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# Algebra in Dutch education, 1600–2000

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*Algebra became part of mathematics education in the Netherlands in course of the seventeenth century. At first in the form of classic algebra, but by the end of the century, the influence of the notation of Descartes was noticeable. In the eighteenth century, algebra was part of the basic curriculum of the Foundation of Renswoude. In the second half of the nineteenth century, algebra was seen as useful for a technical career. The number of topics in school algebra grew, but eventually algebra became mainly a subject in which complicated calculations were performed, which did not seem to serve a purpose outside the subject. At the end of the twentieth century, school algebra in lower secondary became a fairly informal way of solving ‘practical’ problems.*

**Keywords:** Algebra education, Leiden Engineering School, Foundation Renswoude, HBS.

## INTRODUCTION

When we consider examples of mathematics curricula in Dutch education it is obvious that the content of geometry didn’t change very much for a long time. Some problems on geometry in the lecture notes from the Leiden Engineering School, dating from 1620 (Krüger & Van Maanen, 2014), were recognisable for a student in secondary education around 1960. With algebra it is a very different matter.

In 1600 Simon Stevin wrote the *Instruction* for the Duytsche Mathematique, as the Leiden Engineering School was called. In the *Instruction* Stevin specified the content of the course. He mentioned arithmetic, specifying calculations with decimal numbers, geometry, surveying techniques and fortification, but no algebra (Krüger & Van Maanen, 2014).

In the second half of the 18th century the Foundation of Renswoude provided talented orphans with an education for technical professions. A large part of the curriculum consisted of mathematical subjects: arith-

metic, algebra, geometry (*Euclid*) and trigonometry (Krüger, 2014a). Algebra included operations with algebraic forms, formulating and solving linear and quadratic equations and also geometric, arithmetic and harmonic series.

About a hundred years later, in 1863, national legislation for secondary education was introduced by mr. J. Thorbecke, the liberal prime minister. The ‘Hogere Burger School’ (higher secondary school for citizens) or HBS as it was called, provided general education, with science and mathematics as important subjects and prepared students for admission to the Polytechnic School in Delft. The HBS was meant for sons of the middle class. Mr. Thorbecke advised on the content of mathematics for the HBS, the only subject on which he gave such an advice. He advised that algebra should contain quadratic equations, arithmetic and geometric series and the binomial theorem, not very different from the algebra in the Foundation of Renswoude. The HBS attracted many students. Towards the end of the 19th century the number of topics in school algebra had expanded considerably (Krüger, 2014b).

By the end of the 20<sup>th</sup> century the word ‘algebra’ had disappeared from textbooks, though algebra itself was very much part of the curriculum and caused a lot of debate in mathematics education (Drijvers, Goddijn, & Kindt, 2006).

*Which role did algebra have in mathematics education from 1600 until 2000?*

## METHOD

Information on algebra in curricula from the seventeenth, eighteenth and nineteenth century was collected in the course of research on factors and actors which influenced the content of mathematics curricula in the past, from 1600 to the present (Krüger & Van Maanen, 2014). Archives, manuscripts and textbooks

were an important source of data. For the twentieth century there are many sources: textbooks, articles in teachers' periodicals, the archives of the Ministry of Education, research journals, other publications on education research and curriculum development and the authors' own archives provide abundant data. For this paper only changes in the algebra curriculum are taken into account.

## ALGEBRA IN THE CURRICULUM: 1600–2000

### Seventeenth century: Leiden Engineering School, 1600–1681

Towards the end of the sixteenth century the Republic of the Seven United Netherlands had become rather successful in its war with Spain. The war was mainly fought around fortified towns, with help of new military techniques, which required mathematically well trained engineers. The expanding population in Holland and the strong economy also caused demand for military and civil engineers, surveyors and building masters. Prince Maurice of Nassau (1567–1625), one of the two commanders of the army and former student of Leiden university, arranged with the curators of Leiden university that an Engineering School would be attached to the university. Simon Stevin (1549–1620) wrote the formal curriculum, the *Instruction*, in which he stated the aim, the teaching language, the content, learning activities etc. (Krüger, 2010). The formal aim was to train military engineers as quickly as possible, the teaching language would be Dutch, hence the name 'Duytsche Mathematique' and the curriculum consisted of mathematical theory and practical exercises in surveying and in the basics of fortification. However, in a copy which the curators had made of the *Instruction* to have it printed, they slightly modified the aims into 'training in engineering or other mathematical professions'. The Duytsche Mathematique was going to be a course for engineers and also other mathematical practitioners.

Stevin did not mention algebra in the *Instruction*; evidently algebra was not yet useful for an ordinary engineer. But he mentioned the possibility of further study for those who had mastered the basic course in Duytsche Mathematique. Further study may have included cossic algebra, with different symbols for the unknown, its square, its cube, etc. and numerical values for known quantities (Figure 1), as used by Stifel in his *Arithmetica integra* (1544). The introduction by Viète, in 1591, of vowels for unknowns and consonants for known quantities, was an improvement, but the close link with geometry remained. Descartes used a more advanced notation in his *Géométrie* (1637) and allowed powers higher than third degree, which made algebra more useful (Figure 2).

The first teachers at the Engineering School were Ludolf van Ceulen, a respected mathematics teacher and advisor and Simon Fransz. van Merwen, a Leiden surveyor; not much is known about their interpretation of the curriculum. They both died in 1610 and were succeeded by Frans van Schooten Sr. (1581/2–1645), who was a student of Van Ceulen and Van Merwen and who had assisted Ludolf van Ceulen. His lecture notes in the library of Leiden university, *Mathematische Wercken* (BPL 1013) date from ca. 1620 (Van Maanen, 1987). They show the interpretation by Van Schooten of the *Instruction* and were also used by his successor, Frans van Schooten Jr. (1615–1660).

These lecture notes do not contain algebra, but the library of the university in Groningen possesses a manuscript by Van Schooten Sr., also a teaching text, on cossic algebra (Hs 443), which probably dates from ca. 1623 (Figure 1). The problems which are discussed in this manuscript were also used by Frans Jr. (Dopper, 2014). It is likely that Frans van Schooten Sr. taught cossic algebra in private lessons or to advanced students of the Engineering School. Frans van Schooten Jr. gave public lectures on algebra, using the cossic notation, though he knew the work of Viète and of Descartes very well. He produced a Latin translation

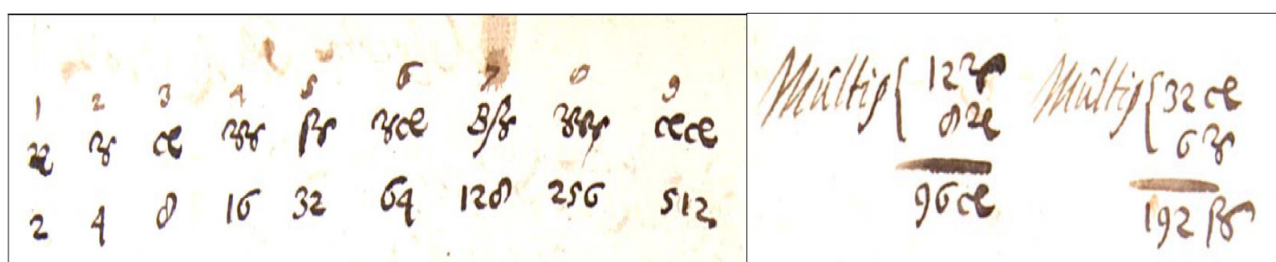


Figure 1: Hs 443, cossic symbols and examples of multiplication, by Frans van Schooten Sr.

of *Géométrie*, which made the work available to a much larger public.

Frans van Schooten Sr. was a mathematical practitioner, who worked as a teacher, surveyor and cartographer, also for the army. He had a good knowledge of mathematics and of the practice of surveying, mapping and fortification. Frans van Schooten Jr. was a very respected and well-known mathematician, but not so much a mathematical practitioner. He attracted a circle of talented private students, with whom he discussed Descartes' work. Dopper (2014) suggests that in his public lectures Van Schooten Jr. used cossic symbols instead of Descartes' notation for didactical reasons.

So there was some cossic algebra taught at the Engineering School, probably by Frans van Schooten Sr., certainly by Frans van Schooten Jr. At the start of the 17th century geometrical solutions for problems were common; gradually geometrical problems were more often translated into equations, algebra gained importance (Van Maanen, 1987, p.174–175). Van Schooten Jr. made occasional use of Cartesian notation in his *Mathematische Oeffeningen (Mathematical Exercises, 1659)*, published shortly before he passed away. In 1672 Abraham de Graaf (1635–ca. 1717), a teacher of mathematics, astronomy and navigation in Amsterdam, published *De beginselen van algebra volgens de manier van Renatus des Cartes (The principles of algebra according to Renatus des Cartes)*. In this book he treated the rules for operations, linear and quadratic equations and he gave many geometrical applications, but also other examples of use of algebra, using Cartesian notation (Figure 2).

So gradually algebra, with Cartesian notation, became more common.

However, surveyors, an important group of potential students for the Engineering School, were not particularly interested in new mathematical techniques, they did not feel a need to use algebra (Muller & Zandvliet, 1987; Morgenster, 1707; Van Nispen, 1662).

$$\begin{aligned}
 &xx \infty ax + bb. \\
 &x^3 \infty axx + bbx - c^3. \\
 &x^4 \infty ax^3 - bbxx + c^3x - d^4, \&c.
 \end{aligned}$$

Figure 2: Cartesian notation in Algebra, by A. de Graaf (1672)

## Eighteenth century

Several mathematicians contributed to the development of algebra during the seventeenth and eighteenth century (Grattan-Guinness, 2000; Struik, 1995). Examples are Wallis (1616–1703), who published his *Algebra* in 1685, Newton (1642–1727), Euler (1707–1783) and Lagrange (1736–1813). When in 1761 teaching started in the Foundation of Renswoude in Utrecht, algebra naturally was part of the curriculum. The Foundation was the outcome of the legacy of a Dutch millionaire, Maria Duyst van Voorhout, baroness of Renswoude. She bequeathed her capital to three orphanages, in Delft, The Hague and Utrecht, on condition that the money should be used to educate talented orphan boys, to prepare them for a technical profession (Krüger, 2014a). In her own words (translation by author):

...select some of the most talented and suitable boys, at least 15 years old, to set them apart from the orphanage in order to teach them Mathematics, Drawing or Painting Art, Sculpture or Stone Cutting, practices in building dykes to protect our Country against floods or similar Liberal Arts....” (Utrecht Municipal Archive: HUA 771, inv. 1).

The Foundation thus aimed to educate selected boys for a wide range of technical professions. This meant that a large part of the teaching was on mathematical topics, with drawing as the second most important subject. In Utrecht the governors of the Foundation left the decisions on what mathematics to teach to the mathematics teacher. However, they took care to find an excellent teacher, with a good reputation for knowledge content and for teaching. Laurens Praalder (1711–1793) had no academic background; he had been teaching mathematics, surveying, navigation and other subjects, in the North of Holland and as mathematician and examiner at the Naval School in Rotterdam. In 1761 he accepted the position of mathematics teacher of the Foundation in Utrecht. Praalder indeed was an excellent teacher, who integrated mathematical theory with practical work and did physical demonstrations with his students. Through his ex-

tensive network he managed to find useful apprenticeships for many of his students. He also attracted private students.

The curriculum was divided into three phases, the first phase lasted ca. 2 years and was the same for every student; algebra was one of the topics in this phase. Praalder used his own teaching notes for lessons, but the students were also provided with books on all topics taught. During the first years of teaching in the Foundation, the governors bought mathematics books for the first and second phase, after consulting with Praalder. For geometry the choice was fairly simple; an edition of *Euclid* by Warius and a standard work on geometry and surveying by Morgenster, this book also contained chapters on trigonometry and logarithms. For algebra the choice was more difficult, as is clear from the different publications which were purchased during the first three years, for 11 students. Eight copies of a Dutch translation of *Elemens d'Algèbre* by Clairaux, eight copies of *Elements of algebra* by Hammond and six copies of *Algebra* by Venema, a Dutch author. After some years Venema became the standard book for most students, it offered a small amount of theory and many exercises. As Praalder taught the theory from his own notes, the students could use the book by Venema to make the exercises. Theory treated the principles of algebra, linear and quadratic equations and series.

A similar program for algebra was taught in 1776 by professor Hennert at the university of Utrecht, though ten years later he had diminished the amount of algebra taught (Bos, 1984). In 1789 the first three military academies were established in the Netherlands, their mathematics curriculum was similar to that of the Foundation. One of the students of the Foundation was appointed as a tutor to the military academy in Breda in 1789 (Krüger, 2014a).

Evidently algebra was considered useful as a tool for solving problems; geometrical and other problems. Formulating and solving linear and quadratic equations formed a major part of the content in teaching. Negative solutions of quadratic equations were sometimes ignored, so were complex roots. For most students this would be sufficient knowledge for their future career.

After Praalder retired, his colleague at the preschool of the Foundation, Dirk de West, was asked to take

over the mathematics teaching in the Foundation. De West used copies of Praalder's teaching notes, including the four bands on algebra, for his lessons. In later years four copies of Euler's *Elements of algebra* were bought, probably for the more advanced students, who could not be taught further by De West.

### Nineteenth century

Characteristics of mathematics in the nineteenth century were: an increasing specialisation, more emphasis on the distinction between pure and applied mathematics and a striving for mathematical exactness (Grattan Guinness, 2000; Struik, 1995). The algebraic properties of numbers, the fundamental theorem of algebra, roots of equations, complex numbers and quaternions were some of the topics in which much development took place. The mathematics curricula of secondary schools were somewhat influenced by these developments.

In the Netherlands secondary education remained a matter of private initiative until 1863 (Krüger, 2014b). Industry and commerce required an educational system of good quality, which would offer subjects such as mathematics and science, modern languages and economics. Also the Royal Academy for Civil Engineers in Delft and the Royal Military Academy in Breda had problems with the relatively large number of students who entered with insufficient knowledge of mathematics and science. In 1863 the HBS, with a curriculum of five years, was Thorbecke's answer to these problems. The programme for algebra should enable the pupils to do well in the physics lessons. Also after the first three years of HBS the pupils should be able to pass the entrance exams of the Royal Military Academy and after passing the final exams a student should be ready for the Polytechnic School, the successor of the Royal Academy for Civil Engineers. The mathematics teachers at the HBS had to have a university degree or an equivalent teaching degree.

Mathematics was one of the 18 subjects taught in the HBS and one of the 16 subjects in the final exams. Mathematics was subdivided into several topics: arithmetic, algebra, plane and solid geometry, trigonometry and descriptive geometry. The final exams on mathematics consisted of four exams: arithmetic/algebra, geometry, trigonometry, descriptive geometry. Mathematical and science subjects took ca. 33% of teaching time, the same amount as the four languages and literature. Mathematics received ca. 18% of the

teaching hours, more than any other subject (Steyn Parvé, 1868).

Until about 1890 most students at the HBS did not take the final exams. Many students left school after the first three years, to sit for entrance exams for a range of professional institutions or to enter the workforce. The content of mathematics in the first three years consisted of arithmetic, algebra and geometry, usually 2 hours each in every year. During the first three years of algebra the pupils were taught the main operations, also with fractions; roots, powers, negative and fractional exponents; linear and quadratic equations and sometimes also equations of degree higher than two, exponential equations and indeterminate equations. In year four and five there was more difference between schools in the choice of topics. Newton's theorem (an extension of the binomial theorem), factorizing, combinations and permutations, infinite series, convergent series, continued fractions, limits, theory of irrational numbers and complex numbers were topics which were taught, but not in all schools. The exam syllabus, introduced in 1870, mentioned arithmetic and geometric series, Newton's theorem, indeterminate and exponential equations. Teachers and textbook authors included topics outside the prescribed exam syllabus for various reasons, i.e. the topic occurred in entrance exams of some institutions or the topic was of educational or didactical value. There were many different textbooks available, most of them written by teachers of HBS or gymnasium (the former Latin school).

By 1899 the authors of a popular textbook on algebra explained that they had not included the theory of irrational numbers, as there was no time to treat the topic with the necessary exactness (Derksen & De Laive, 1899). Gradually, the topics which were not in the final exams would be treated in fewer schools.

In the Polytechnic School algebra would be used in analytic geometry and in analysis. There were occasional complaints about lack of skills of first year students, but overall the results of the first year students in these topics were satisfactory (*Onderwijsverslagen*, 1866–1876 [Education reports to Parliament, 1866–1876]).

### Twentieth century

Groen (2000) presents a concise overview of changes in the mathematics curriculum of the HBS between

1900 and 2000. In 1937 a new mathematics curriculum for the HBS was introduced, after many fruitless attempts to modernize the curriculum. Differentiation and integration, number theory, functions and graphs were some of the new topics. However, the exam syllabus did not change and so the topics in the final examinations remained the same. As the final exams had gained considerable importance since the start of the HBS, this meant that the new curriculum did not have much influence on the lessons. In 1958 a new curriculum was introduced, this time including a new exam syllabus. Analytic geometry, differentiation and integration were part of the new curriculum; descriptive geometry was at long last abolished. There was less emphasis on extensive calculations in algebra.

In 1963 new legislation introduced a new structure for secondary education; the HBS was replaced by new types of school. The pupils should receive general education and preparation for entrance into higher education (Van Kemenade, 1981). The new curriculum started in 1968; it was influenced by the ideas of 'New Math', as happened in many countries. For a short time algebra was fairly abstract, with emphasis on formal language and sets. Algebra was mainly taught in the first three years of secondary schools; pupils in higher years were supposed to make use of algebraic techniques in differentiation, integration and analysis. In the 1970's, new teaching methods for lower secondary schools were introduced, exercises in algebra were based on realistic situations (contexts); the use of tables, graphs and simple formula's was emphasized (Kindt, 2000). In 1998 new exam syllabi for upper secondary schools were introduced, these contained topics such as graphs, functions, combinatorial analysis, matrices, analysis and differentiation and integration. It was a very full curriculum, but as at the same time the graphic calculator was introduced into the schools, the curriculum designers assumed that the use of a graphic calculator would save much time. This curriculum was supposed to improve transition to higher education and also to provide general education. One of the many reasons it didn't work as expected was the lack of algebraic skills with which students entered upper secondary school. Another reason was the lack of training of teachers in the use of graphic calculators in teaching (Krüger, 2014b).

## CONCLUSION

From the mid-seventeenth century algebra was recognized as a valuable topic in mathematics education. The content expanded from mainly arithmetic with letters and modelling problems in linear and quadratic equations to a variety of topics at the end of the nineteenth century. Algebra in the curriculum of the HBS was considered useful for physics; it also was part of the preparation for technical studies. In the twentieth century the emphasis shifted, at first from elaborate calculations to a more formal approach of structures, following that with an emphasis on global understanding and modelling of ‘realistic problems’. At the end of the twentieth century there were many complaints about (lack of) algebraic skills, made by teachers from universities and colleges, by high school teachers and gradually also by other people who felt the need to make their point. It is interesting to note that this type of complaint was also made in the nineteenth and the first part of the twentieth century. It is evident however that the introduction on a large scale of new learning materials, graphic calculators and software for algebra, is bound to have a large impact on the learning of algebra (Drijvers & Van Reeuwijk, 2006; Goos, Galbraith, Renshaw, & Geiger, 2003). These developments should have a large impact on teacher training and teaching methods as well. These aspects received insufficient attention from nearly all those who were involved.

## REFERENCES

### Archival sources and manuscripts

- The Archive of Utrecht (HUA). HUA 771, inv. 1 (copy of testament of Maria Duyst van Voorhout).  
*Onderwijsverslagen*, 1866–1876. Staten Generaal Digitaal.  
 Available at [www.statengeneraaldigitaal.nl/](http://www.statengeneraaldigitaal.nl/)  
 Schooten F. van, sr. (ca.1622). *Mathematische Wercken*. BPL 1013, Library of Leiden university.  
 Schooten, F. van Sr., & Schooten, P. van (1623–1679). *De cos rekening*, UBG Hs 443, University Library of Groningen.

### Publications

- Bos, H.J.M. (1984). Johan Frederik Hennert, wiskundige en filosoof te Utrecht aan het eind der achttiende eeuw. *Tijdschrift voor de geschiedenis der geneeskunde, natuurwetenschappen, wiskunde en techniek*, 7, 19–31.  
 Derksen, H. A., & Laive, G. L. N. H. de (1899). *Leerboek der algebra met vraagstukken. Eerste deel*. Zutphen, The Netherlands: W. J. Thieme & Co.

- Dopper, J. (2014). *A life of learning in Leiden. The mathematician Frans van Schooten (1615–1660)*. PhD thesis, University of Utrecht, The Netherlands.  
 Drijvers, P., Goddijn, A., & Kindt, M. (2006). Oriëntatie op school-algebra. In P. Drijvers (Ed.), *Wat a is, dat kun je niet weten* (pp. 7–23). Utrecht, The Netherlands: Freudenthal Instituut.  
 Drijvers, P., & Reeuwijk, M. van (2006). Algebra en ICT. In P. Drijvers (Ed.), *Wat a is, dat kun je niet weten* (pp. 137–159). Utrecht, The Netherlands: Freudenthal Instituut.  
 Goos, M., Galbraith, P., Renshaw, P., & Geiger, V. (2003). Perspectives on technology mediated learning in secondary school mathematics curriculum. *Journal of Mathematical Behavior*, 22, 73–89.  
 Graaf, A. de (1672). *De beginselen van algebra volgens de manier van Renatus des Cartes*. Amsterdam, The Netherlands: Jan Rieuwertsz.  
 Grattan-Guinness, I. (2000). *The rainbow of mathematics*. New York, NY: Norton & Company.  
 Groen, W. (2000). Honderd jaar leerplanwijzigingen. In F. Goffree, M. van Hoorn, & B. Zwaneveld (Eds.), *Honderd jaar wiskundeonderwijs* (pp. 223–238). Leusden, The Netherlands: NVvW.  
 Kemenade, J.A. van (1981). Algemene inleiding. In J.A. van Kemenade (Ed.), *Onderwijs: bestel en beleid* (pp. 7–31). Groningen, The Netherlands: Wolters-Noordhoff.  
 Kindt, M. (2000). De erfenis van al-Kwarizmi. In F. Goffree, M. van Hoorn, & B. Zwaneveld (Eds.), *Honderd jaar wiskundeonderwijs* (pp. 57–69). Leusden, The Netherlands: NVvW.  
 Krüger, J. (2010). Lessons from the early seventeenth century for mathematics curriculum design. *BSHM Bulletin*, 25, 144–161.  
 Krüger, J. (2014a). The power of mathematics education in the 18<sup>th</sup> century. In *Proceedings of CERME8*. Antalya, Turkey: ERME and METU. Available at [www.cerme8.metu.edu.tr/wgpapers/wg12\\_papers.html](http://www.cerme8.metu.edu.tr/wgpapers/wg12_papers.html)  
 Krüger, J. (2014b). *Actoren en factoren achter het wiskunde-curriculum sinds 1600*. PhD thesis, University of Utrecht, The Netherlands. Available at <http://dspace.library.uu.nl/handle/1874/301858>  
 Krüger, J., & Maanen, J. van (2014). Evaluation and design of mathematics curricula: Lessons from three historical cases. In *Proceedings of CERME 8*. Antalya, Turkey: ERME and METU. Available at [www.cerme8.metu.edu.tr/wgpapers/wg12\\_papers.html](http://www.cerme8.metu.edu.tr/wgpapers/wg12_papers.html)  
 Maanen, van J.A. (1987). *Facets of seventeenth century mathematics in the Netherlands*. PhD thesis, University of Utrecht, The Netherlands.  
 Morgenster, J. (1707). *Werkdadige Meetkonst*. Amsterdam, The Netherlands: Johannes Loots.  
 Muller, E., & Zandvliet, K. (1987). *Admissies als landmeter in Nederland voor 1811*. Alphen aan den Rijn: Canaletto.

- Nispen, M. van (1662). *De beknopte lantmeetkonst*. Dordrecht, The Netherlands: Mattheus van Nispen.
- Schooten, F. van (1659). *Mathematische oefeningen, begrepen in vijf boecken*. Amsterdam, The Netherlands: Gerrit van Goedesbergh.
- Steyn Parvé, D.J. (1868). Overzicht van het middelbaar onderwijs bij het einde van 1867. *De Economist*, 17, 273–320.
- Struik, D.J. (1995). *Geschiedenis van de wiskunde*. Utrecht, The Netherlands: Het Spectrum.