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# The history of mathematics in the lower secondary textbook of Cyprus and Greece: Developing a common analytical framework

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In this paper, we examine the ways the history of mathematics is integrated in the national textbooks of Cyprus and Greece. Our data-driven analyses suggest that the references identified can be clustered in four categories: (a) biographical references about mathematicians or historical references regarding the origins of a mathematical concept, (b) references to the history of a mathematical method or formula containing a solution or proof, (c) mathematical tasks of purely cognitive elements that require a solution, explanation or proof and (d) tasks that encourage discussion or the production of a project that would connect the history of mathematics with life outside mathematics.

**Keywords**: History of mathematics, textbooks, Cyprus, Greece.

## INTRODUCTION

A number of recent studies highlight the importance of introducing elements from the History of Mathematics (HoM) in lessons as an alternative and supplementary means to establish more efficient teaching and learning of the subject (see, for example, Fauvel & van Maanen, 2000; Siu & Tzanakis, 2004). While recognizing the various pedagogical benefits of such an incorporation -as, for example, the development of more positive attitudes towards the subject (Farmaki et al., 2004), better conceptual understanding of mathematical ideas (Fasanelli et al., 2002), the elimination of racial discriminations (Michalowicz et al., 2002; Strutchens, 1995) and so on - this paper describes and discusses the developed framework that facilitates our analyses for the textbooks of the two countries under scrutiny, Cyprus and Greece, as well as some of our findings.

# COMPARING MATHEMATICS TEXTBOOKS ACROSS COUNTRIES

Textbooks constitute the main teaching resource for most countries around the world (Mullis et al., 2012), especially for highly centralised educational systems, as are those of Cyprus and Greece. Textbooks play a significant role in the ways mathematics lessons are designed and delivered; their influence over the content of the lessons; the instructional approaches; the quality of activities assigned for pupils, in classrooms and for homework; and, the learning outcomes and achievement (Alajmi, 2012; Törnroos, 2005; Weiss et al., 2003; Robitaille & Travers, 1992). Drawn from the well-known trichotomic distinction of the curriculum in the intended (vision and intentions as specified in official curriculum documents and/or materials), the implemented (teachers' interpretation and enactment of the curriculum according to their perceptions and knowledge) and the attained (learning experience as perceived by learners and their resulting learning outcomes), mathematics textbooks could be seen as a mediator between the first two types (or what Schmidt, et al. call the potentially implemented curriculum). From this perspective, Mesa (2004, pp. 255-256) talks about "a hypothetical enterprise: What would students learn if their mathematics classes were to cover all the textbook sections in the order given? What would students learn if they had to solve all the exercises in the textbook?" Rezat and Straesser (2014) take this argument a step further and distinguish between three perspectives on textbooks analyses. The first perceives textbooks as curriculum materials that offer supplementary ideas for teaching. From another perspective, textbooks are approached as artifacts that are employed for the preservation and transition of acquired skills. Analyses from this perspective focus only on opportunities to teach and to learn. Lastly,

textbooks can be seen as *instruments*. Such analyses take into consideration the actual use of textbooks in lessons.

Many studies make use of various frameworks for analysing mathematics textbooks, which include, among other things, their physical characteristics (i.e. size, number of pages, volumes, and so on), the structure of the lessons, the topics covered, the presentation of particular mathematical concepts, and the level of the cognitive demands of tasks (Bayazit, 2013; Alajmi, 2012). As far as cross-national comparisons of textbooks are concerned, a specific interest is shared between scholars, which shows how the cultural expectations and goals of different educational systems are manifested in the instructional materials produced (Haggarty & Pepin, 2002). Cross-national textbook analyses typically fall under three broad categories; namely, horizontal, vertical, and contextual (Charalambous et al., 2010). Horizontal analysis focuses on textbooks' general characteristics (i.e. content, structure, etc.), while the textbook is examined as a whole. Such examples include Stevenson's and Bartsch's (1992) analyses of Japanese and American textbooks, and the work of Campbell and Kyriakides (2000), who have investigated Cypriot and English elementary textbooks as part of the national curriculum of the two countries. In vertical analysis, the interest is on how the textbooks under examination treat certain mathematical concepts. From this perspective, Ding and Li (2010) discuss the ways the distributive property is presented in US and Chinese elementary textbooks, while Alajmi (2012) examines how elementary textbooks in the US, Japan, and Kuwait address fractions. Finally, contextual analysis is concerned with the role of the textbooks in classroom activities, either by the teacher or the pupils (see Remillard, 2005). Only few comparative studies employ more than one of the types of analysis above, as, for example, Haggarty's and Pepin's (2002) work with English, French and German textbooks, and their function in classrooms, and Charalambous and colleagues' (2010) framework developed for analysing learning opportunities provided by Cypriot, Irish, and Taiwanese textbooks.

# THIS STUDY

In this paper, we analyse references included in the lower secondary textbooks of Cyprus and Greece in regards to the HoM. Both countries have highly centralised educational systems (Charalambous et al., 2010; Saiti & Eliophoto-Menon, 2009) in which schools are considered segments of the domain of government, and not of the community. The national curricula and continued textbooks are prepared by each country's respective Ministry of Education, more specifically, by the Pedagogical Institute, a department of the Ministry of Education. In both the Cypriot (MoEC, 2010) and the Greek intended curriculum (MoERA, 2002), explicit references are made to the importance of the HoM. Nevertheless, the purpose of this paper is to examine how the general curricular references to the HoM are actually transformed into learning opportunities in the Cypriot and Greek textbooks for both teachers and pupils.

In our study, textbooks are treated as artifacts (Rezat & Straesser, 2014) since we examine their content in relation to the HoM, and not in relation to other teaching material, or the ways by which they are utilized in classrooms. Furthermore, our approach could be regarded as a comparative vertical analysis (Charalambous et al., 2010), which focuses on how the two sets of textbooks, Cypriot and Greek, treat a particular concept, namely the HoM.

For the purposes of this project, we analysed the national textbooks of Cyprus and Greece for lower secondary education (grades 7, 8, and 9). The educational system of Cyprus is currently reforming its curriculum for all subjects and grades; consequently, only the textbooks for lower secondary education were available to us once our study began, therefore, we chose to focus our analyses on the three lower secondary grades. The Cypriot textbooks were launched between 2012 and 2013 by the Pedagogical Institute of Cyprus, and the Greek ones were introduced in 2007 by the Pedagogical Institute of Greece.

At the first stage of data collection, all references to the HoM in both textbook series were identified. We were interested in both encyclopedic pieces of information and mathematical tasks inviting pupils to interact with them and provide solutions or answers. We worked independently with the two data sets (Cypriot and Greek), trying to find ways of clustering the references identified. Due to the long distance, in person meetings were not possible; however, after individual progress we had online meetings to discuss our ideas, which were, interestingly, similarly handled. After combining our working ideas, we applied



Η δουλειά του γερμανού μαθηματικού Georg Cantor (1845 – 1918) σημαδεύει τη δημιουργία της θεωρίας συνόλων. Πριν από την έρευνα του Cantor η έννοιά τους γινόταν δεκτή σιωπηρά βασισμένη σε ιδέες από την εποχή του Αριστοτέλη. Το μαθηματικό του έργο είναι πολύ σπουδαίο. Θεωρείται δημιουργός της θεωρίας συνόλων και των απειροσυνόλων. Οι ιδέες του Cantor, που ήταν πολύ πρωτοποριακές για την εποχή του, όχι μόνο δεν έγιναν άμεσα αποδεκτές, αλλά βρήκαν και μεγάλη αντίδραση.

#### Figure 1: An example of a biographical reference

a data driven analysis (Kvale & Brinkmann, 2009) as well as the constant comparison process outlined by Strauss and Corbin (1998), which led to clustering the references in two broad categories. Further analyses resulted in the division of the two categories into two subcategories each. The first category was concerned with references that did not pose questions to pupils; they'd rather provide historical pieces of information. In this category, we could find (a) simple biographical references about mathematicians or historical references concerning the origins of a mathematical concept and (b) references to the history of a mathematical method or formula including a solution path or proof. Figure 1 shows an example of the first subcategory and provides biographical information about



Figure 2: An example of a historical reference to a method and its solution process

	No question(s) for pupils		Asks pupils to interact with it	
	Simple historical/biographical references	Show solution/proof of a method/formula	Mathematical tasks	Encourage discussion/ project
Cyprus	27	2	28	2
Greece	41	4	26	7

Table 1: Distribution of the historical references in the two textbook series

the life and work of the German mathematician Georg Cantor (Cypriot textbooks, grade 7, part A, p. 27).

Figure 2 shows an example of a historical reference to a method and its solution process. In particular, it explains how Ancient Egyptians used a piece of rope with 13 knots and 12 equal line segments to create right angles, a method called in the book, "the reverse of the Pythagorean theorem" (Greek textbooks, grade 8, p. 128).

The second category comprised tasks that invited pupils to interact with them and provide some sort of answer. This included (a) mathematical tasks of purely cognitive elements that require a numerical solution, explanation or proof and (b) tasks that encourage discussion or the production of a project that would connect the history of mathematics with life outside mathematics. Examples of these subcategories are illustrated in Figures 3 and 4 respectively. Figure 3 shows the method applied by the ancient Greek mathematician, Thales, for the calculation of the distance between a ship and the coastline. Furthermore, pupils are asked to prove Thales' method and explain how he could be sure that the distance was right (Greek textbooks, grade 9, p. 197).

Figure 4 provides an example of a task that requires pupils to carry out a small project comparing and contrasting the number systems of Mayans and of Babylonians-Sumerians, as well as discussing the difficulties of these two systems, which eventually led to a need to establish the positional decimal system (Cypriot textbooks, grade 6, part A, p. 85). It is worth stating that there are no previous references to the two ancient number systems in the textbooks. Pupils are asked to conduct their own research and find information about them, since the textbook implies that there is no further instruction about these systems. Table 1 shows the distribution of the HoM's references per country. No significant differences could be observed as regards the two broad categories and the references of the HoM from each country,  $\chi^2$  (1, N = 137) = 0.986, p = 0.32.

## **DISCUSSION AND CONCLUSIONS**

Clearly, both countries value the HoM, a fact not only expressed in their curricula, but also reflected in their national textbooks. This could be attributed to a common cultural-historic heritage and the profound role of ancient Greek mathematics in the development of contemporary mathematical thinking. Nevertheless, the two countries' educational traditions share much in common, historically speaking (Koutselini-Ioannidou, 1997). In the case of their mathematics textbooks, Cypriot textbooks, launched a few years after the Greek ones, include several slightly modified examples. For instance, in the Greek textbook of grade 8 (page 10) we find a mathematical riddle from the tombstone of the ancient Greek mathematician Diophantus, inviting passers to solve it and calculate the age of his death. In the textbook, an equation showing the solution to the riddle follows. In the Cypriot textbooks (grade 7, part A, p. 173) although the same riddle is included, no solution is presented and pupils are asked to calculate Diophantus' age.

About half of the HoM's references in the Cypriot series and more than half in the Greek one constitute biographical and historical information that do not ask pupils to interact with them in any way. From our experiences as former pupils in the two educational systems, such information typically remains unexploited in classrooms, in a similar way to those arguing that the mathematics curriculum so overloaded with topics that teachers do not have much time to dedicate to the HoM. Also, despite comments of authors like Lawrence (2008) and Jahnke and colleagues (2002), who see advantages of the HoM in lessons as an

# «ΝΑ Θ ΑΠΟ ΤΗΝ ΙΣΤΟΡΙΑ ΤΩΝ ΜΑΘΗΜΑΤΙΚ

#### Υπολογισμός της απόστασης ενός πλοίου από τη στεριά

Αν ένα πλοίο βρίσκεται στη θέση Α στη θάλασσα, εμείς στεκόμαστε στη θέση Β στη στεριά και θέλουμε να υπολογίσουμε την απόσταση ΑΒ, τότε:

- Ξεκινάμε από το σημείο Β και περπατώντας πάνω στην παραλία κάθετα στην ΑΒ διανύουμε μιαν απόσταση ΒΓ. Στο σημείο Γ βάzουμε ένα σημάδι, π.χ. στερεώνουμε ένα ραβδί και συνεχίzοντας πάνω στην ίδια ευθεία διανύουμε την απόσταση ΓΔ = ΒΓ.
- Στο σημείο Δ αφού βάλουμε ένα σημάδι, π.χ. μια πέτρα, κάνουμε στροφή και περπατώντας κάθετα στη ΒΔ σταματάμε όταν βρεθούμε σ' ένα σημείο Ε, από το οποίο τα σημεία Α και Γ φαίνονται να είναι πάνω στην ίδια ευθεία.





Τη μέθοδο αυτή, λέγεται, ότι εφάρμοσε πριν από 2.500 χρόνια περίπου ο Θαλής ο Μιλήσιος.

Πώς ήταν σίγουρος ο Θαλής ότι ΑΒ = ΔΕ; Μπορείτε να το αποδείξετε; Αναzητήστε τις πέντε προτάσεις που απέδειξε ο Θαλής και σημειώστε ποια απ' αυτές χρησιμοποίησε για να υπολογίσει την απόσταση του πλοίου από τη στεριά.

Figure 3: An example of a mathematical task

- Να μελετήσετε, να παρουσιάσετε και να συγκρίνετε το αριθμητικό σύστημα των Μάγια με το αριθμητικό σύστημα των Βαβυλωνίων – Σουμερίων.
  - (α) Να γράψετε τρεις αριθμούς με τα σύμβολα των Μάγια και των Βαβυλωνίων.
  - (β) Να εξηγήσετε τις δυσκολίες που παρουσιάζονταν και οδήγησαν στην ανάγκη καθιέρωσης του δεκαδικού συστήματος.



#### Figure 4: An example of a project-based assignment

opportunity for pupils to realise how mathematical knowledge of the past has influenced our modern everyday life, both series include very few project-based tasks. Once again, from our experiences, Cypriot and Greek secondary mathematics teachers perceive such tasks as not "mathematical" and often ignore them, as a result. It is worth mentioning that both series are not restricted to ancient Greek mathematics; they include references from many civilizations, and this is an opportunity considered important for the elimination of racial discriminations (Michalowicz et al., 2002; Strutchens, 1995).

In closing, we cannot but emphasize the significance of collaboration between researchers from different countries in respect to comparative enquiries. Scholars must be aware that things with the same name might have a different meaning and function across nations, and they ought, therefore, to avoid assuming that colleagues elsewhere share the same understanding. Furthermore, we would like to invite colleagues from other countries to draw on our emerged framework of analysis about the types of historical references in mathematics textbooks, and to examine the ways in which HoM is included in other curricula, textbook series, and classrooms around the world.

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