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The contribution of history of mathematics on students' mathematical thinking competency

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This paper presents a small subset of results from a pilot study conducted at a small private university in 2012. The study sought to identify changes in students' mathematical thinking competency. In this paper, I present summaries of pre-instruction and post-instruction think-aloud interviews for one of 18 items on the think-aloud instrument, for two of the participants. I propose that history and philosophy of mathematics courses do have the potential to impact students' mathematical thinking competency and further studies such as the one presented here must be undertaken to expand what we know and how we can use the knowledge to enhance the teaching and learning of mathematics.

Keywords: History and philosophy of mathematics course, think-aloud protocol, mathematical thinking competency.

INTRODUCTION

In the past, much of the available research on history of mathematics courses has been primarily focused on how to teach such courses (e.g., Miller, 2002). Recently, however, scholars have attempted to "describe ways in which a history of mathematics course can help prospective teachers of mathematics develop knowledge they will need for teaching" (Huntley & Flores, 2010, p. 603). Although investigations into the development of mathematical knowledge for teaching constitute important contributions, there exists a need to examine the potential impact of a history of mathematics course on the more general undergraduate mathematics major population. I propose that what is missing from the research literature regarding the alleged influence of the history of mathematics is inquiry on the actual impact on learning, or as considered here, on students' mathematical thinking competency as a result of studying mathematical ideas from the perspective of the historical and philosophical development of those ideas.

The field of mathematics education boasts fewer empirical examples about how the study of the history and philosophy of mathematics contributes to student thinking about and understanding of essential mathematics concepts than it does theoretical studies on the same. The outcomes of the study discussed here hold promise for increasing awareness of what the history of mathematics contributes to learning and thinking about mathematics (if anything) and for contributing something further to the empirical base of previously well-known theoretical studies.

Based upon research that I have conducted with prospective mathematics teachers (Clark, 2012), I conjectured that the think-aloud task interview results would in fact reveal changes in the undergraduate students' mathematical thinking competency, using the definition provided in Danish KOM-report (Niss & Højgaard, 2011, pp. 52–53). In particular, I am interested in identifying evidence for which participants engaged in different aspects of this competency, such as:

> ...being able to *recognise, understand* and *deal with the scope of given mathematical concepts* (as well as their limitations) and their roots in different domains; *extend the scope* of a concept by *abstracting* some of its properties; *understand* the implications of *generalising* results; and be able to *generalise* such results to larger classes of objects. (emphasis in original; Niss & Højgaard, 2011, p. 53)

That is, the primary goal for the research was to link students' experiences from a history and philosophy of mathematics course to changes in their competency for thinking about mathematics with regard to important fundamental mathematical concepts. Thus, this qualitative, exploratory study set out to document and analyse changes in the students' mathematical thinking competency on three topics of interest (in-

finity, the complex number system, and the axiomatic structure in mathematics).

The results of the present research may enable instructors of history and philosophy of mathematics (HPhM) courses to consider the power of the course on improving mathematical understanding and could find it beneficial to revise courses to capitalize on this implication. Additionally, researchers interested in the "value-added" impact of a course on the history (and philosophy) of mathematics for prospective mathematics teachers may be able to make a stronger case for the inclusion of a history of mathematics course in teacher preparation programs that do not currently require it.

METHODOLOGY

The importance of such an empirical study—even an exploratory one—on changes in how students articulate their mathematical thinking as a result of an HPhM course is timely. The study is exploratory along two dimensions. The first dimension entails piloting the mathematical task think-aloud protocol, created to allow students to articulate their thinking about the mathematical ideas of infinity, the complex number system, and the axiomatic structure of mathematics. The second dimension, which is the focus of this paper, is the description of students' mathematical thinking pre- and post-instruction in the HaPM course and an attempt to qualify changes that occurred in their mathematical thinking competency with respect to these fundamental ideas in mathematics.

I heavily draw on Corbin's and Strauss' (2008) definition of qualitative analysis as "a process of examining and interpreting data in order to elicit meaning, gain understanding, and develop empirical knowledge" (p. 1). The nature of data analysis required me to search for changes between a student's thinking during the think-aloud interview post-instruction when compared to their pre-instruction thinking, and to qualify those changes using most of the aspects of the mathematical thinking competency (Niss & Højgaard, 2011): recognise, understand, and deal with the scope of the given concept; extend scope by abstracting; understand implications of generalising results; and generalising results to large classes of objects. For this reason, each student's pair of interviews was coded individually, according to their own phrasing, references (e.g., to the HPhM course material), and examples.

Participants

The particular context for the study was selected for two reasons, First, I was already familiar with faculty members at the university (one in mathematics education, one in mathematics), which would facilitate my multi-day visits to collect data, get to know the students in the course, and to conduct the think-aloud interviews. Second, students from a variety of majors took the history and philosophy of mathematics (HPhM) course at Private Christian University (PCU; a pseudonym). Thus, a course with a diverse student population as the one at PCU (e.g., first-year through fourth-year undergraduate students; mathematics and non-mathematics majors) was a valuable convenience sample to use.

Students were recruited from the HPhM course at the beginning of Fall 2012. The course instructor introduced the opportunity to participate in the research study during the first class session. Then, at the end of the second class session, I introduced myself and explained that the primary goal of the research was to investigate changes in students' mathematical think-

Participant	Year in school	Major	Highest level mathematics course taken in high school; college mathematics courses taken
Jenny	Senior	Elementary Education	Precalculus (high school); Intro to Mathematical Thinking (PCU)
Tabitha	Senior	History (Secondary History Education)	Intro to Mathematical Thinking (PCU)
Darren	Sophomore	Mathematics or Music (undecided in Fall 2012)	Advanced Placement Calclulus (high school); currently enrolled in two math courses (PCU)
Michael	Junior	Mathematics (recently changed major to Mathematics Education)	Mathematical Analysis II (high school); Intro to Mathematical Thinking (PCU)

Table 1: Student participants (Private Christian University, Fall 2012)

ing about mathematics concepts (e.g., the concept of infinity) that occur as a result of studying the history and philosophy of that concept. Finally, I explained the nature of the think-aloud interviews and answered students' questions about the research, interview process, and their potential participation. Of the 19 students enrolled in the course, four students volunteered to participate in the pre- and post-instruction "think aloud" interviews. Brief descriptions of the four participants are given in Table 1. The names that appear in bold indicate the participants on whom I focus for the results discussed in this paper.

Data collection

Mathematical task interviews were conducted using a think-aloud approach, in which student participants were asked to articulate their methods, interpretations, and thought processes while working on mathematical tasks. During the think-aloud process, the researcher intervenes as little as possible, with the exception of limited prompts such as "talk about what you're thinking" or simply "keep talking" (Young, 2005). The think-aloud interviews were conducted before and after classroom instruction and subsequent study of the three concepts of interest. Smartpen technology was used during each interview. The Smartpen was used by students during the interviews and enabled me to capture audio as well as written documentation of the students' responses to each of the tasks. Interview transcripts, once transcribed, were analysed for qualitative differences in students' mathematical thinking competency, as a result of studying historical details and philosophical perspectives of fundamental mathematics topics. In addition to the audio and written documentation, the course instructor's instructional plans and documentation related to the three selected topics comprised the third data source (e.g., lecture notes, assigned reading, homework assignments, class session activities).

Think-aloud interview instrument

A three-part interview instrument was constructed for this research, which was composed of 18 items. The instrument included items about the concept of infinity (five items), the complex number system (six items), and axiomatic structure in mathematics (seven items). However, due to space limitations, I discuss only one of the items in this paper: Item 1 (complex number system) asked students to *draw a Venn diagram to show the relationships among the different types of numbers that comprise the number system.*

In the following discussion, I present a summary of the pre- and post-instruction responses for both Tabitha and Michael. Additionally, I identify which aspects of the mathematical thinking competency were found in their responses. Finally, I provide a profile for Tabitha and Michael (based upon their pre- and post-responses) in an attempt to identify changes in their mathematical thinking competency resulting from their experience with a history and philosophy of mathematics course.

SELECTED RESULTS

The four students who participated in this study generated a surprising amount of rich data in their responses to the 18 items during the think-aloud interviews. In an effort to make sense of the various ways to describe the changes in students' mathematical thinking competency, this paper presents responses for only one item of the think-aloud instrument from two of the four participants.

The important results of this study are found in the qualitative analysis of the participants' pre- and post-instruction think-aloud responses. Tabitha's and Michael's responses to one item are presented to display the potential for HPhM course work to impact students' mathematical thinking competency. For this paper I selected an item in which the HPhM course might impact students' competency differently, given the range of student abilities and experiences represented.

To illustrate the scope of changes in the mathematical thinking competency for Michael and Tabitha, I present a summary of the key content of their responses, along with identification of aspects of the competency exhibited in their response. When necessary, I also provide a continuum rating to quantify the extent to which the aspect was evidence (e.g., "naïve or beginning understanding", "stronger example").

The case of Tabitha

To begin, I provide the summary of Tabitha's pre-responses to Item 1 (complex number system), with elements of the mathematical thinking competency (Niss & Højgaard, 2011) given in bold as well as an image from Tabitha's think-aloud interview (Figure 1):

(a) She expressed types of numbers as dyads: positive and negative; real and fake, or "unreal" numbers; ra-

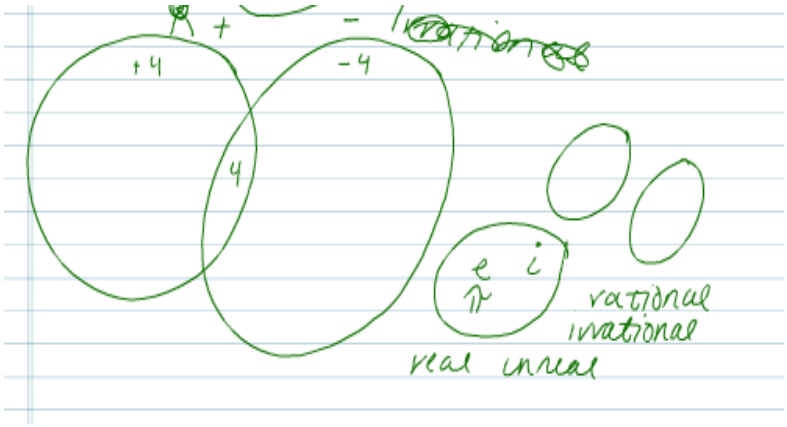

Figure 1: Tabitha's pre-response Venn diagram

tional and irrational, which belong to real numbers (though her Venn diagram did not reflect this) *[recognise a given math concept]*;

(b) Numbers that are not numbers: *e, i, π*; she later changed her mind: *π* is a number—but not *[recognise a given math concept (naïve or beginning understanding)]*;

(c) She first stated that *i* means "infinity" and then changed her mind and stated that *i* stands for "irrational" *[(attempt to) recognise a given math concept]*;

(d) +4 and − 4 mean the same thing, since they each just reduce to 4 *[(attempt to) recognise a given math concept]*; and

(e) She tried to seek connections throughout her discussion *[extend concept by abstracting properties]*.

Next, I summarised Tabitha's post-responses to Item 1:

(a) She identified whole numbers and fractions and those numbers that cannot be written as fractions: "so irrational" *[recognize and understand a given math concept]*; she also asked: "can prime numbers be irrational?" *[awareness of types of questions]*;

(b) In her Venn diagram she needed two non-intersecting circles one for rational, one for irrational *[recognise and understand a given math concept]*;

(c) She again listed dyads of numbers: rational/irrational; prime/composite; imaginary/complex *[recognise and understand a given math concept]*;

(d) Her post-response included imaginary versus real numbers (Figure 2) *[recognise and understand a given math concept; extend concept by abstracting properties]*;

(e) She also added infinity and zero to her diagram, noting that these particular numbers represented divine connections for her *[extend concept by abstracting properties; deal with scope of given mathematical concept in different domains]*.

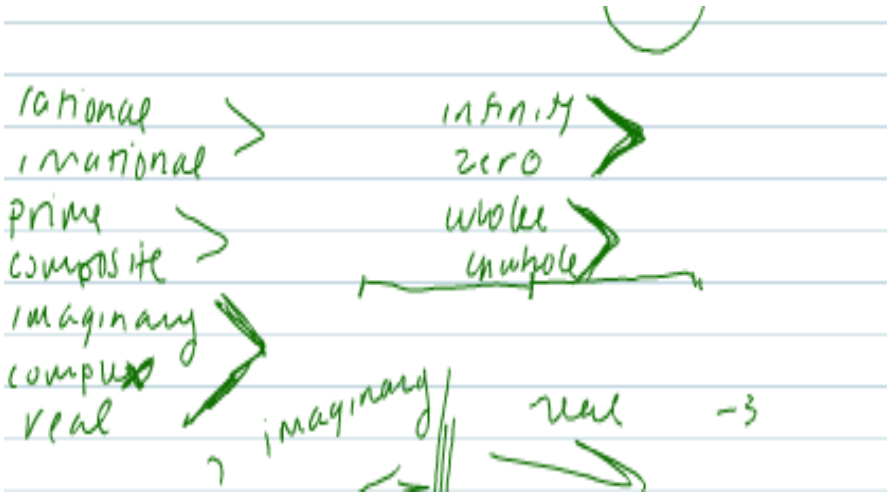

Figure 2: Tabitha's list if types of numbers needed for her Venn diagram (post-response)

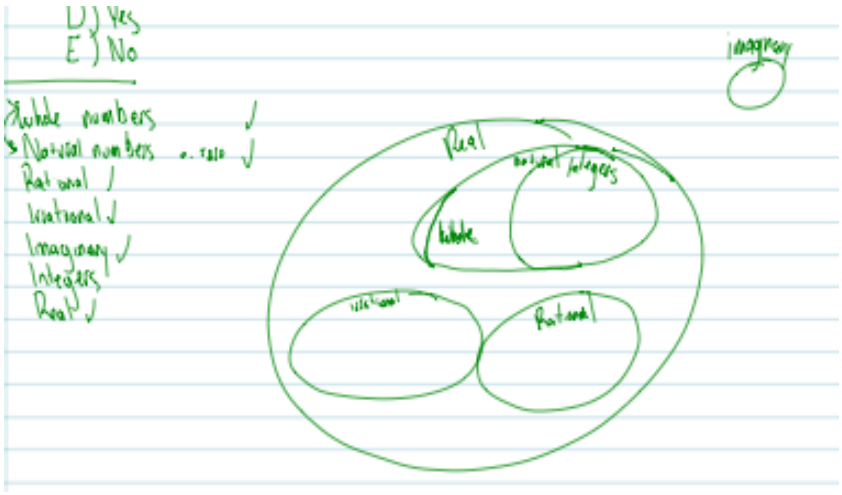

Figure 3: Michael's pre-response Venn diagram

Finally, I constructed a mathematical thinking competency profile for Tabitha (Item 1, complex number system), based upon the elements from the mathematical thinking competency (in bold in the summary). This profile shows growth from pre- to post-instruction, in that the variety of aspects from the mathematical thinking competency developed from several instances of only two aspects to multiple instances of four aspects of mathematical thinking.

Tabitha: Item 1 (complex number) pre-response

- Recognise a given math concept (four instances, including two at a beginning or naïve level of understanding)
- Extend concept by abstracting properties

Tabitha: Item 1 (complex number) post-response

- Recognise and understand a given math concept (four instances at varying levels)
- Extend concept by abstracting properties (two instances)
- Deal with scope of given mathematical concept in different domains
- Awareness of types of questions

The case of Michael

Michael, a mathematics major who recently changed his program of study (major) to mathematics education, possessed stronger mathematics content knowledge than Tabitha. However, the history and philosophy of mathematics course experience did appear to impact his mathematical thinking competency. Michael's pre-response summary and the identification of the relevant competency aspects for Item 1 (complex number system) included:

(a) He first listed kinds of numbers, including whole, natural, rational, irrational, imaginary, and integers (of the last category Michael asked: "where do these belong?") *[recognise and understand a given math concept (transitional understanding, not completely sophisticated)]*;

(b) He asked: Are you going to be paying attention to how big my circles are?...because there is this whole debate on if there are more rational or irrational numbers" *[extend concept by abstracting properties]*

(c) He was careful to make sure that the circles representing the sets of rational and irrational numbers did not overlap in the Venn diagram, but he did want the two circles to touch (see Figure 3) *[recognise and understand a given math concept (transitional understanding)]*.

In his post-response to Item 1:

(a) Michael began the discussion of his Venn diagram by beginning with the top of the hierarchy for the number system as he understood it: "...[we have] real numbers, then we have imaginary, then you'd have... complex numbers, which is a little of both" *[recognise, understand, and deal with the scope of a given math concept]*;

Figure 4: Michael's post-response Venn diagram

(b) He listed sets of numbers as he did before: whole, natural, integers; and real, irrational, algebraic, transcendental, and transfinite *[recognise, understand, and deal with the scope of a given math concept]*; and

(c) He felt there was more that he could do with classifying types of numbers: "I feel like I could do this forever; just drawing circles and breaking them up... because even with all of these you could define them into primes and which primes are seen more often, and..." *[extend concept by abstracting properties]*.

Using the summary above, I constructed Michael's mathematical thinking competency profile for this item:

Michael: Item 1 (complex number) pre-response

- Recognise and understand a given math concept (two instances; though not completely correct on each instance (e.g., "transitional understanding"))
- Extend concept by abstracting properties (one instance)

Michael: Item 1 (complex number) post-response

- Recognise, understand, and deal with scope of a given math concept (two instances)
- Extend concept by abstracting properties (stronger example than in pre-response)

Michael's understanding of the complex number system before participating in the HPhM course was fairly strong. Yet even with his understanding of the complex number system, his mathematical thinking competency was strengthened by the end of the course.

DISCUSSION

The pre- and post-response summaries, identification of mathematical thinking competency aspects, and construction of a competency profile for Tabitha's and Michael's work on Item 1 (complex number system) attempt to show that the HPhM course had some influence on students' mathematical thinking competency—at least from the perspective that the students themselves noted this while cognitively addressing the content before them. For example, Michael's ability to discuss and organise the different numbers of the complex number system (although "complex" was purposely not given to participants in the item or during the think-aloud interview) was already strong before the HPhM course. However, after the course, Michael revealed a more complete or nuanced understanding of the complex number system, which he stated was due to his reading of a required course text, *Zero: The Biography of a Dangerous Idea* (Seife, 2000). Classroom observations also revealed that Seife's book was used during classroom discussions and assignments.

Tabitha's post-instruction responses do reflect a better understanding of the types of numbers comprising the complex number system. However, except for a brief reference to learning about Venn in a class lecture, it is difficult to explicitly link the change in her thinking about the complex number system to the HPhM course. That said, I believe the course did influence her knowledge because as a Secondary History Education major, it is unclear from where she would

have gained such knowledge during the semester of study. Instead, an attempt to identify changes in Tabitha's mathematical thinking competency shows promise for characterising the impact of HPhM courses on tertiary students' mathematical experience.

CONCLUSION

This paper reports only a very small subset of results from a pilot study conducted in 2012 and its purpose is to highlight the potential for history and philosophy of mathematics courses to enhance students' mathematical learning. In the current attempt, I tried to qualify the profile of two students' mathematical thinking competency, using student responses to identify aspects of the competency (Niss & Højgaard, 2011), for a single item from the interview instrument developed for this project. In the next step I will analyse responses to several additional items from the interview instrument. Then, in a future publication, all four student cases (Darren, Jenny, Michael, and Tabitha) will be presented, along with supporting data from classroom observations and instructor documentation, in order to reveal a more extensive landscape of changes in students' mathematical thinking competency resulting from their experience in a history and philosophy of mathematics course. In the interim, it is my hope that this initial report will promote discussion about similar empirical studies.

REFERENCES

Clark, K. M. (2012). History of mathematics: illuminating understanding of school mathematics concepts for prospective mathematics teachers. *Educational Studies in Mathematics, 81*(1), 67–84.

Corbin, J. M., & Strauss, A. L. (2008). *Basics of qualitative research: Techniques and procedures for developing grounded theory.* Thousand Oaks, CA: Sage.

Huntley, M. A., & Flores, A. (2010). A history of mathematics course to develop prospective secondary mathematics teachers' knowledge for teaching. *PRIMUS, 20*(7), 603–616.

Miller, C. C. (2002). Teaching the history of mathematics: A trial by fire. *PRIMUS, XII*(4), 334–346.

Niss, M., & Højgaard, T. (Eds.) (2011). *Competencies and mathematical learning: Ideas and inspiration for the development of mathematics teaching and learning in Denmark.* IMFUFA tekst, Nr. 485–2011. Roskilde, Denmark: Roskilde University.

Seife, C. (2000). *Zero: The biography of a dangerous idea.* New York, NY: Penguin Books.

Young, K. A. (2005). Direct from the source: the value of 'think-aloud' data in understanding learning. *Journal of Educational Enquiry, 6*(1), 19–33.