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The role of history of mathematics in fostering argumentation: Two towers, two birds and a fountain

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This paper aims at discussing how a context of argumentation can be created in the classroom through the development of tasks based on the history of mathematics. These were proposed to a class of 8th grade. Through this experience, we found that students use different types of arguments that show different degrees of formality and kinds of reasoning; express and justify their ideas; and interpret and understand the opinions that are presented to them. The history of mathematics proved to be an enabling tool for mathematics learning, particularly for building a community of mathematical discourse, in which mathematical argumentation is reflected.

Keywords: History of mathematics, argumentation, Pythagorean theorem.

INTRODUCTION

The integration of the history of mathematics in mathematics classes has, over time, attracted the interest of researchers, mathematicians and mathematics teachers (Fasanelli, 2000). In fact, in recent years, such an integration has been prominently featured in literature and in educational curricula of different countries. In parallel, there are several studies that highlight the value of argumentation in mathematics (Pedemonte, 2002; Knipping, 2008) as well as a number of specific references in the curriculum guidelines that advocate the development of students' ability to communicate the way they reason. Taking as starting point these principles, we intend to show how the integration of tasks based on the history of mathematics in the context of the classroom can foster the development of mathematical reasoning, influencing students' ability to reason.

HISTORY AS A PROVIDER OF MATHEMATICAL ARGUMENTATION

A number of arguments justify the benefits of integration of the history of mathematics in teaching and learning mathematics. Tzanakis and Arcavi (2000) point out five fields in which such an integration may be particularly relevant to support, enrich and improve the teaching and learning process: the learning of mathematics; the nature of mathematics and mathematical activity; the didactical background of teachers; the affective predisposition towards mathematics; the appreciation of mathematics as a cultural endeavor. On the other hand, Jankvist (2009) proposes two sets of categories in which to place the arguments for using history (the “hows”) – history as tool or history as a goal – and the different approaches to doing this (the “whys”) – illumination, the modules and the history-based approaches.

Discussing and developing mathematical arguments implies creating conditions for students to learn how to reason mathematically, since a classroom culture that promotes the argument stimulates the participation of students in their own learning (Krummheuer, 1998). In this sense, the history of mathematics can play a useful role in creating a community of mathematical discourse, namely fostering contexts for argumentation. The different points of view that arose from different historical contexts provide an opportunity for students to argue and to develop the art of arguing, to justify their own opinions and to present their thoughts to their peers (Fasanelli, 2000). Thus, students develop not only deeper mathematical skills but also other sorts of skills such as the ability to discuss, analyze and “talk about” mathematics (Tzanakis & Arcavi, 2000). In fact, according to Siu (2007), using

the history of mathematics in the classroom does not necessarily make students obtain higher scores in a particular subject, but it can make learning mathematics a more meaningful and lively experience, thus helping to learn more and more easily. It contributes to establish a context of teaching and learning that provides both teachers and students with different ways of using and acquiring knowledge about what is taught in its diversity. These different forms may arise, for example, by proposing and solving certain problems, since they allow not only the observation of the historical evolution of the concepts, but also promote reasoning and comparison of resolution strategies. Such a process promotes the exchange of ideas and reasoning schemes (Lakoma, 2000), which allows not only the development of argumentation situations, but also the establishment of connections with previously acquired knowledge or other areas knowledge, fostering, in particular, the students familiarity and personal involvement in mathematics (Tzanakis & Arcavi, 2000). Therefore, the history of mathematics appears as a way promoting communication development and, in particular, mathematical reasoning in the context of the classroom.

ARGUMENTATION IN MATHEMATICS CLASS

Argumentation refers to techniques or methods for establishing a speech statement, i.e. processes that produces a (not necessarily deductive) logical assertion on a given subject. The value of arguments in mathematics classes, arises not only associated to the idea of explanation and justification – to convince the other – but also to the discussion and evaluation of different ideas expressed in the classroom, for example while addressing a given task.

The model proposed by Toulmin makes possible a local analysis of the arguments. This model aims at capturing the “logical form” of a speech (Pedemonte, 2002). Toulmin (2008) describes the basic structure of rational arguments as a pair *data/conclusion*. The passage from data to conclusion can be put in question, and often is justified by a *warrant*. Although, sometimes the distinction between data and warrants is unclear, their functions are different: data transmit a set of information; warrants authorize the step of inference. This distinction allows Toulmin to provide the skeletal elements of a pattern that we call *simple argumentation form*. However, this elementary scheme may not be sufficient to analyze certain argu-

mentative discourses. Toulmin, thus, adds three auxiliary elements of discourse analysis: *modal qualifiers*, *conditions of rebuttal* and *foundation*. In particular, in mathematical argumentation these auxiliary elements arise, respectively, as indicators of the strength of the argument, as exception conditions and warrants to support the inference. We designate this by *complex argumentation*.

A classification of the different arguments produced is given by Reid and Knipping (2010), who establish four categories of arguments: empirical, generic, formal and symbolic. Moreover, they state that within these four categories several subcategories can be identified. Other cases may lie at the borderlines between them.

METHODOLOGY AND CONTEXT

This experience, as part of a broader research study, followed a qualitative methodology, involving students in a class of 8th grade and the respective teacher. It aimed at examining how the integration of history-related tasks in the context of the classroom may influence the students’ ability for reasoning and argumentation.

The study had several phases: organization of the task; implementation (along two-class periods of ninety minutes each) and data collection; analysis of the arguments produced by students, local and global analysis of different discursive interactions, identification of difficulties experienced by students and their own assessment.

The task – *two towers, two birds and a fountain* – was structured in different parts. Thus, for Part I, students were asked, in small groups, to solve the following problem: Two towers, measuring 30 meters and 40 meters of height, respectively, are placed at a distance of 50 meters. Between the two towers there is a fountain to where two birds, flying down from the towers at the same speed, arrive at the same time. What is the distance between the fountain and each of the two towers? The problem is found on Fibonacci’s *Liber Abaci*. In Part II, students were asked to read and review strategies of resolution of this problem by Fibonacci himself and Gaspar Nicolas (a Portuguese mathematician of the 16th century).

Data collection was carried out through observation field notes, as well as audio and video records. A documental analysis was also carried on using documents produced by the students: resolutions of the tasks performed and their critical reflection upon them. The teacher put a strong emphasis on group work and discussion with the whole class, interacting with students whenever considered necessary.

The analysis of the arguments produced by students was done using local analysis as proposed by Toulmin and the classification of arguments proposed by Reid and Knipping (2010) model arguments. The evaluation made by the students was based on the five domains mentioned by Tzanakis and Arcavi (2000).

PRESENTATION OF RESULTS

We present in the sequel some examples of the work done by the students during the resolution of task's Part I as well as a brief reference to how students reacted when reading and discussing the different arguments documented in the historical primary sources (Part II).

Type of arguments (in solving the problem)

In Part I all groups tried to solve the problem and different types of argument were observed. Using unrepresentative examples, the arguments produced by students were not restricted to a simple enumeration. In some cases they can even be framed in the subcategory *crucial experience*. Taking into account that at a certain moment the two birds are flying down from the two towers at the same speed and arriving at the center of the fountain at the same time, and that both birds start from the top of two towers, 30 and 40m high, respectively, the students observed that “the [tower] 40 is higher than 30, so it must be closer to the fountain so that the birds arrive at the same time and with the same speed.” (Nelson, group G2). Moreover, given the difference between the heights of the two towers, 10 meters, the students felt that this value would influence the distance from the fountain to each of the towers. The fact that students take into account the data of the problem, noting what should be the position of the fountain (closer to the highest tower), suggests that they are addressing the problem in general. In fact, the initial argument of this group lies between the empirical and the generic, as it is the result of a refutation.

Nuno: We depicted the towers [pointing to the towers drawn on paper] and the space between them is 50. If we put the fountain in middle, which is 25, gives this [pointing to the bird that is in the 30m tower] going down faster than this one [pointing to the bird that was in the 40m tower], because the building is taller.

Argumentation: local analysis (in solving the problem)

Yet in Part I, through the various types of arguments used, it was possible to carry out a functional reconstruction of various statements made in the different groups. Observe an excerpt of the dialogue between the teacher and the G4 group:

- 1 Teacher: What have you done?
- 2 Diana: We have depicted two towers with two birds each. A 40m high and another with 30m and the distance between them was 50m. Afterwards we put the fountain here. 20m from this [pointing to the tower with 40m] and 30m from that [pointing to the tower with 30m, Figure 1], because the difference between them [the two towers] was 10m. So the fountain has to be closer 10 meters to one of them than to the other.
- 3 Teacher: That is...
- 4 Diana: It is closer to this one [pointing to the tower 40m]. This one [pointing to the tower 40m] is 10m higher than that [pointing to the tower with 30m], therefore the fountain must be 10m farther to walk the same distance. And here [pointing to the tower 30m] has as least 10m, then [the bird] will walk less, then the fountain must be farther.
- 5 Filipa: That is, the fountain has to be closer to this [pointing to the 40m tower].

The excerpt shows that this group considers the following solution for the problem: the fountain must be located 20m from the highest tower and 30m from the lower (§2) one. The determination of these values was based on two preliminary conclusions, (§4) and (§5), which acted as new data in the argumentative discourse, to obtain the final conclusion. In this reasoning chain it is still possible to identify a warrant (§2) “because the difference between them [the two towers] was 10m. So the fountain has to be closer 10

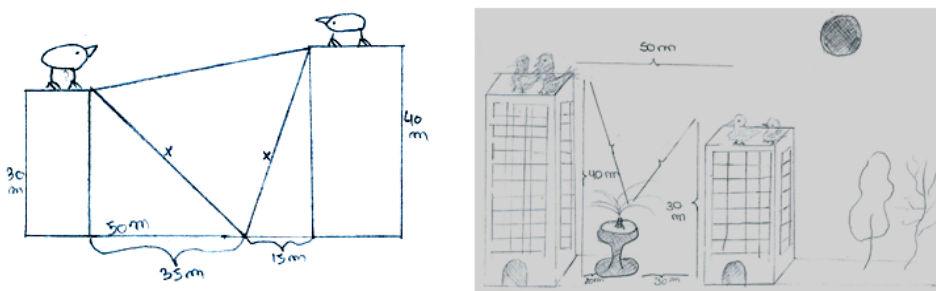


Figure 1: Illustration of the problem made, respectively, by groups G1 and G4

“meters to one of them than to the other”, whose role for this group of students was to legitimize the final conclusion. For the Toulmin model, the argumentative discourse corresponds to a simple form of argumentation.

Almost all groups obtained the wrong values mentioned above. Group G1, however, initially proposed a set of different values. Their initial solution took the form of a conclusion obtained after a refutation, although later they also refute the conclusion.

Nuno: We depicted the towers [pointing to the towers, Figure 1] and the space between them is 50. If we put the fountain in the middle, which is 25, results in this [pointing to the bird in the lowest tower] coming down faster than this one [pointing to the bird in the other tower], because the building is taller. As this is higher, we added 10m ahead [for 40m along the tower], so that they come down at the same speed [and arrive at the same time], we get 35 [25 + 10] with this [pointing to the tower 30m] and x [pointing to the way the bird would go] and use the Pythagorean theorem.

Through this passage, we can conclude that the first solution figured out by the students was 25m, i.e. the fountain would distance 25m of each of the towers. However, students refute this possibility, because the bird on top of lowest tower would reach quicker the center of the fountain, since the other tower was “a higher building”. Although, at this moment, they do not mention the problem data given, they certainly consider it: the birds fly at the same speed and arrive at the same time to the center of the fountain. Therefore, considering the negation of the claim refuted as a new data, they claim that the fountain should “move forward” 10m, i.e. come 10 meters closer

to the highest tower. Accordingly, they add 10m to 25 (which was given as half of the distance between the two towers). Thus, for those students, the fountain should be located 35m away from the lowest tower and 15m from the highest one. It should be noted that, although not explicitly mentioned, the choice of 10m is related to such being the difference in height between the towers. According to the Toulmin model, this argumentative discourse is a complex form of argument.

In the final phase of the excerpt shown, students insist they still need to apply the Pythagorean theorem. Later this group will come to refute the consensual conclusion: that fountain is located 35m ahead from the lowest tower and 15m from the highest one. Note that this group using the same method, will refute the conclusions of the others. Thus, at the end of Part I no group had come up with the correct solution.

Reading and interpreting the reasoning argument present in primary sources

Although students have not found the correct solution, in Part II they were faced with three new resolutions coming from historical primary sources. They read, reviewed and discussed each of these resolutions, and arrived to the actual solution. They compared the resolutions, not only among themselves, but also with their own attempts. From what students wrote we are able to undertake a local analysis of the arguments, but also to identify the difficulties shown and the assessment they made of their work, namely, the confrontation of different resolutions.

From the claim accompanying the arithmetic resolution by Fibonacci, which reads “(...) in geometry it is clearly demonstrated that the height of either tower multiplied by itself added to the distance from the tower to the center of the fountain multiplied by itself is the same as the straight line form the center of the fountain to the top of the tower multiplied by itself; this therefore known (...)”, the students were

able to solve the problem algebraically. Observe the following:

- 1 Emanuel: 900, here 1600 (writing next to the respective towers, Figure 2).
- 2 Teacher: Then what does he say? He adds what?
- 3 Emanuel: The distance from the tower to the center of the fountain multiplied by itself.
- 4 Teacher: You have already the square of the distance from the lower tower to the center. How much it is?
- 5 Emanuel: 900.
- 6 Teacher: And what does he do? He add 900 to...
- 7 Emanuel: The square of the distance from the lowest tower to the center of the fountain.
- 8 Teacher: You know how much is this distance?
- 9 Emanuel: It is x !
- 10 Teacher: It is x ?
- 11 Emanuel: Can be x , because I don't know the length (marks on the drawing x)
- 12 Emanuel: It'll be this (pointing to 900) plus x squared (writing next to the figure $900 + x^2$). Then there is also this 1600.
- 13 Teacher: And you agree with what is there?
- 14 Emanuel: (After a silence) Yes, it is the Pythagorean theorem.
- 15 Teacher: Let's continue. Could you do the same for the other triangle?
- 16 Daniel: Yes.
- 17 Emanuel: This is not x ! (pointing the distance from the fountain to the highest tower).
- 18 Teacher: So here is...
- 19 Emanuel: $50 - x$. [The distance from the center of the fountain to the highest tower]

(Next, they write the expression $1600 + (50 - x)^2$ to denote the square of the distance from the highest tower to the center of the fountain).

- 20 Teacher: Yes, and what can you do with these two expressions?
- 21 Emanuel: We have to match! (they reply very promptly).
- 22 Teacher: And why did you match?
- 23 Emanuel: Must be the same distance, because the flights were of equal lengths.

(Writing $900 + x^2 = 1600 + (50 - x)^2$). And now we solve.

Through this passage, we can conclude that the students translate, in current notation, the information given. Although the purpose of this task was to analyse the process of resolution presented by Fibonacci, this group, from reading the first part of the resolution of Fibonacci, chose to translate it algebraically, obtaining an equation. Thus, for this group, it was enough to solve the equation and thus get the solution.

After solving algebraically the equation, group G2 continued to read and analyse the Fibonacci resolution. They noted that the method used by Fibonacci was the false position, a method often used by the Egyptians to solve problems. Students refer this fact, because the procedure used by Fibonacci corresponds to choose a measure for the distance from the center of the fountain to the highest tower, which is not the problem solution. Then this is revised in order to obtain the correct solution. Students observe that Fibonacci appointed for 10 the distance from the fountain to the highest tower, introducing new data in the problem; naturally the distance from the lowest tower to the fountain is 40, since "the distance between the towers is 50 steps". Students justify Fibonacci's choice of this value (10) as being related to the difference between the heights of the towers. Next, students report that Fibonacci squared these values (heights of the towers and the distance from each tower to the fountain), as he "knew that what was shown in the figure was the Pythagorean theorem". Students continue their interpretation, noting that actually the result has to be the same. Therefore, they report that Fibonacci put the fountain 5 steps more ahead of the lowest tower. Implicitly, they account for the fact that the birds fly at the same speed and arrive at the same time to the center of the fountain, which shows that the birds travel the same distance. By the same procedure, which corresponds to the application of the Pythagorean theorem, students say that Fibonacci got a new value for the difference between the distance

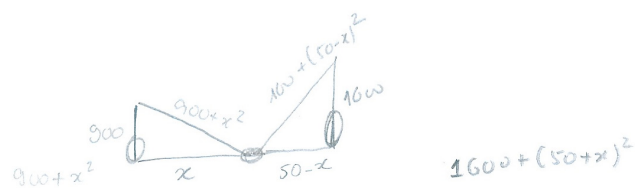


Figure 2: Illustration of the problem made by group G2

travelled by birds. Again the values obtained are not equal, and thus conclude “the fountain was not well positioned”. They state that Fibonacci noted that when he “moves” the fountain 5 steps ahead the difference between the distance travelled by each bird was 300, i.e. the difference decreased in 500 steps. Since the birds travel the same distance, students report he used “the rule of three to see how many steps they had to walk so that the difference stays equal to 0”. However, in Fibonacci’s resolution there is no reference to this rule, but rather a numerical recipe – “multiply 5 by 300, and divide by 500”.

In the local analysis of the arguments produced, it is observed that those are presented sequentially, which did not happen when students attempted to solve, initially, the problem. It was also noted a common concern with effective warrants and even with the corresponding foundations, which could legitimate the inference steps.

The second strategy to solve the problem is based on the similarity of triangles, Figure 3, efz and gem where m is the intersection point of ef and the parallel to df which contains g . Fibonacci started solving the problem by showing geometrically that the point z is the center of the fountain. Then he proceeded numerically. In this first phase, the students were challenged to read the beginning of Fibonacci’s resolution and to find a justification for point z be the center of the fountain.

After some discussion students observed that e is, by construction, the midpoint of the segment ga , and ez , and also by construction, a segment perpendicular to the segment ga , and thus the triangles aez and gez are geometrically identical, since both sides are identical

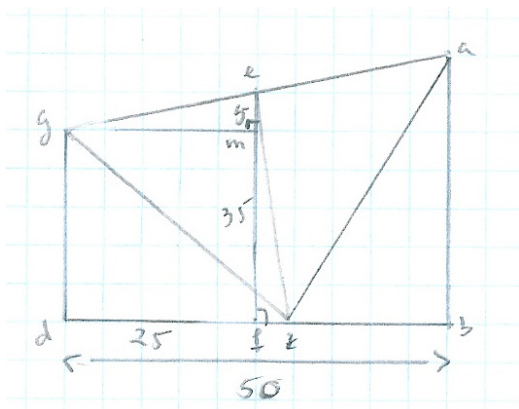


Figure 3: Illustration of the problem made by group G2 to the second strategy

and form an angle (90°). With respect to the arithmetic resolution, although the students were able to identify its main elements, they did not understand how Fibonacci arrived, in current notation, to the expression $[35x(35 - 30)] / 25$.

Daniel: Subtract this 35 by 30 will give 5; drawing this parallel line will get this 5 [pointing to the Figure 3]. Then multiplying 35 by 5 gives 175. Then dividing by 25 will [pointing to the segment]. But to divide by 25 will give the result of this [pointing to fz]. I’m not understanding why it gives the result of fz here [pointing to fz], if it is to change this [pointing to 5 and 35] for this [pointing to 25].

This excerpt reflects the difficulty expressed by students. Although the figure can identify the values reported by Fibonacci, it cannot justify the solution. Involving the whole class in the discussion the teacher points to the relationship between the triangles efz and gem .

- 1 Teacher: And these two triangles are equal?
- 2 Andrea: Which ones? This [pointing to the triangle egm] and this [pointing to efz]?
- 3 Teacher: Yes.
- 4 Daniel: No!
- 5 Emanuel: They do not have the same measures.
- 6 Teacher: Are they similar?
- 7 Teacher: [after a silence] What is required for two triangles to be similar?
- 8 Daniel: They must have two equal angles...
- 9 Nelson: Or the sides proportional...
- 10 Teacher: Or two proportional sides and the angle formed by them equal.
- 11 Daniel: They have a right angle [pointing to egm]... and also this [pointing to efz].

This extract, shows that students observed that triangles, gem and efz , are not geometrically identical (§4), since they have different sides (§5). Questioned whether they were similar (§6), students pointed out two cases of similar triangles, (§8) and (§9), and the teacher a third one (§10). Students observed the existence of the two right angle triangles (§11). For them to be similar triangles it would be necessary to find

another pair of geometrically equal angles. Later, the students find that angles gem and fze are geometrically identical and therefore the triangles are similar.

The resolution proposed by Gaspar Nicolas, is written in the form of a recipe without any algebraic symbol. Although he considered the towers with 80 and 90 fathoms and the distance between both to be 100 fathoms, the students understood easily the calculations but question the procedure: “why to subtract 6400 to 8100”, “why to add this difference to the square of the distance between the two towers” and “why to divide that result by 200”. The teacher suggested a possible correspondence with the algebraic process used previously. With two unknowns, students were able to establish a correspondence between Nicolas’s and their own algebraic process.

CONCLUSION

The experiment shows that different sorts of arguments were produced by different groups of students. Many of them were not developed sequentially, i.e. in a deductive way. The need for sharing ideas and exchange opinions was responsible for the emergence of not only parallel arguments, but also of new data which was suitably inserted into the argumentative chain. In simple forms of argumentation, we also observed the existence of inference steps for which the corresponding warrants or foundations were not explicitly presented. On the other hand, when analyzing the resolution strategies of Fibonacci and Gaspar Nicolas, students exhibit a strong concern for seeking warrants or some sort of formal explanations (eventually not explicit in the resolution shown) to legitimize certain inference steps. The analysis of the forms of argument used shows that students were not only able to express their ideas, but also to interpret and understand the ideas presented to them, as well as to participate constructively in the discussion. In assessing the experience, students highlighted the importance of solving the same problem through different processes and of becoming able to compare different resolution strategies. They also stressed the exercise influenced the predisposition to this discipline, while providing a broader view of its nature: “the geometrical resolution was the most difficult, but we learned that it was possible to solve the same problem geometrically”; “for me the most interesting was the resolution of Gaspar Nicolas, because I liked to

try understand his thought and to solve an equation in words”.

Experience has shown the potential of integrating the history of mathematics in the context of the classroom. This entails the need for a broader consideration of this problematic and its importance. Similarly, it seems necessary to outline a number of guidelines that promote this integration and scales up its potential namely, in what concerns the development of mathematical reasoning. This experience confirms that the development of reasoning in the mathematics class is a complex process: it requires a careful selection of the tasks, and entails the need for promoting a suitable environment for them, focused on the students’ mathematical maturation.

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