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► **To cite this version:**

Peter Collignon. Using calculus in economics: Learning from history in teacher education. CERME 9 - Ninth Congress of the European Society for Research in Mathematics Education, Charles University in Prague, Faculty of Education; ERME, Feb 2015, Prague, Czech Republic. pp.1811-1816. hal-01288386

**HAL Id: hal-01288386**

**<https://hal.science/hal-01288386v1>**

Submitted on 15 Mar 2016

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# Using calculus in economics: Learning from history in teacher education

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*Historical awareness has an impact on teaching and learning mathematics. It includes knowing the historical development, the questions under investigation and the answers given to these problems. In this paper, the focus lies on calculus and its applications in economics. It shows how far the knowledge of a changing scientific understanding can be beneficial in teacher education. The paper covers this issue from an epistemological, historical and educational perspective and suggests a constructivist view for educational purposes.*

**Keywords:** Calculus, constructivism, economics, epistemology, teacher education.

## INTRODUCTION

Within teacher education, an expansion of scientific understanding can be achieved, which promotes an adequate epistemological view and modifies naive-realistic ideas. For this purpose, covering mathematical economics using methods of calculus is an unusual but promising approach. According to Fischer & Malle (1985, p. 107), the absence of a law-of-nature-character is necessary to allow learners the free use of mathematics describing “reality”. In this way the modelling perspective demonstrates a human distance to reality. Jablonka (1996, p. 34f) states, that this view assumes an understanding of the underlying mathematical concepts separate from the context. In the following text, this consideration will be modified by discussing the usage of a certain mathematical concept (calculus) for modelling (economic) circumstances, which originally did not contribute to its genesis. However, this approach should go along with an adequate historical awareness of the evolution of the mathematics involved.

In most cases first-year students have not experienced the formal treatment of scientific issues in the class-

room as a modelling process. Even more there is little or no experience with a systematic processing of economics applications. Dealing with economics in terms of mathematical modelling offers general education and provides insights into epistemological concepts and helps to foster an enlightened understanding of science. In view of the above and considering also the potential of a constructivist understanding of science due to Ernest (2007) and Lyotard (1979) it is promising to use a mathematical tool in connection with a subject that it is originally not meant for. This approach aims at prospective teachers primarily, but it applies to their educators as well.

## APPLIED MATHEMATICS IN TEACHER EDUCATION

Mathematical modelling is listed as one of the six important competencies in Germany’s “Bildungsstandards Mathematik”. This is a consequence of the postulate of integrating more reality-based problems in lessons and lectures on mathematics. The focus is not the application of a given algorithm, but the mathematization of facts and problems whose relation to mathematics may be not initially obvious. Circumstances of the so-called “real world” are to represent formally by abstraction so that the representatives enable a quantitative analysis. The solution found for the model may be interpreted as a proposal for the solution of the real problem.

In many cases, such an approach is presented as a multiple passage of a modelling cycle that shows the interaction between “reality” and model as an idealized scheme. Critical validations should successively lead to a revised design of the model as an interpretation of reality.

Beginning their studies, prospective teachers have years of experiences in applying mathematical meth-

ods. For example, elementary calculus, stochastic and analytical geometry are known in principle. Introductory lectures at university address those issues again and there is the chance to add new aspects corresponding to the didactic spiral principle. This includes applications in general and should cover topics from mathematical modelling in particular.

### **MATHEMATICAL MODELLING IN ECONOMICS**

In current mathematics education discussion the modelling cycle of Blum and Leiss (2005, p. 19) is widely accepted as a “model of modelling”, but also different approaches are considered, e.g. an interpretation of the modelling process in the sense of taking the spiral principle or descriptions including a coexistence of cyclic and progressive courses into account. Meyer and Vogt (2010, pp. 142ff.) argue for an increased emphasis on the processual character and the inclusion of an appropriate terminology. Möller (2014) analyses the concept of the “rest of the world” and exposes the problems of the disjoint separation of mathematics and reality.

#### **Facets of the reliance on mathematics**

The field of economics shows a multitude of aspects that differ from those of the natural sciences. There are much less “canonical” formalizations than in physics, e.g.

Human action is involved, which refers to needs and wants. The scarcity of goods and services forces people to economize. Economic actions go along with conflicts and competition; they require decisions under uncertainty. Therefore, they are fraught with risks. Economic activities are goal- or benefit-oriented, often for-profit.

Economic issues, due to their reliance on mathematics, must take all aspects of the modelling process into account. Using mathematics within the “exact” and natural sciences often appears canonically, and in school they are rarely taught as a result of a modelling process. An in-depth understanding of their non-canonical nature, which manifests itself in the different models of the physics of classical mechanics, Maxwell’s electrodynamics, the theory of relativity, quantum mechanics to recent developments in cosmology remains a domain of advanced students and experts.

Nevertheless, mathematical modelling is often demonstrated in the context of physical-technical issues. The decision for a certain model is closely related to the finding and definition of suitable parameters and their functional relationship. Frequently the number of parameters involved and their skillful assignments are looked upon as a quality criterion.

While early graders get involved in proportionalities relating to quantities, on the secondary levels the notion of function comes to the fore. It requires good education to impart the features of modelling in such a framework. This applies to teacher training, too.

At this point the comprehension of economic topics carries a valuable educational potential. The fundamentals of the conditions and circumstances – with respect to the subject of the application as well as the mathematical methods used – gives insight into the processes of gaining knowledge.

The values of goods and services are defined by their position on a common one-dimensional discrete scale; a price is assigned. So already the introduction of the quantity “money” is an early normative modelling item (cf. Möller, 2007, pp. 3ff.). It allows a comparison and measuring with monetary units, which implies the definition of an order relation for all the goods and services. Here a descriptive aspect becomes apparent: Aspects of economic objects can be characterized (via prices). Once the concept of functions is available, rates may be expressed by functional dependencies and forecasted under appropriate model assumptions. In accordance with and in extension to Henn (2000) the descriptive aspect can be interpreted as a summary of description, declaration and prognosis.

#### **The role of calculus**

Even if modelling using functional relationships is regarded as reasonable, there is initially no need to apply methods of calculus. Under what conditions and for what motives does this happen, at all? In order to answer this question one must distinguish between

- a use of a calculus not reflected upon (cf. Doorman & van Maanen, 2009, p. 4), which offers the possibility of using methods taught in school to discover certain properties of functions (local extrema, inflexion points, monotony etc.);

- a deliberately chosen approach, which allows an analysis of the model with methods of infinitesimal calculus in the first place with the aim of gaining knowledge.

The latter begins with the substitution of discrete quantities, such as money or lot sizes, with continuous ones. In this way, the real numbers (as well as real intervals) are involved. Functional dependencies described by tables or in terms of numerical sequences must now be expressed by functions defined on (connected) subsets of the real numbers. Often the mapping rules are chosen in a way, that allows an analysis according to infinitesimal calculus, i.e., they are continuous, differentiable etc. Providing this, it is possible

- to use methods from calculus on the mathematical model level;
- to interpret the available mathematical concepts relative to the real situation, i.e., an economic interpretation.

Corresponding to the second point, we often observe an interaction between the mathematical practice and the subject to be mathematized: Several economic concepts were generated or at least clarified by the use of a certain kind of mathematics. Marginal costs, defined by the first derivative of the cost function, are an example. The history of physics and its partial co-evolution with calculus offers a variety of relevant analogies.

### **STUDENTS' PREVIOUS KNOWLEDGE ABOUT THE CO-EVOLUTION OF CALCULUS AND PHYSICS**

Modern calculus textbooks are characterized by an axiomatic representation and by offering physical applications. The representations are supplemented by graphics or pictures to support their understanding. The main concepts include the real numbers, the notion of a function and a limit. They are proposed by a formalised representation following a trias that is definition, theorem and proof, supplemented by a few examples illustrating the propositions. In this way the representation of calculus made its way during the first half of the last century (see also Jost, 1998). This outcome is still regarded as “modern” and results from a long historical non-linear development with

discontinuations. Thus it may be an ambitious task for first-year students to look into history of calculus self-contained. In the case of master’s students an independent research is more appropriate. The following sections outline the knowledge students should have to participate in a discussion about further applications in social sciences, especially in economics.

The discovery of the Archimedean palimpsest (cf. Netz & Noel, 2007) revealed his geometric view consisting of several singular perceptions of areas bordered by parabolas. His way can be characterized as singular efforts but he did not deduce a method which could have led to a general view. He took first steps onto the way to Riemannian integration, but however, he did not develop a general method like Riemann did.

New steps towards a theoretical approach of analysis were taken more than a thousand years later within occidental mathematics. Further efforts were made within the influence of natural sciences in particular the planetary movements as well as the ballistic investigations. These considerations corresponded to the notion of dynamics, e.g. the velocity, and both concepts involve the dependency between a geometric position and the time. Here Johannes Kepler (1571–1630) used the lists made available by Tycho Brahe (1546–1601) (lists that put the loci of the planets in dependency of the time) and deduced his laws of planetary orbits.

Newton (1643–1726) further developed and refined the theory of planetary movements by using an infinitesimal calculus that emphasized dependencies of time. However he did not yet apply yet our modern concept of a function. Around the same time Leibniz (1646–1716) generated an infinitesimal calculus by introducing infinitesimal quantities, not using consequently a functional approach either. However, both of them and some other contemporary scientists provided a basis for their followers in the 18th century, see also Sonar (2011).

Euler (1707–1783) picked up these conceptions and was the first to introduce the functional concept in a way in which we use it still nowadays. This means the infinitesimal calculus reached the next level and is since called modern. During the following decades the focus was on computational aspects but there was still a lack of theoretical foundations. The real numbers had no theoretical foundation and the concept of a limit was still missing.

Karl Weierstrass (1815–1897) bridged both gaps (real numbers and limits) using the epsilon-delta-statement for the definition of a limit and contributed a theoretical approach to the real numbers. The latter was refined later on by Dedekind and Cantor who defined the real numbers by Dedekind-cuts. Cantor contributed the concept of Cauchy sequences (equivalence classes of Cauchy sequences) and the continuum hypothesis (cf. Hairer & Wanner, 1996, pp. 172ff.). Their contributions established a completion of the rational numbers to the real numbers so they obtained a complete field of numbers.

Teacher students should be also aware of the following: In the classroom, but also in contemporary university lectures, especially in teacher education, one can observe some kind of (anti-) didactical inversion (cf. Freudenthal, 1983, pp. 305ff.), interpreted in terms of history. There is an introduction to the real numbers on secondary I level, which mainly consists of some examples of irrational numbers. As a matter of fact this is part of the recent history of calculus and it is, strictly speaking, mathematics from the 19<sup>th</sup> century. Regarding the concept of functions, the situation is similar: It is introduced before a discussion about infinitesimal concepts takes place on secondary II level. The figure below presents a very rough scheme that reflects teacher students' often observed reception of the history of calculus and the (experienced) order of teaching.

### EDUCATIONAL POTENTIAL

There is no controversy about the necessity of gaining insight into economics and economics applications using mathematical concepts. However, this is defined differently or non-specifically. According to May (2001, p. 3),

... economic literacy can be paraphrased as the qualification (knowledge, competencies, skills,

attitudes etc.) to manage living conditions determined by economics.

At the same time we find the statement:

The intention of teaching economics is ... the (economically) responsible citizen.

Engartner (2010, p. 15) emphasizes the social meaning of economic literacy:

Only an appropriately qualified citizen, educated in matters of responsibility, is in a position to follow the rapid process of social change at least rudimentarily.

### Orientation towards the "basic experiences"

Sociological and economic issues affect the reality of young adults' lives to a larger degree than topics from physics do. From the mathematics educational point of view they are matters of the "basic experiences" of Winter (1996): Mathematics education is providing general education by

- realizing phenomena of nature, society and culture;
- knowing (and appreciating) mathematical issues, represented by language, symbols, images and formulas;
- acquiring heuristic competencies.

Mathematics education has a general and life preparatory function (Heymann, 2013, pp. 131ff.). Mathematical modelling offers the opportunity to realize this in two ways: firstly, on the basis of concrete facts to be modelled, then again by modelling itself on a meta-level. This provides access to epistemological issues.

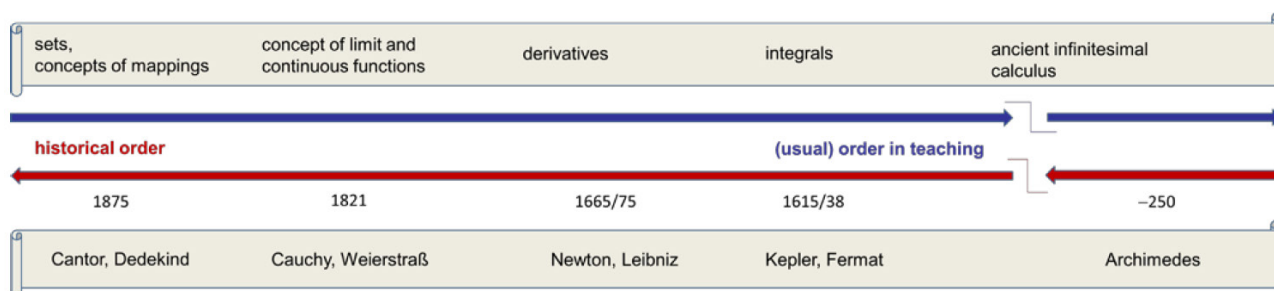


Figure 1: Didactical inversion – in terms of history



### Epistemological aspects

Theory of science is not science philosophy, and the latter is not the same as epistemology. Especially the German language is differentiated here, although not uniformly (Poser, 2012, pp. 15ff.). It is not the role of an introductory mathematical lecture to explicate this in detail. But particularly with regard to teacher education one should not ignore the observation, that students, but also teachers, often show a naively realistic understanding of science. Zeyer (2005) confirms their socialization through their academic training in terms of an unconsidered positivistic attitude. In this context a proposal from Duit (1995, p. 905) deserves consideration.

Knowledge acquisition is regarded as an active construction on the basis of existing ideas. The active, self-directed and self-reflective learner is in the center and the idiosyncratic processes of construction are always embedded in a particular social context.

In case of the natural sciences and pure mathematics there are justified objections to a strong emphasis of relativism, even from a didactic point of view. Regarding economics, the situation is quite different. It is obvious, that the economic denominations listed above are closely connected to human action. The implications and theories deduced do not have the significance of “laws of nature”. This makes it easier for students who did not have any contact with epistemological questions until this point to identify the reference objects of many scientific discourses as models.

The history of economic sciences shows various paradigm shifts – even within the radical meaning of Kuhn (1992) –, and even the contemporary ones are more accessible to laymen than the development of physics in the twentieth century. This supports an open-minded modelling activity in economic contexts in which the distinction between “wrong” and “correct” solutions becomes less important and is put into perspective. The main idea of the paradigm and the paradigm shift relates to the facts to be modelled and the mathematics used at the same time. For example, the economic marginal analysis uses infinitesimal methods originally physically and geometrically motivated. In economics, however, quantities of discrete nature are often modelled as continuous ones with intent to apply methods of differential calculus, which emerged from completely different requests. Lectures and seminars on

calculus offer the opportunity to point to the transfer of methods from natural to economic sciences and discuss how far this is an historic attempt to make economy “accurate”.

Considering everyday experiences, and apart from the necessity of laboratory conditions, Newtonian mechanics may be a “canonical” modelling. Anyway, the use of infinitesimal methods in economics is based on model assumptions which are replaceable in a more obvious way, e.g. by the application of methods from discrete mathematics.

Brodbeck (1998, pp. 22ff.) refers to these observations as “social physics” and continues:

The mechanical approach in economics shall primarily allow the application of mathematical and experimental methods in analogy to physics (...).

### Prospects

Jablonka (1996, p. 187) concludes her meta-analysis of approaches to mathematical modelling and applications by declaring:

The lack of examples from which we can learn where to ask more questions and to try different ways and to illuminate the problem from different perspectives and to weigh the outcome and evaluate critically clearly shows that a reflection beyond the scope of an analysis of purposes and efforts has little place in mathematics lessons, which target the comprehension of mathematical procedures and theories.

As a resource to change the viewing angle self-consistent, and as an instrument to recognize and evaluate alternatives, to search for reasons and in the exchange of arguments, reflection is not knowledge, but an attitude in the assessment of mathematical methods to be aimed at an intentional creation of situational activity, which results in the insight, that recipients or persons concerned are authorized and applying operators are obligated, to adopt a critical attitude.

Teaching experience meets the high expectations linked to the potential of economic modelling against the background of the history of calculus outlined here. Qualitative investigations planned in this context will provide further indications.

## REFERENCES

- Blum, W., & Leiß, D. (2005). Modellieren im Unterricht mit der „Tanken“-Aufgabe. *Mathematik lehren*, 128, 18–21.
- Brodbeck, K.-H. (1998). *Die fragwürdigen Grundlagen der Ökonomie. Eine philosophische Kritik der modernen Wirtschaftswissenschaften*. Darmstadt: Wissenschaftliche Buchgesellschaft.
- Doorman, M. & van Maanen, J. (2009). A historical perspective on teaching and learning calculus. *Australian Senior Mathematics Journal*, 22(2), 4–14.
- Duit, R. (1995). Zur Rolle der konstruktivistischen Sichtweise in der naturwissenschaftlichen Lehr- und Lernforschung. *Zeitschrift für Pädagogik*, 41(6), 905–923.
- Engartner, T. (2010). *Didaktik des Ökonomie- und Politikunterrichts*. Paderborn, Germany: Schöningh.
- Ernest, P. (2007). Epistemological Issues in the Internationalization and Globalization of Mathematics Education. In B. Atweh, A. Calabrese Barton, M. C. Borba, N. Gough, C. Keitel-Kreidt, C. Vistro-Yu, R. Vithal (Eds.), *Internationalisation and Globalisation in Mathematics and Science Education*. Dordrecht, The Netherlands: Springer.
- Fischer, R., & Malle, G. (1985): *Mensch und Mathematik. Eine Einführung in didaktisches Denken und Handeln*. Mannheim, Germany: Bibliographisches Institut.
- Freudenthal, H. (1983): *Didactical Phenomenology of Mathematical Structures*. Dordrecht, The Netherlands: D. Reidel Publishing Company.
- Hairer, E., & Wanner, G. (1996). *Analysis by its History*. New York, NY: Springer.
- Henn, H.-W. (2000). Warum manchmal Katzen vom Himmel fallen oder von guten und von schlechten Modellen. In H. Hischer (Ed.), *Modellbildung, Computer und Mathematikunterricht* (pp. 9–17). Hildesheim, Germany: Franzbecker.
- Heymann, H. W. (2013). *Allgemeinbildung und Mathematik*. Weinheim und Basel, Germany: Belz.
- Jablonka, E. (1996). *Meta-Analyse von Zugängen zur mathematischen Modellbildung und Konsequenzen für den Unterricht*. Berlin, Germany: Transparent H. & E. Preuß.
- Jost, J. (1998). *Postmodern Analysis*. Berlin, Germany: Springer.
- Kuhn, Th. (1992). *Die Entstehung des Neuen* (4. Aufl.). Frankfurt am Main, Germany: Suhrkamp.
- Lyotard, J.-F. (1979). *La condition postmoderne: rapport sur le savoir*. Paris, France: Les Éditions de Minuit.
- May, H. (2001). *Didaktik der ökonomischen Bildung*. München, Germany: Oldenbourg.
- Meyer, M., & Voigt, J. (2010). Rationale Modellierungsprozesse. In B. Brandt, M. Fetzer, & M. Schütte (Eds.), *Auf den Spuren interpretativer Unterrichtsforschung in der Mathematikdidaktik* (pp. 117–148). Münster, Germany: Waxmann.
- Möller, R. D. (2007). Zur Modellierung ökonomischer Kontexte in der Grundschule. In *Materialien für einen realitätsbezogenen Mathematikunterricht* (Bd. 11, pp. 1–8). Hildesheim, Germany: Franzbecker.
- Möller, R. D. (2014). Gibt es Mathematik im Rest der Welt? In *Tagungsband GDM, 2014*.
- Netz, R., & Noel, W. (2007). *The Archimedean Codex*. London, UK: Weidenfeld & Nicholson.
- Poser, H. (2012). *Wissenschaftstheorie. Eine philosophische Einführung*. Stuttgart: Reclam.
- Sonar, T. (2011). *3000 Jahre Analysis*. Berlin, Heidelberg, Germany: Springer.
- Winter, H. (1996). Mathematikunterricht und Allgemeinbildung. *Mitteilungen der Gesellschaft für Didaktik der Mathematik*, 61, 37–46.
- Zeyer, A. (2005). Szientismus im naturwissenschaftlichen Unterricht? Konsequenzen aus der politischen Philosophie von John Rawls. *Zeitschrift für Didaktik der Naturwissenschaften*, 11, 193–206.
- Ziegler, B. (1998). *Geschichte des ökonomischen Denkens. Paradigmenwechsel in der Volkswirtschaftslehre*. München, Germany: Oldenbourg.