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The aim of this study is to compare how Japanese and Australian teachers utilise opportunities to promote students' switching between mathematical modelling cycles based on the dual modelling cycle framework (DMCF). This study found that teachers need to change how they assist students when transitioning from one modelling cycle to another not only based on differing levels of student ability, but to account for differences between countries as well. The Japanese students had more sophisticated visualisation skills than the Australian students when working with the geometric structure of an ordinary helix on the side face of a cylinder. However, both groups of students benefited from use of the DMCF to develop their understanding of the mathematical problem as they moved between modelling cycles.

Keywords: International comparisons, dual modelling cycle framework (DMCF), ordinary helix, modelling teaching, mathematics education.

INTRODUCTION

Problem solving in the traditional mathematical classroom has tended to be an individual task. Students work in isolation as they go about problem solving. When teaching problem solving skills, teachers have relied on the work of Polya (1945). His approach included the hermeneutic of solving a similar, simpler problem which provided each student with an approach that they could use to find a solution for their problem. In more recent years there has been a realisation that to effectively function in society students need to develop the skills of being more flexible and creative problem solvers. Mathematical modelling provides an opportunity to develop these skills, as it is designed for group work that promotes collaborative interactions. This approach is usually set in the context of real world problems where information is incomplete or ambiguous, promoting questioning and the posing of conjectures (Brown & Walter, 2005). As a result it allows for multiple solution paths, permitting discussions around the *best* solution rather than *the* solution. In these situations research indicates that modellers' attempts to find a solution usually results in their shuttling between the real and mathematical worlds (e.g., Stillman, 1996; Stillman & Galbraith, 1998; Borromeo Ferri, 2007; Matsuzaki, 2007, 2011). According to Busse and Kaiser (2003), modellers construct their own *subjective figurative context* from the modelling task, and the modellers' perception of the task context can affect their modelling progress. When the modellers' modelling processes have stalled, evidence suggests that students move from their initial modelling task to a similar and simpler modelling task where some traction is considered possible. In this paper, we explore a theoretical extension to this approach to mathematical modelling, as limited research exists on how to facilitate the teaching of mathematical modelling when responding to a diversity of modeller abilities.

Saeki and Matsuzaki (2013) proposed a new theoretical modelling framework called the *dual modelling cycle framework (DMCF)* (see Figure 1). This DMCF re-conceptualises the modelling cycle explicated by Blum and Leiß (2007). In the case of solving an initial task located on the first modelling cycle, one modelling cycle is enough if modelling is proceeding successfully. If problem solving is unsuccessful or the

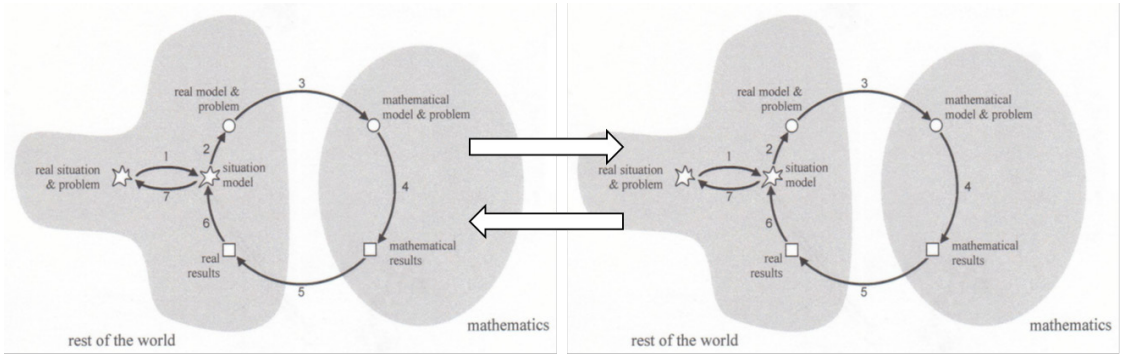


Figure 1: Dual modelling cycle diagram (Saeki & Matsuzaki, 2013, p. 94)

modeller does not know enough to solve the task, the modeller can be assisted by trying to solve a similar task as was proposed by Polya (1945) in his earlier work. One rationale for using two separate modelling cycles when changing from an initial modelling task (TASK1) to a similar and simpler modelling task (TASK2) is that there may be cases when doing so leads to more success.

Research by Saeki and Matsuzaki (2013) has identified that to support successful outcomes for all modellers using the DMCF, the most important point is for teachers to support switching between the first modelling cycle and the second modelling cycle by providing a similar and simpler task. Matsuzaki and Saeki (2013) implemented experimental modelling lessons for undergraduate students in Japan and identified three stages: transition from the first modelling cycle to the second modelling cycle, modelling within the second modelling cycle, and transition from the second modelling cycle back to the first modelling cycle. Kawakami, Saeki and Matsuzaki (2012, 2014) implemented DMCF-based modelling lessons with Year 5 elementary school students in Japan and classified six types of students' responses. They also described modelling lessons in terms of a first trial of two tasks, one in each modelling cycle; a second trial of TASK1 based on TASK2, and a final trial of TASK1 through classroom discussion. DMCF-based modelling lessons were also implemented with Year 6 students in Australia (Lamb, Kawakami, Saeki, & Matsuzaki, 2014), permitting international comparisons. The aim of this paper is to compare how teachers can assist students in switching between modelling cycles while supporting a diverse range of student capabilities within two different countries.

CHARACTERISTICS OF MATHEMATICS LESSONS IN JAPAN AND AUSTRALIA

In this paper, we explain the differences in mathematics teaching in Japan and Australia in order to make international comparisons between the two modelling lessons. Some of these differences are based on work by Mok and Kaur (2006), where characteristics of mathematics lessons are explained with a focus on the '*learning task*'.

The teaching strategy used by Japanese teachers is one that supports each student's level of ability. Teachers lead lessons by considering the needs of each student and providing a variety of activities to suit. Furthermore, many Japanese teachers have adopted problem-discovery oriented teaching methods based on Yamamoto's (2007) work, which outlines three stages when detailing such methods: (1) initial learning activities, (2) discovery of a problem that must be solved, and (3) solution of the problem. A characteristic of this method is to emphasize the children's change of awareness. Consequently this style of lesson can be challenging for the teacher. Thus this view of mathematics teaching matches the modelling teaching practice described above (Kawakami et al., 2012, 2014).

On the other hand, the teaching strategy adopted by Australian teachers relies on mathematical tasks based in daily-life contexts where students make links to their daily life activities (Mok & Kaur, 2006). Supporting this Australian teaching strategy, Stillman, Brown, Faragher, Geiger and Galbraith (2013) analyzed the goal of mathematics by analyzing textbooks and curricula in secondary classrooms in Australia from a socio-cultural perspective. This led to three findings: (1) textbooks were used as a foundation for teaching materials, (2) teaching materials were based

Figure 2: Oil tank image URL<http://blog.goo.ne.jp/kobeooi/e/b021c971381154725 fc3ee4a3d645aa8> [18 Mar 2014] (Note: Picture reversed in class)

in contexts that enhanced students' understanding of the world, and (3) assisted the development of a critical disposition towards the surrounding world that requires decisions to be made. Thus teaching tasks emphasized daily-life contexts that evoked a need for decision making. With this in mind we were conscious of the need for context based problem-solving in Australian schools and we found data for this perspective (Lamb et al., 2014).

EXAMINING THE HEART OF THE DUAL MODELLING CYCLE FRAMEWORK

We developed DMCF-based teaching material for elementary school students to assist them in understanding the geometric structure of an ordinary helix on a side face of a cylinder. The students were initially provided with a picture of oil tanks with differing diameters (see Figure 2). The students were then provided with an *Oil tank task* (TASK1) and a *Toilet paper tube task* (TASK2), displayed in Figure 3.

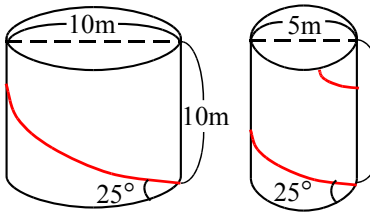
In our earlier research using the same task as above, we found that modellers who could not solve TASK1 were able to advance their modelling of this task by modelling a similar but simpler task, TASK2 (Kawakami et al., 2012; Lamb et al., 2014). Students who could solve TASK1 but were encouraged to engage with TASK2 developed a more advanced understanding of TASK1. By actively switching between TASK1 and TASK2, most students were able to solve TASK1 (see Kawakami et al., 2012; Lamb et al., 2014 for details). These tasks helped students understand the geometric structure of an ordinary helix on the side face of a cylinder. This structure is important because it forms the foundational knowledge necessary to solve the oil tank task in higher grades (using either the Pythagorean theorem or trigonometric ratio). Student investigation of the 3D model leads to understanding the rectangle model and the parallelogram model (see Figure 4).

The DMCF aims to deepen students' mathematical understanding by switching between two modelling cycles, as indicated in Figure 5. There are two kinds of switching that lead to in-depth engagement in the tasks.

The first is a teacher's intentional switching to facilitate student understanding. It is therefore very important for teachers to design an approach to switching through the use of teaching material before implementing the lesson. Teacher's intentional switching is done twice. The first instance of switching is the transition from the first modelling cycle to the second modelling cycle. In this transition, the teacher used the toilet paper tube to present an opportunity for students to work with a concrete object. The second instance of switching uses feedback from the second modelling cycle to return to the first modelling cycle. In this transition, it is necessary for all students to recognize that they have returned to TASK1. Therefore

***Oil tank task* (TASK1)**

There are several types of oil tanks. Their heights are equal but their lengths of diameters are different. Is the length of the spiral stairs on these oil tanks equal or not? As conditions, angles to go up spiral banisters are all the same.



***Toilet paper tube task* (TASK2)**

It is impossible to open along the actual spiral stair of the oil tank. We can use a toilet paper as a similar shape to an oil tank as it can be opened along its slit to show the 2D shape. Con

Figure 3: Teaching material based on DMCF

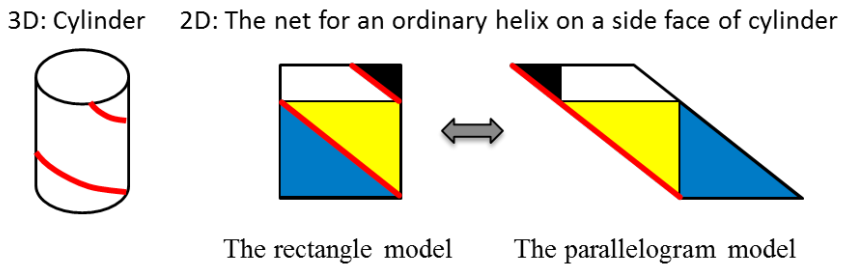


Figure 4: The geometric structure of an ordinary helix on the side face of a cylinder

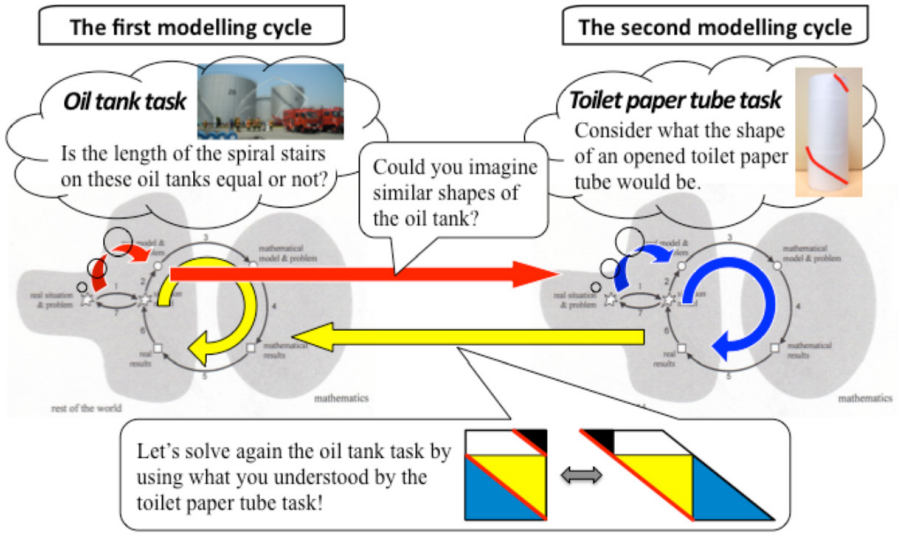


Figure 5: Dual modelling cycle diagram of switching between two modelling cycles

we offered the information of TASK1 to students again and let them predict whether the spiral stairs would be the same or not. It is necessary for the teacher to prepare methods of switching that accommodate differing levels of student ability. In the case of the Australian school, significant student difficulties necessitated a substantial change in the switching methods used (see description on pages 7 and 8).

The second is students' intentional switching to solve TASK1. This switching is important as it provides an opportunity for students to develop ideas by themselves. Hence, teachers have to prepare a range of alternative approaches to stimulate the transition between modelling cycles that correspond to differing student needs. It is important to note that one of the problems in this process is that some students lose track of which modelling cycle they are in. When this is the case it is necessary for teachers to guide students in understanding their position in the modelling sequence and the correct direction they need to take to move to TASK2 or back to TASK1.

THE MODELLING LESSONS IN JAPAN AND AUSTRALIA

Case of Japan

The Japanese experimental class consisted of three 45-minute lessons (see Kawakami et al., 2012). The class included 33 Year 5 students (aged 10 or 11) from a Japanese private elementary school.

Showing the Oil tank task

At the beginning of the lessons, the teacher showed photographs of two oil tanks and asked the students if the length of the spiral stair was equal or not (see the oil tank photograph, Figure 2). In order to simplify the *Oil tank task* (Figure 3), the teacher asked which part of the spiral stair should be measured, its banister or its steps. Through discussion with the students they agreed to measure the length of banister at the side of the oil tank. Then the teacher showed 3D models of the oil tanks displayed in Figure 3 and asked students what they could do to solve the *Oil tank task*. The students responded by producing 2D drawings.

Seventeen students (52%) were able to draw the mathematically correct 2D rectangle models of the oil tanks

from the 3D models, representing the spiral staircase as straight lines on their models (see Figure 6). The remaining students were not able to draw the mathematically correct models, as the representation of the staircase was not connected on their models (see Figure 7). Some students rounded their paper to check whether the staircase would be connected or not. However they did not make the full link to the mathematical structure of the spiral shown in Figure 4.

Teacher's intentional switching (1):

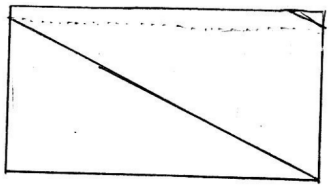


Figure 6: Mathematically correct models

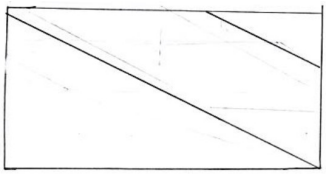


Figure 7: Mathematically incorrect models

Guiding to the Toilet paper tube task:
At the beginning of the second lesson, in order to switch from the first cycle of the *Oil tank task* to the second cycle of the *Toilet paper tube task*, the teacher asked the students to consider what objects were similar in shape to the oil tank, but smaller in size. The student responses included pencils, toilet paper tubes and so on. The teacher provided an actual toilet paper tube for each student and asked them to open the toilet paper tube. The students were asked to compare the length of each staircase and to produce a 2D drawing of the toilet paper tube (see the *Toilet paper tube task*, Figure 3). Almost all the students identified cutting the tube along the slit and were able to subsequently draw the parallelogram model.

Teacher's intentional switching (2):
Returning to the Oil tank task:
At the beginning of the final lesson, in order to switch from the second cycle of the *Toilet paper tube task* back to the first cycle of the *Oil tank task*, the teacher asked the students to compare the length of the staircase with reference to findings from the *Toilet paper tube task*. The students tried to solve the *Oil tank task* independently. Fifteen students made the parallelogram model of the oil tank and tried to calculate the length of the staircase.

During the final lesson, the teacher observed four different methods used to find solutions: (1) calculating the length of spiral stair (five students), (2) measuring the length of spiral stair in rectangle models and parallelogram models (four students), (3) translating the staircase in the rectangle models (three students) and (4) translating the staircase in the parallelogram models (six students). As a result of class presentations, students gained access to classmates' solutions and were able to solve the *Oil tank task* using parallelogram models or rectangle models of the oil tank.

Case of Australia
The Australian experimental class consisted of two 60 minute lessons (see Lamb et al., 2014). The class included 23 Year 6 students (aged 11 or 12) from an elementary school.

Showing the Oil tank task
At the beginning of the lessons, the teacher showed photographs of two oil tanks and framed the problem within the context of a fireman needing to climb to the top of one of the tanks as quickly as possible to extinguish a fire. The teacher then asked the students if the length of spiral stair was equal or not. The teacher showed 3D models of the oil tanks displayed in Figure 2 and asked the students to produce 2D drawings of the 3D models.

No student was able to draw the mathematically correct models of the oil tanks from the 3D models of the oil tanks. Eleven students (48%) drew the rectangular representation of the oil tank. However, in each case the students drew the staircase as a curved line on their 2D model. The remaining students (52%) were unable to draw a 2D model, tending instead to copy the 3D model provided for them.

Teacher's intentional switching (1):
Guiding to the Toilet paper tube task
In order to switch from the first cycle of the *Oil tank task* to the second cycle of the *Toilet paper tube task*, the teacher showed the students a toilet paper tube and asked them to predict what the toilet paper tube would look like when cut along the slit. No student was able to draw a mathematically correct 2D model. Six students (26%) drew a shape close to a parallelogram in which the spiral stair was curved. Other students pro-

duced shapes similar to a roll. To assist the students in finding the relationship between the *Oil tank task* and the *Toilet paper tube task*, the teacher asked them to cut the toilet tube vertically and confirm that the shapes created were parallelograms and rectangles.

As most students in the class were not able to first visualize and then draw 2D models from the 3D models of the oil tank or the toilet paper tube, at the beginning of the second lesson, the teacher used a concrete aid to demonstrate how the 2D models related to the 3D model. A rectangular piece of cardboard, rolled into a cylinder and marked with a red line, was cut at an angle and unrolled to illustrate the relationship between the 3D model and the 2D parallelogram model. Furthermore, the teacher cut a similar cylinder vertically and unrolled it to show the 2D rectangular model.

Teacher's intentional switching (2): Returning to the Oil tank task

In order to switch from the second cycle of the *Toilet paper tube task* back to the first cycle of the *Oil tank task*, the teacher again asked the students whether the spiral stair was the same or different for each oil tank. The students tried to solve the *Oil tank task* collaboratively. One group made another 3D model of the oil tank by using concrete 3D models. They opened the model and measured the length of spiral stair in the 2D rectangle model and the 2D parallelogram model. A student in the group explained, "I think they are all the same because the parallelogram and rectangle are almost the same size, so I expect they are the same".

During the last lesson the students were able to explain that both staircases were the same length for both the rectangle model and the parallelogram model through cutting and placing the pieces of their concrete 2D models on the whiteboard. The teacher demonstrated both models represented the same 3D model (Figures 8 & 9).

DISCUSSION

Using the *dual mathematical modelling cycle framework* (DMCF) to examine the same problem in both Japanese and Australian classes allows for comparisons to be made. The results from this study indicate that the teachers needed to change the method used to switch between modelling cycles intentionally to account for different levels of student ability. The Japanese elementary students in this study had more sophisticated visualisation skills and were able to move between 2D and 3D models of an ordinary helix on the side face of a cylinder as well as visualise the shape of the staircase in a 2D model. This allowed them to calculate the length of the spiral stairs and to compare the rectangle and parallelogram models, facilitating their understanding of the problem. As the Australian students had more difficulty with the problem, the teacher changed two of the switching methods. The teacher asked students to cut up the toilet tubes to confirm that parallelogram and rectangle models were equivalent, and demonstrated how concrete 3D models were related to 2D parallelogram and rectangle models. The changed method for the modelling lesson still depended on the students' understanding and promoted class discussion. It also remained grounded in the context, with a focus on the need to find the fastest route to the top of the oil tanks.

Use of the DMCF and its emphasis on switching between modelling cycles benefited both Japanese and Australian students by deepening their understanding as they moved between 2D and 3D models and the two cycles. The approach encouraged all students to participate at their ability level and to gain access to more sophisticated modelling approaches during whole class discussions.

Our future work will be to compare Japanese and Australian students' international switching by analysing the students' protocols, activities, and worksheets.

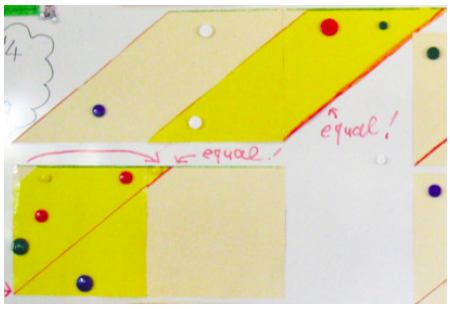


Figure 8: Solutions by both models

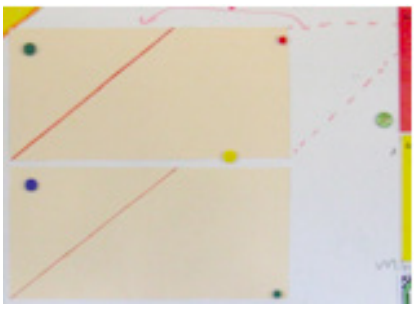


Figure 9: Relationship between both models

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