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# A comparative analysis of mathematics textbooks from Kosovo and Albania based on the topic of fractions

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*This study analyses the presentation of fractions in Kosovar and Albanian mathematics textbooks designed for students of one to fifth-grade. Physical characteristics of the selected textbooks, the presentation of fractions and the nature of the problems were analysed as well. Findings showed that Albanian mathematics textbooks covered more lessons on fractions than Kosovar textbooks. Textbooks from both countries focus mostly on part whole and operator construction. Also, the majority of problems focused on pure mathematics and single procedures. Few problems in Kosovar textbooks required problem explanation, while problem solving as a cognitive requirement was used more in Albanian textbooks. A common deficiency in all texts was the lacks of problems requiring special requirement.*

**Keywords:** Textbooks, fraction representation, subconstruct, problems.

## INTRODUCTION

Numerous studies have shown that there are enough similarities but also differences in curricula and textbooks of countries with different history, culture, language, economy and geographical stretch (Delaney, Charalambous, Hsu, & Mesa, 2007; Charalambous, Delaney, Hsu, & Mesa, 2010; Alajmi, 2012; Erbaş, Alacaci, & Bulut, 2012; Cheng & Wang, 2012; Özer & Sezer, 2014). It is shown also that in addition to the culture of a country, language and structure of the school, students' performance in certain countries is affected particularly by the educational background of teachers and textbooks (Cheng & Wang, 2012).

Kosovo and Albania have been carrying out innovative reforms to increase the quality of the system of education. Unfortunately, studies that show the degrees

of student achievement as a result of these reforms are very limited. While, the data for improved student achievement on internationally comparable standardized tests in Albania (PISA, 2012) are welcome, for Kosovo, participation in international assessment programs still remains a challenge. Thus, both countries are making the efforts to reform school curricula and textbooks to achieve the educational objectives of European Union (EU) and Organization for Economic Co-operation and Development (OECD). Since majority of the population in Kosovo and Albania speak the same language, have the same culture and tradition, regarding the improvements on education there are efforts to unify Curriculum for Pre-University Education System, with the assumption that this unification will have impact on improving quality in schools and students' learning. By considering the textbooks as an important part of classroom life, as very important tools for quality assurance (Pehkonen, 2004; Nicol & Crespo, 2006) and as an important indicator that enable students to reflect on school curricula (Erbaş, Alacaci, & Bulut, 2012), it seems important for us to explore how fractions as one of the key concepts presented in elementary mathematics textbooks are used in Kosovo and Albania.

## BACKGROUND AND RESEARCH QUESTIONS

Many researchers identified several factors which contribute to students' learning fractions. Teachers can improve students' fraction learning by placing more emphasis on examining and improving the design of their instruction (Brown & Quinn, 2007; Pitsolantis & Osana, 2013). Keijzer and Terwel analysed how low-achieving students learn fractions in the context of Realistic Mathematics Education (2004) and Reimer and Moyer found that the virtual manipulative helped students in the class to learn

more about fractions (2005). However, the textbooks are the major written source which play an important role in guiding teachers in lesson planning and present them to their classes (Li, 2000; Alajmi, 2012). Textbooks, as an important resource in support of teaching and learning have received increased attention from the international education community over the last decades (Nicol & Crespo, 2006; Fan & Zhu, 2007; Charalambous, Delaney, Hsu, & Mesa, 2010; Alajmi, 2012; Erbaş, Alacaci, & Bulut, 2012). The TIMSS survey found that in almost all countries, the majority of teachers use textbooks as the main written source in deciding how to present a given topic in their classrooms (Beaton, et al., 1996). Textbooks influence what teachers teach, how they teach it, and how they measure the learning opportunities (Alajmi, 2012; Özer & Sezer, 2014). We could consider that textbooks help teachers with their workloads because the books provide ready and sensible structures for lessons and enough exercises for pupils (Pehkonen, 2004). They are an instrument for learning as well as an object of learning, and textbooks address both students and teachers (Erbaş, Alacaci, & Bulut, 2012). Because of their importance, many researchers focused on reviewing mathematical textbooks regarding the presentation of a particular mathematical concept or they focused on the cognitive demands of the material in the texts (Kieren, 1976; Behr & Harel, 1990; Alajmi, 2012). Fractions are among the most complex mathematical concepts that children learn in primary school. It was documented that teaching and learning of fractions has traditionally been one of the most problematic areas and one of the most serious obstacles in primary school mathematics (Behr, Post, Harel, & Lesh, 1993; Charalambous & Pitta-Pantazi, 2007). The misconceptions that students have about fractions, both in terms of fractions as numbers and how to operate with fractions relates particularly to the way in which fractions are represented and how they are taught (Barmby, Harries, Higgins, & Suggate, 2009). Various theoretical models have been proposed for understanding fractions in the elementary school (Behr et al, 1993; Charalambous & Pitta-Pantazi, 2007; Nicolaou & Pitta-Pantazi, 2011). Early subconstruct theories postulated that integrating the qualities of multiple perspectives were crucial to the understanding of fractions (Moseley, 2005). According to Behr and colleagues (1993), rational numbers can be interpreted as a part-whole comparison, an indicated division or quotient, a ratio, as an operator and as a measure. The part-whole construct refers to a contin-

uous quantity or a set of discrete objects partitioned into parts of equal size. This perspective emphasizes situations which are related to the comparison of parts with the total amount. On the other hand the quotient construct considers a fraction as a result of a division of two whole numbers. The ratio perspective is based on comparing separate quantities and usually presented as  $a:b$  or  $a/b$ . Whereas the ratio is a relation between two sets, an operator is a relation between two states of a set. The operator construct reflects a function that transforms line segments, figures, or numbers. And lastly the measure construct identifies fractions as numbers or associating fractions with the measure assigned to some interval (Kieren, 1976; Behr & Harel, 1990).

Using multiple representations makes the fraction concept more concrete and understandable. But, it was shown that even in cases where students seem to understand the conceptual meaning (Hiebert & Carpenter, 1992) of the different perspectives of fraction representations they still struggle to connect fraction ideas to real world problems (Moseley, 2005; Alajmi, 2012). Textbooks should focus not only on problems that have low-level cognitive requirements, such as recall and reproduction (Li, 2000). Also, they should not present only the pure mathematics contexts but they should emphasize real-world problems dealing with fractions in all lessons (Alajmi, 2012).

This study reviews the similarities and differences of the presentation of fractions and problems of fractions in textbooks in Kosovo and Albania. Specifically, the study addressed the following questions:

- 1) What are the physical characteristics of first-to fifth-grade mathematics textbooks in the selected series in Kosovo and Albania?
- 2) What similarities and differences can be observed in the representation of fractions in the mathematics textbooks in Kosovo and Albania?
- 3) How are problems involving fractions introduced in these textbooks?

## METHODS

### Textbook sampling

Two elementary mathematics textbook series from Kosovo and Albania were chosen for this study.

Kosovo has a national curriculum. In this system only one textbook series is available, and it is published and distributed at no cost to students by the Ministry of Education, Science and Technology - MEST. Thus, all public primary schools use the same mathematics textbook series. Albania also has a national curriculum but any of fifteen commercial series of textbooks may be used in Albanian schools. Teachers together with the parents' council analyse each of these series of books and choose which textbooks will be used throughout the school year. For this study we selected the series of EduAlba publishing house, because these textbooks are used as secondary textbooks by some schools in Kosovo. Focusing on the fraction content, this study examined textbooks from both countries in detail, with respect to two aspects: presentation of fractions and the mathematics problems provided for students' practice. In this study we have taken into account only the basic textbooks and not the student workbooks or other materials.

### Data Analyses and Procedures

In this study a framework and the coding system was developed in two stages. First we analysed the textbooks' physical characteristics, such as page size, number of pages and number of pages containing fractions while using a horizontal approach which gives readers an initial introduction of the textbooks. Secondly, for the vertical-analyses (Charalambous et al., 2010) we read each lesson relevant to fractions from each textbook carefully in order to analyse the meaning of tasks (Cheng & Wang, 2012). The meaning of tasks here indicated a meaning of a specific task, a definition, a picture, a question, an example, practical exercise or a problem that serves as a particular purpose in the lessons. Based on the meaning of tasks we developed the coding system which includes the following two dimensions: conceptual and contextual. Under first dimension we analysed the fraction meaning and presentation based on Kieren (1976) model of the five subconstructs: the part-whole, the ratio, the operator, the quotient and the measure construct. For the second dimension, Li's (2000) framework was used to analyse textbook problems provided to students. The problems selected from each textbook were those exercises or questions that did not have solutions or answers. Each problem was coded based on the three dimensions below:

- 1) Mathematical features: Single procedure (S) and Multiple Procedure (M);
- 2) Contextual features: Purely mathematical context in numerical or word form (PM) and illustrative context or story (IC);
- 3) Performance requirements:
  - 3.1 Response type: no explanation or solution process required (numerical answer or numerical expression only) (NES) and explanation or solution process required (ES)
  - 3.2 Cognitive requirement: conceptual understanding (CU); Procedurals practice (PP); Problem solving (PS) and Special requirements (SR).

Cognitive requirement mean 'the kind and level of thinking required from students in order to successfully engage with and solve the task (Stein, Smith, Henningsen, & Silver, 2000, p. 11), while special requirement (SR) refers to the problems that contain special or mixed cognitive requirements. For example, the problem asks students to write up a problem based on given information (Li, 1990).

After we selected the pages with fractions' content from each textbook and we coded them independently, the tasks that were coded differently were discussed and recoded again for compatibility. The researcher's agreement was 97 %.

## RESULTS

### Physical characteristics

Textbooks in Kosovo differ from Albanian textbooks. Textbooks in Kosovo are larger in size (28.5 x 20.5 x 0.8 cm) in each grade level. Albania textbooks have smaller sizes (27 x 20.5 x 1 cm). For the first time fractions are introduced in the second grade texts both in Kosovo and Albania. The number of pages devoted to fractions increased by grade to grade level in all two textbook series. In Kosovar textbooks, the percentage of pages grew from 2.34% on the second grade to 10.73% on the fifth grade, while in Albanian texts the percentage of pages grew from 3.15% to 24.89 %. Based on these facts the number of tasks per page related on fractions is bigger in Kosovar texts (5–6 tasks per page) than in Albanian textbooks (2–3 tasks per page). Also, the number of lessons on fractions increased each year from three lessons in the second grade to fourteen lessons in the fifth grade in Kosovar

textbooks while in Albanian text books it increased from one lesson in the second grade to eleven lessons in the fifth grade. Addition and subtraction of fractions with same denominator for the first time are presented in third grade in the Albanian textbooks, though in Kosovar textbooks they are presented for the first time on the fifth grade. Also, in the Kosovar textbooks decimal numbers are not covered in the textbooks, whereas in the Albanian textbooks they are introduced in the fifth grade. The greatest difference in these textbooks is related to the quantity of real world problems included. It was found that the Albanian textbooks emphasize real world problems more than their Kosovo counterparts. Only two real world problems were found in the Kosovo fifth grade textbook, while in Albanian textbooks there are two real world problems in the second grade, twelve in the third grade, twenty-six in the fourth grade level and thirteen real world problems in the fifth grade level.

**Fraction meaning and presentation**

Based on the model of the multiple representations of fractions, the fraction constructs were identified in all textbooks (Table 1). Table 1 shows that the part-whole construct is used from the 2<sup>nd</sup> grade, when fractions appear for the first time, up to 5<sup>th</sup> grade in both series of textbooks. It was identified that the part-whole construct was used in a higher percentage in 2<sup>nd</sup> grades and 3<sup>rd</sup> grades, while in 4<sup>th</sup> and 5<sup>th</sup> grades the percentage of tasks presented by this construct is reduced. It seems that the ratio and quotient constructs were used only in a few tasks. In Kosovar textbooks there are only two tasks presented with the ratio construct, while in Albania textbooks, the ratio construct appears in 3<sup>rd</sup> grade and 4<sup>th</sup> grade, with a total of six tasks. It was found that only 5 tasks in the 5<sup>th</sup> grade Albanian textbook used the quotient construct. In Kosovar textbooks there were no tasks presented with the quotient construct. The dominant

construct among tasks identified in 4<sup>th</sup> and 5<sup>th</sup> grade textbooks in both series of textbooks was the operator construct. Most of the tasks in 4<sup>th</sup> and 5<sup>th</sup> grade of both series of textbooks used the operator construct (about 70% in 4<sup>th</sup> and 87% in 5<sup>th</sup> grade Kosovar textbooks, whereas in Albanian textbooks, about 91% and 94% of all tasks). The measure construct was identified in three Kosovar textbooks (grade 3, 4 and 5) while in Albania only one task in the textbook of the 3<sup>rd</sup> grade was identified.

**Classification of problems with fractions**

Table 2 shows the percentages of problems classified in terms of problems' requirements (Li, 2000). It was found that there is a significant difference between the types of problems presented in the textbook series of the two countries.

From the third to fifth grade in Kosovar series of textbooks we have a higher percentage of problems or practice exercises which requires a single procedure (SP) in the solution process, while in Albania in third grade texts 76% of the tasks require multiple procedures (MP). Also we founded that 50% of problems require single procedures and 50% multiple procedures in the Albanian textbook of the 4<sup>th</sup> grade. In contrast, the Albanian textbooks of the 5<sup>th</sup> grade contained about 85% of the problems that required a single procedure for the solution. In Kosovar 3<sup>rd</sup> grade textbook, the majority of problems are presented through illustrative contexts (76.47%). In contrast, Albanian third grade textbooks have more pure mathematics problems (52.78%) than illustrative context exercises (47.22%). In the 4<sup>th</sup> and 5<sup>th</sup> grade textbooks we have approximately the same percentage of tasks with PM and IC. What is also important to mention (see Table 2) is the number of tasks that require only the numerical result (NES) of the assignment and no explanation on how the solution was reached (ES). Only in the Kosovo

Country	Kosova										Albania										
	1		2		3		4		5		1		2		3		4		5		
Grades	N#	%	N#	%	N#	%	N#	%	N#	%	N#	%	N#	%	N#	%	N#	%	N#	%	
Part whole	/	8	66.67	18	78.26	8	21.62	7	9.72	/	13	92.86	26	37.68	5	7.35	1	1.0			
Ratio	/	0	0	0	0	2	5.41	0	0	/	0	0	5	7.25	1	1.47	0	0			
Operator	/	4	33.33	4	17.39	26	70.27	63	87.5	/	1	7.14	37	53.62	62	91.18	94	94.0			
Quotient	/	0	0	0	0	0	0	0	0	/	0	0	0	0	0	0	0	5	5.0		
Measure	/	0	0	1	4.35	1	2.70	2	2.78	/	0	0	1	1.45	0	0	0	0			
Total	/	12	100	23	100	37	100	72	100	/	14	100	69	100	68	100	100	100			

Table 1: The fraction presentation based on the model of the five subconstructs

	Grade		Mathematical features		Contextual features		Performance requirements					
			S	M	PM	IC	Response type		Cognitive requirement			
							NES	ES	CU	PP	PS	SR
Kosova	2	N#	-	-	-	5	-	-	6	-	-	-
		%	-	-	-	100	-	-	100	-	-	-
	3	N#	4	3	4	13	17	-	15	-	1	-
		%	57.14	42.85	23.52	76.47	100	-	93.75	-	6.24	-
	4	N#	18	3	16	21	36	-	27	3	-	-
		%	85.71	14.28	43.24	56.75	100	-	90	10	-	-
	5	N#	31	3	37	18	43	5	35	15	1	-
		%	91.17	8.82	67.27	37.72	89.58	10.41	68.62	29.41	1.96	-
Albania	2	N#	-	-	-	5	-	-	5	-	-	-
		%	-	-	-	100	-	-	100	-	-	-
	3	N#	6	19	19	17	42	-	6	10	-	-
		%	24	76	52.78	47.22	100	-	37.5	62.2	-	-
	4	N#	18	18	22	25	46	-	29	3	20	-
		%	50	50	46.81	53.19	100	-	55.77	5.77	38.46	-
	5	N#	45	8	35	9	33	-	30	13	10	-
		%	84.91	15.09	79.55	20.45	100	-	56.60	24.53	18.87	-

**Table 2:** Problems Classified According Li's Framework (Li, 2000)

5<sup>th</sup> grade textbook about 10.4% have problems that required the explanation. Regarding cognitive requirements, it was identified that in textbooks from Kosovo as well as in Albanian textbooks, the largest number of problems requires conceptual understanding (CU) and procedural practice (PP). There are very limited requirements for problems which ask for problem solving in Kosovar textbooks. There was only one problem in each of the 4<sup>th</sup> and 5<sup>th</sup> grade Kosovar textbooks, while in the Albanian textbooks it was a greater percentage of problem solving (about 38% and 19% of problems in the 4<sup>th</sup> and 5<sup>th</sup> grade respectively).

A common deficiency in all analysed texts is the lack of problems requiring special requirements (SR). Textbooks do not provide assignments in which students have the opportunity to practice more complex cognitive requirement or require contextualising fractions in everyday life.,.

**DISCUSSION**

This study has analysed the physical characteristics of the textbooks, fraction presentation and nature of problems used in Kosovar and Albanian textbooks. In both series of textbooks, fractions are presented for the first time from the 2<sup>nd</sup> grade. The number of pages covered by fractions is larger in the Albanian

textbooks than in the Kosovar textbooks. But, it was observed that there is a difference in terms of the number of tasks, associated figures, tables or diagrams per pages. While in the books from Kosovo there are five or six tasks per page, in Albanian books every page has at most 3 tasks. Also, there are differences in terms of content. An introduction to fractions as decimal numbers is presented only in the Albanian textbooks. It seems that Kosovar textbooks covered less content and less representations for fractions. Thus, low fraction content coverage can have a negative effect on the student's abilities to use fractions in the future. According to Brown & Quinn (2007), students should spend the majority of their time learning fractions before the 6<sup>th</sup> grade if we want to devote the time to the developing of their conceptual understanding of fraction relationships. Both series of textbooks used mostly two constructs for fraction presentation (part-whole and operator) and were focused more on rules and procedures, even though using multiple representations would assist students in developing more interconnected and viable representations knowledge for fractions (Behr et al., 1993; Charalambous & Pitta-Pantazi, 2007; Moseley, 2005).

It was found that the majority of problems in both series of textbooks are presented in purely mathematical contexts and that almost in all problems

single-procedures are required. Also, it was found that the largest number of tasks requires contextual cognitive requirement with illustrative context. But, as Hiebert and Carpenter (1992) argue, there is a need to have a conceptual understanding, as well as the necessary skills for procedural practice, in order to be successful in mathematics. Only few problems were found in Kosovar textbooks that demanded problem explanation and problem solving. Based on Li's point of view, focusing only in the problems that have low-level cognitive requirements, such as recall and reproduction (Li, 2000) doesn't help students to be good problem solvers. On the other hand, Albanian textbooks pay more attention to the idea of including more practice examples that require problem solving compared to Kosovo. However, it is important to mention a common deficiency in all analysed textbooks, which is the lack of practice exercises that require special requirements. In this case connecting fractions to students' lives and with real word problems is an important issue. Students should be provided with opportunities to see fractions in their real lives and use these meanings in the learning process. Therefore, the texts should assist teaching and learning in order to enable the development of the meaning of fractions, understanding them (Moseley, 2005; Alajmi, 2012) and solving problems from everyday life contexts.

The study has highlighted some differences that textbooks from different countries have, and aims to encourage authors of textbooks to cooperate at the international level in order to improve the quality of mathematics textbooks. A comparative study of mathematic textbooks from Kosovo, Albania and other countries will help to share the best practices for fraction introduction in elementary textbooks. The recommendation of this study is to increase both the number and the variety of problems in all textbooks, especially those with a high level of cognitive demand. Finally, taking in consideration that the elementary mathematics textbooks are the main instructional resource and learning tool, the teachers' and students' views on textbooks are crucial for their improvement.

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