regularity in a cultural activity involving language, physical actions and artefacts. An example is the give-3 activity where a child is asked to fetch three plates from the shelf. A child can take part in such activities before being able to do it alone. During the learning process what children have objectified can vary significantly. Following Radford (2005), we use the concept 'layers of objectification' to classify alternative objectifications of the same activity. The original use by Radford was to study how students in school objectify the generality of geometrical number patterns. Later on Lorange and Rinvold (2014) have applied the theory to the study of students' strategies in expanding fractions to a common denominator. One layer of objectification of give-3 is when a child is using 1–1 matching to take one plate for each member of his family. A more advanced layer of objectification for the same activity is to remember the number word three and take one plate for each of the three first counting words. Different layers correspond to different strategies for doing the task depending on the regularities, skills and semiotic tools the child has grasped. Counting aloud, silently or using their fingers, are variants which give rise to more finely grained layers of counting. Layers are not necessarily linearly ordered. We have seen the same child demonstrate different layers of objectification of counting within one learning session. An example is perfect counting of six toy cars and recital of the counting words one-two-three when asked how many noses the boy has. Fragile and uncertain mastery and understanding in a learning phase is one possible explanation, but also social uncertainty in responding to demanding tasks together with other children.

Children who master the give-N task for all N, or in practice 5 or 6, are called cardinal principle knowers or CP-knowers. According to Sarnecka and Negen (2012), recent empirical studies have found that CP-knowers show an implicit understanding of succession and equinumerosity. The competency of succession is described as knowing that adding one item to a set, means moving one word forward in the counting sequence. Sarnecka and Carey (2008)

found that CP-knowers performed well on two tasks intended to operationalize this competency, and that the non-CP-knowers performed only slightly better than chance. Sarnecka and Wright (2013) came to a similar conclusion for a task intended to measure the competency of equinumerosity. The latter means that two sets with a 1–1 correspondence must be labeled with the same number word. These results indicate that objectification of give-N is an important milestone in the learning of numerosity.

THE PARALLEL INDIVIDUATION SYSTEM

Children younger than one year have been shown to be sensitive to small numerosities. This ability does not, however, mean that the children posit the concepts of the numbers one, two and three. Visual discrimination does not by itself gives understanding of language. *Verbal subitizing* (Benoit, Lehalle, & Jouen, 2004) in which a child immediately gives a number word when shown a set of one, two or three concrete objects has to be learnt. This learning process does not have to be difficult, but is complicated by the fact that many children meet the number words only in counting and not in number talk. Some children may never have been shown for instance three dolls and told that this is three dolls. Subitizing is believed to be based on what is called the parallel individuation system in the brain. According to Sarnecka and Negen (2012, pp. 247–248), this system represents and tracks individuals (objects, sounds, or events). Concrete objects and their properties are represented, so the parallel individuation system is not an abstract number system.

> This system privileges spatiotemporal information to initiate a mental index, or object file, for each item. Although inherently non-numerical in nature, these representations afford numerical content by retaining information about numerical identity – mentally stored items can be compared on a one-to-one basis with visible objects in the scene to detect numerical matches or mismatches. (Hyde, 2011)

Learning the colour red is mediated by pointing to red things and saying for instance "this apple is red". We expect that verbal subitizing can be learnt similarly, but that generalization is somewhat more demanding than for colours. The parallel individuation system does not let the child see twoness, but the mental image of for instance two dolls. The possibility of comparing a given set of concrete objects to some standard visual representation makes generalization possible. Mental images also let the child solve some matching tasks without numerical thinking.

METHODOLOGY

In the preliminary case study we followed twelve children in a Norwegian kindergarten for a period of three months. The kindergarten was chosen because it is located in reasonable distance from the university college where the researchers work, and has a preschool teacher interested in cooperating in this project. The participants were all the children between three and five years with no known learning difficulties who wanted to take part and whose parent granted permission. At the start of the study the ages varies from 3 years 2 months to 5 years 7 months. The children took part in weekly intervention sessions of 10 to 20 minutes duration. In the first part of the study as many as seven children took part in each session. Later on some sessions had only three or even two children. All sessions were videotaped and observed by at least one of the researchers. An experienced male preschool teacher conducted all the sessions. Sometimes an assistant was present too, whose contribution was limited. Before each session the researchers gave the preschool teacher an outline of what to do, but he had quite a bit freedom and was not given precise instructions. The content of the sessions were different kinds of counting tasks and activities. Most often one child at a time completed a task or answered a question. In the paper we study samples with the four youngest children, two boys and two girls. They were Siri: 3 years 2 months, Nina: 3 years 6 months, Ole: 3 years 7 months, and John: 3 years 10 months. No testing or systematic assessment of the children was done, but all of them had some previous experiences with counting. Already in the first session we noted that Ole and John were relatively poor in counting.

ANALYSIS OF LEARNING INTERVENTIONS

We present selected excerpts from three sessions, focussing on one task from each of them. Each excerpt is a dialog between the teacher and one child. Information about movements, gestures and other relevant facts is put in square brackets. The tasks are analysed from both teacher's and child's perspective.

The pure give-N task

In this task a piece of paper with a numeral printed on it is laid on the floor. One of the children is asked to say which number it is, and then the teacher repeats the number. One child is asked to collect that number of plastic bears from a bucket and then place the bears beside the numeral. We follow the first give-N task given to Siri.

Teacher: It's the number four. Can you fetch that many bears?
Siri: [Collects two bears without any signs of counting and puts them on the floor]
Teacher: How many bears have you put on the floor? [He moves one of them a bit and makes counting gestures towards each bear] How many is this?
Siri: [after some silence] Two [without visible or audible counting]
Teacher: one – two [while pointing to each of the bears]
Siri: two – three

The teacher thinks that counting will help the child, but Siri resorts to a more primitive objectification layer of counting. When the teacher says one – two, she turns back to the practice of reciting the number words. The teacher emphasises counting, and he probably expected her to count aloud before answering "two". But, her answer was satisfactory. Both verbal subitizing and counting internally are legitimate strategies. He could have challenged her better by taking away the bears and giving her a give-2 task.

The collecting plates to the bears task

One week later Nina is part of the group in place of Siri. Now the children are given a situation which is modelling an everyday activity. This enables them to rely on information and strategies they already know.

Teacher: In one of the houses live a mammy bear, a daddy bear and a tiny baby bear. [moving the mammy bear] Now you must come and eat. Can you Nina come and fetch a plate to each of the green bears. The green bears must have one plate each. [while holding a transparent plastic bucket with small circular plastic pieces]
Nina: [Takes four plastic pieces from the bucket, one at a time, without looking at the bears. She loses one piece on the floor, begins to retrieve it, but stops.]
Teacher: They must have one plate each.
Nina: [She lays down one plate for each bear] porridge [takes the last plate and gives it to a bear belonging to another family of bears]

Nina is not looking at the bears while collecting plates, so she either takes them randomly or relies on some other information. The former is less probable, since she is taking time collecting the plates. The individuality of the bears can be remembered by the names mammy, daddy and baby or their medium, large and small sizes. This is within the limit of the parallel individuation system, so 1–1 matching based on memory is possible. That she takes one plate too many, may indicate that she relies on numerical information, but has not developed a firm grasp of numerosity.

As number words are not mentioned, this is not really a give-N task. It could have been so if the teacher had introduced number words, for instance by asking the child how many bears there are in the family and how many plates are needed. Later on in the same session Ole gets a give-2 task in which the number word 'two' is used both before and after the task.

Teacher (T): The bear family got visitors. [He places two bears beside the green family] The mammy bear asked if the two visitors also wanted porridge. Can you put on plates to them Ole?
Ole and T: [Ole takes some plastic pieces from the bucket. Simultaneously, the teacher repeats "place plates to the two, two".]
Ole: [Ole looks at the bears and returns some plates to the bucket until he has two. Then he walks to the bears, takes one plate in each hand and places one plate in front of each bear.]
Teacher: Excellent. There they got two plates.

That Ole first collects several plates in the bucket, may be because the visual presence of five bears overrides the number word 'two'. The teacher's repetition of the question initiates a change to collection by visual 1–1 matching. One week previously Ole was successful with give-2 tasks without direct visual support. The available visual resources lead Ole into a more primitive layer of objectification of give-2.

In the same session, John, the mathematically weakest child, gets a give-4 task.

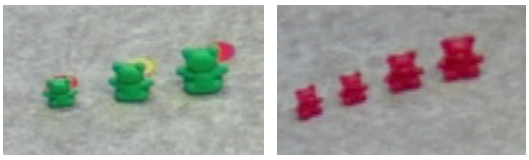

Figure 1: The green and red bear families

Teacher: John, the red ones must also have plates. [points to each of the plates in front of the green bears]
John: No. [takes three plates one by one from the bucket, staring into the bucket, places one plate carefully in front of three of the bears]
Teacher: How many have you there? Can you count them?
John: [points twice to the plate in front of the first bear]
Teacher: You must count them. [Counts to three aloud while both he and John point to each of the three plates] How many more plates do you need then? One [while John is taking one plate from the bucket]
John: [Places the plate in front of the fourth bear, then looks at the teacher]
Teacher: Good!

Since John is weak in counting and numerosity, it is unlikely that he relies on numerical information when taking out three plates. The sizes of the red bears (Figure 1) indicate that the family has two adults and two children, but this information does not seem to be used. A likely explanation is that the parallel individuation system is utilized, but that the capacity limit of three makes John unable to take four plates at first. The second time he correctly takes one plate, but this does not necessarily mean that John is a 1-knower. The teacher's utterance 'one' comes while John already is about to take one plate, so it is likely that John's action is based on spatiotemporal memory.

The garage task

In a session more than one month later the teacher has placed a heap of cars on the floor beside a garage. Ole, John and Siri are sitting on chairs placed so that they see both the garage and the heap of cars. The garage task is a challenging give-N task giving less support and meaning. Parking places and cars are part of daily life, and children enjoy playing with toy cars. However, it is not natural at all to park a given number of cars, randomly chosen from a larger set. The children's interest in toys cars can also be a distracter. The attention of some children was lead toward the cars themselves rather on the numerical information they were given.

Teacher: Then I first want that John takes two cars [taps three times with his hand on top of the garage] and puts them on top here. Can you take two cars and put them on top of the car house?
John: [gets up, walks to the heap and almost without hesitation takes three cars one by one into his left hand. After a very short break he takes another car and holds the cars to his chest. Then he continues and takes a last car. Then he takes a long break while looking at the last car.]
Teacher: Two cars do we want up there [taps three times with his hand on top of the garage] two cars.
John: [rises and walks to the garage, then places all the five cars one by one carefully on top of the garage, then turns in the direction of his chair.]
Teacher: [gestures John to return] Can you count how many cars there are here? Can you count them together with me? [Takes John's finger] one – two – three – four – five [while he moves John's finger to each of the cars]
John: [starts moving towards his place]
Teacher: Wait a moment. How many should you have? Two cars
John: [Begins to take up one car from the heap.]
Teacher: [Stops him] We must take away some cars. [Takes away three cars] Can you count now? How many cars are placed on top of the roof now?
John: one – two [while pointing to the cars one by one. Then looks at the teacher]
Teacher: Two, yes. It was this you should have. Exactly, two cars

Possible explanations for John's inappropriate response to this give-2 task are that he ignores the word 'two', or that he interprets it as 'many'. He may also take the cars he liked, or he takes into account

how much space is available on top of the garage. The teacher's question of counting the cars is intended to draw attention to the numerical aspects of the task. John, however, again tries to return to his place, so this does not seem to be of any help. The next action by the teacher is to remind John that it is two cars he should have. This initiates John to start a new give-2 attempt by picking up a car. Unfortunately that attempt is refused by the teacher before we know what John would have done. John seems to have objectified give-N as a procedure for selecting some objects and putting them in a given place. If John had not been stopped, he may have taken up two cars and been rejected again because five cars already are on top of the garage. The teacher's strategy of reducing a larger set into a set of two members is far above the level of the child, and shows that he is not able to take the perspective of the child.

DISCUSSION

The most striking observation from the selected excerpts is the challenges the preschool teacher meets in mediating the children's learning. The same is true also for the rest of the videotaped interventions. When he tries to meet their needs, his response is often irrelevant and leads the children into more primitive layers of objectification. The level of tasks is not adapted to what each child has managed earlier. In order to help children struggling with the most basic tasks, the teacher should systematically remember or make written assessments of what each child has achieved so far regarding counting and give-N. More basically, the teacher needs to be aware that the child experiences cultural procedures and does not possess conceptual knowledge of numbers.

One conclusion is that give-N tasks are of different kinds and of varying difficulties. In the most demanding give-N tasks numerosity is given by a number word, and the child has to rely on that information in order to carry out the task. For the most demanding give-2 and give-3 tasks, verbal subitizing is a possibility if mastered by the child. In the easier tasks, the objects are visible to the child or have names supporting 1–1 matching while the child collects the objects. In the follow up study we plan first to give a child the easy type, but will also challenge the child with pure give-2 and give-3 tasks as preparation for the mastery of give-4. While working with the easy tasks, the teacher has to support the connections to audible and visible counting, but also give praise and appreciation to children who can tell the number of objects in sets by internal processes. Both internal counting and verbal subitizing are competences which are valuable in the learning of numerosity. Beyond the limits of the parallel individuation system, counting is indispensable. This is in accordance with Sarnecka and Negen (2012, p. 252), who claim that it is not possible to learn the meaning of large numbers in the same way as the small numbers have been learned. Similarly, Sarnecka and Carey (2008, p. 664) report that children who only master give-1, give-2 or give-3 (subset-knowers), do not use counting when solving these tasks, even when asked to do so.

In a follow up study we will take into account both the findings from this paper and other results which were not included here due to considerations of space. We will try both collaboration between children and more physical activities. Give-N has been described as collecting a set of physical objects. A variant of give-N is to repeat some physical movement a given number of times. This is used in collecting N objects, as the child fetches an object N times. Non-collecting examples are jumping N times or clapping N times. Board games in which the player moves a piece as many steps as a dice shows, is another possibility. Such a game may be tried with children who do not master the collection form of give-2 and give-3, by using a dice showing only one, two or three. To show the actual number of spots on a dice is an easier alternative than communicating N by a number word. Three different approaches can be combined by first asking a child to say the number of spots on a dice, and then apply counting to move a piece that many steps on a playing board.

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