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Mathematical exclusion with the every day

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Problem solving can involve using mathematics to solve everyday problems. In this study, we examine an interaction between a teacher and a class of six-year olds in Sweden around an open-ended problem, from an everyday context. Using Bernstein's ideas about vertical and horizontal discourse, a mixture of everyday and mathematics understandings is identified in the interaction. This mixture seems to result in confusion for both the teacher and the children over what should be the focus. This paper raises issues about how the connection to the everyday in problem solving could reduce children's opportunities to learn mathematics.

Keywords: Real-life problems, Bernstein, vertical discourse, horizontal discourse, young children.

INTRODUCTION

Problem solving is often considered to provide a purpose for students to learn mathematics (Dahl, 2014). For example, van Oers (2001) discussed how Freudenthal (1973) promoted the use of real-life contexts in his realistic mathematics education – “the realism of mathematics then is seen in the applicability of self-invented mathematics in a meaningful problem, and for many people this seems to mean a real-life problem” (p. 64). Nevertheless, van Oers (2001) queried whether it was possible for higher levels of mathematics to arise from real-life problems:

Despite the enormous innovation this view could produce in the content and activities of the mathematics classrooms, it entails a serious danger by focusing too exclusively on the real life quality of the contexts from which the mathematical thinking originates. (p. 64)

In high school classrooms, research has shown that the use of contexts can result in some students being excluded from mathematical learning opportunities (Meaney & Lange, 2013). This could be because they have difficulties recognising the mathematics in a problem solving task when it is posed in an everyday setting (Zevenbergen & Lerman, 2001). Students also can be uncertain about whether they are expected to ignore their everyday experiences (Gellert & Jablonka, 2009). Similarly, Boaler (1994) raised two related issues. The first is that students do not know how much or how little of the everyday they should use. The second is that students may not realise that the rules of the mathematics classroom require them to suspend their knowledge of reality to make sense of the mathematics classroom reality. Her research found that girls gained poorer marks on problems of which they had had real world experiences.

Although it has been suggested that using contexts could support working class students' learning of mathematics (Lubienski, 2000), previous research has shown that these groups of students are more likely to draw on their everyday experiences than those with middle class backgrounds (Cooper & Dunne, 1998; Gellert & Straehler-Pohl, 2011). For example, Cooper and Dunne (1998) found that “working class children are almost twice as likely as service class children to refer only to their everyday knowledge in answering our enquiry” (p. 128).

As is the case with the research already described, most research on the confusion caused from drawing on everyday knowledge to solve mathematics problems has been done with high school students. Little research has investigated how young children make sense of mathematical problems set in everyday contexts. Therefore, our research question is: how do

young children solve mathematical problems situated in their everyday knowledge?

The research is situated in a Swedish preschool class with mostly six-year-olds. This class is considered a bridge between preschool and school and as such is the first place children have contact with formal, school knowledge and ways of working.

THEORETICAL FRAMEWORK

In order to respond to the research question, we follow Gellert and Straehler-Pohl's (2011) and Dahl's (2014) lead in using the concepts of vertical and horizontal discourse developed by Basil Bernstein. Over several decades, Bernstein developed a systematic sociology of education which included the development of many different ideas. One of these was the distinction between what he labelled the horizontal and vertical discourses:

A vertical discourse takes the form of a coherent, explicit, and systematically principled structure

...

A horizontal discourse entails a set of strategies which are local, segmentally organised, context specific and dependent, for maximising encounters with persons and habitats. (Bernstein, 1999, p. 159)

Horizontal discourse is vital for solving specific issues relevant to the solver. Consequently, it is often related to everyday understanding, gained through practical experiences. However, the knowledge gained through the horizontal discourse is not easily transferable to other situations because of how it is organised and its strong connection to a specific context (Bennett & Maton, 2010). Vertical discourse is considered generalisable to a range of situations. In reviewing research on these concepts, Knipping, Straehler-Pohl and Reid (2012) suggested that vertical discourse is often equated with the knowledge learnt in schools.

Bernstein (1996) stated "to make specialised knowledge more accessible to the young, segments of the *horizontal discourse* are recontextualised and inserted in the contents of school subjects" (p. 169). Therefore, the distinction between vertical and horizontal discourse seems useful because although school mathematics problems are often situated within everyday

contexts, they require generalisable knowledge to be solved. In order to make use of these concepts, they need to be operationalised. Before describing the operationalisation, we provide background to the collected data.

DATA COLLECTION AND ANALYSIS

In the first half of 2013, video recordings were made in one preschool class on four different occasions. Our wider research aim is to investigate what mathematics is or could be in preschools and our video recordings in this preschool class were to be compared with the preschool data. In order to get comparable material, we specifically asked to video problem solving sessions. In the problem solving situations, the children worked in pairs, groups or by themselves, with a sharing session at the end. We had permission from parents for 12 children to be filmed and the films to shared, but not all children were present at each session. The class had 2 teachers who shared being filmed and working with the children whose parents did not give permission for them to be filmed.

The lesson examined in this paper was typical in the format of all lessons, by beginning with a warm up activity, in this case around pairs of numbers that added to ten. Then a problem was posed to the children, who were given time to solve it, this time, individually. The problem in this lesson was about ten children in a small preschool class who needed to be distributed to three different activities, woodwork, baking and painting. The teacher stated that there are no wrong or right answers and that it might be possible to distribute them evenly or it could be that one group had more children or another group had no children at all. The children were given paper on which to record their groups but were told that they could record them in any way that they liked. While the children worked, the teacher moved around the class, talking with each child. At the end of the session, the teacher had the children fold their papers and sit down in a horseshoe. She told the children that they must explain why they have distributed the class in the way that they did. With more or less help from the teacher, each child constructed a story about their distributions.

We discuss three children's interactions with the problem, using photos and transcript extracts. In order to identify whether the teacher and/or the children

Discourse	Characteristics	Examples
Horizontal	Context specific	Filippa’s reason for only splitting the ten children into two groups was that only two teachers would be needed. In her class there were two teachers, suggesting she drew on her own experiences.
	Segmentally organised knowledge	In the presentation session, the teacher queried the children about their distributions. The children did not know that they needed to have a story for their distributions until they did their presentations. The solving of the problem and the telling of their stories were separated both in time and in space and so can be considered as taking place in different sites and thus were segmentally organised.
	Maximising encounters between persons and habitats	When Nicolas presented a paper, on which there was written, 5, 5, 5, the teacher worked hard with the other children to provide a story that allowed for this, so Nicolas could be considered successful. The emotional demands of not having a child fail became the issue to be resolved.
Vertical	Coherent, explicit and systematically principled structure	Lova uses a series of different strategies (using her fingers, making marks on the paper) to determine different combinations of 3 numbers which added up to 10. Although she does not systematically list all of the possible answers, her actions suggest that she is aware of the underlying principles which allows her to determine appropriate solutions.

Table 1: Characteristics of vertical and horizontal discourses

used vertical or horizontal discourses, we looked for particular characteristics as identified in Table 1.

THREE CHILDREN’S PROBLEM SOLVING

We have chosen to discuss the problem solving of Nicolas, Lova and Filippa as they illustrate a range of responses as well as showing different problem solving methods.

Nicolas

In the warm-up activity about ten-friends, Nicolas was

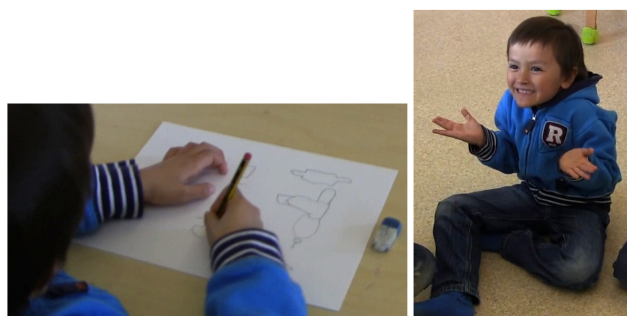


Figure 1: Drawing distribution and showing uncertainty in the presentation

given a card with 9 on it. Teo had 1 and soon located Nicolas. Teo ensured that they are acknowledged by the teacher as having the correct answer. During the individual work, Nicolas sat next to Teo and followed him in spending time copying the symbols (rolling pin, hammer, paint brush) for the three groups. He then wrote 5, backwards, against each of the symbols

(see Figure 1). When the teacher called the children together, Nicolas was the fourth child to show his solution. The following exchange accompanied, Nicolas turning over his paper. When the teacher asked in the first turn, how many he had he shrugged his shoulders (see Figure 1) to indicate that he was uncertain.

Teacher: Nicolas, can you show? You’ve got five, is it fives? Five and five and five in each group. How many children is that? I see five here and five and five. How many children is that? (Nicolas kan du visa? Du har gjort fem, är det femmor? Fem och fem och fem i varje grupp. Hur många barn blir det? Jag får se här fem och fem och fem. Hur många barn blir det?)

A child: Twenty? (Tjugo?)

Children: Fifteen. (Femton.)

Teacher: Fifteen children, how many children were there? (Femton barn, hur många barn var det?)

Nicolas: Ten. (Tio.)

Teacher: Ten, how many children too much? Did you mix in a few more children from another preschool class? You thought fifteen children were better so you could share or? (Tio, hur många barn för mycket? Blandade du in lite barn från en annan förskoleklass? Du tyckte femton barn var bättre så man kunde dela eller?)

One child: It may be that five new children have started in that preschool class. (Det kan ju va så att fem nya barn har börjat på den förskoleklassen.)

Teacher: That I did not know of. So it became five in each group. I'll take it. Thank you very much. (Som jag inte visste om. Så blev det fem i var grupp. Jag tar den. Tack så mycket.)

The discussion initially focused on the numbers, both on how many Nicolas used and how many more this was than in the task. At this point, the conversation seemed to be within the vertical discourse, as the teacher is sequencing the knowledge in a cohesive, structured manner. However, Nicolas' body language (see Figure 1) indicated that he had become aware that his response was incorrect, perhaps because the teacher asked him number questions that she had not asked earlier presenters. The teacher then shifted direction and provided a possible story so that Nicolas' numbers could be considered appropriate, perhaps because she had earlier indicated that there were no right or wrong answers. By producing a story about Nicolas' numbers, the teacher moved back to the context-specific-ness of the horizontal discourse and restored the personal relationships. The difference between 10 and 15 disappeared from the focus and was replaced by a discussion about how 15 was a better number to share and how this number of children could come about. Nicolas made only one contribution to the discussion. It is therefore unclear whether he had understood either the vertical discourse about the difference between 10 and 15 or the horizontal discourse about the ease of splitting the class into 3 groups. The teacher's shifting between the two discourses seems to provide him with no opportunities to develop his problem solving skills either in a context-specific or more generalised format.

Filippa

In the introductory warm-up activity, Filippa had a card with 5 on it. It was not until all the other pairs were formed that Filippa realised that Hugo did not have a partner and therefore their numbers (5 and 5) must equal 10.

In giving the instructions about working individually, the teacher specifically mentioned Filippa. Filippa seemed to have taken the request to heart in that she covered her work with her arms so that Lova could not



Figure 2: Filippa's problem solving

see what she did. After Lova was moved by the teacher, Filippa looked at Teo using his fingers to work out a solution. It was not possible to hear, the teacher's comments to Filippa as she moved around the room but the consequence of it was that Filippa erased the beginning of a symbol for the first group on her paper. Filippa's presentation was the third presentation. Her paper showed two 5s.

Teacher: Filippa, can you show what you have done? Okay, you've written five and five. Five in two groups, it is. Which group was it that there was no one in? Baking, woodwork or painting? (Filippa, kan du visa vad du har gjort? Okej du har skrivit fem och fem. Fem i två grupper blev det. Vilken grupp var det som det inte blev någon i. Baka, snickra eller måla?)

Filippa: Woodwork. (Snickra.)

Teacher: Woodwork there was no group of, no children at all, but why not? Do you remember, how you thought? (Snickra blev det ingen grupp i, inga barn alls i, men varför inte det? Kommer du på det, hur du tänkte?)

Filippa: Because they will do it another time. (För dom skulle göra det en annan gång.)

Teacher: They would do woodwork another time instead so then it became an equal number of children in each group. (De skulle snickra en annan gång istället så då blev det lika många barn i varje grupp.)

Filippa: Otherwise they thought that it became a little too bustling with everything. (Annars tyckte de att det blev lite för stöttigt med allting.)

Teacher: Yes it could be, they would have had to be many teachers. Thank you so much. (Ja det kan ju bli. De hade ju fått va många fröknar. Tack så mycket.)

Filippa: Although they had only two teachers. (Fast de hade bara två fröknar.)

Teacher: Two teachers and then there was one teacher in each group. Brilliantly solved. (Två fröknar och då blev det en fröken i var grupp. Strålande löst.)

As with Nicolas, the teacher presented the drawing and immediately asked which group did not have any children in it. From this question, both the teacher and Filippa built up the story about why there were only two groups. Filippa was complemented by the teacher as having a brilliant solution. In developing the story, it seemed that Filippa drew on her own experiences of only having two teachers in the class and finding moving between too many activities busy and noisy. At no time, did the teacher bring up mathematical understandings. Whereas Nicolas' unexpected answer resulted in the teacher moving into the vertical discourse, the interaction with Filippa remained firmly situated in the horizontal discourse. Unlike the girls in Boaler's (1994) study, Filippa did not seem confused by the familiarity of the context, rather the teacher emphasised that she wanted the context-specific information.

Lova

In the warm-up activity, Lova was the first to stand up and try to find the pair number for her 2. However, it was her partner, Svante, who told the teacher about their pair. As the teacher described the problem of sharing ten children in the three groups, Lova could be seen using her fingers to work out possible solutions (see Figure 3). Before she collected her paper, she shared her solution with Svante who also used his fingers to find a solution.

In working on the problem, Lova moved from using her fingers to putting tally marks next to the symbols for each of the groups (see Figure 3). She seemed to

recognise that there was more than one solution. After she has added one round of tally marks, she counted them before adding the next round. Lova's actions suggested that her interest was in the vertical discourse surrounding the principles connected to adding three numbers together to make 10. When sitting in the horseshoe, Lova was one of the few children who the teacher asked to present her response (see Figure 3), although in this case, the teacher indicated that Lova had a proposal, not a solution.

Teacher: Lova, what proposal do you have? Oh, okay what is there? Can you tell me? (Lova vad har du för förslag. Oj okej vad står där? Kan du berätta för mig?)

Lova: Three, five and one. Three, five and two. (Tre, fem och ett. Tre fem och två.)

Teacher: Let me see, three five and two, okay. It is, let's see here. (Jag får se, tre fem och två okej. Det är, ska vi se här.)

Lova: Three in one group, five in another. (Tre i en grupp fem i en.)

Teacher: But where are the three, is it in the baking group? (Men var är de tre, är det i bak gruppen.)

Lova: Yes. (Ja.)

Teacher: And then it's five in the woodworking group and two in the painting group. (Och så är det fem i snickargruppen och två i målargruppen.)

Lova: Because the woodworking group, it's many more who like to do woodwork, less who like to paint and in between who like baking. (För att snickargruppen, det är mycket mer som tycker om att snickra, mindre som tycker om att måla och mittemellan som tycker om att baka.)



Figure 3: Lova's problem solving

Teacher: So they could choose for themselves in that class, okay. (Så dom fick välja själva i den klassen, okej.)

In presenting her solution, Lova seemed to focus on the numbers, suggesting that she wanted to stay within the vertical discourse. However, the teacher shifted her to the horizontal discourse by asking about which group had three children in it. Lova happily participated by providing details about why she had split the ten children.

DISCUSSION AND CONCLUSION

Previously, van Oers (2001) highlighted how too much attention on children's everyday experiences could be detrimental to their learning formal mathematics knowledge. In this study, we have examined examples from one lesson in a Swedish preschool class which suggests that the issue is more complex.

To acknowledge the preschool class as a bridge between the informal learning in preschools and the formal learning in schools, teaching activities could invite children to participate in both vertical and horizontal discourses. Certainly, the problem solving in this lesson provided a context with plenty of opportunities to work within both the horizontal and vertical discourses. However, the interactions within the three cases suggest that the two discourses, metaphorically speaking, are conflated.

In the case of Nicolas, the teacher made an initial attempt to work in the vertical discourse by discussing the relationship between 5, 10 and 15. In a few, very complex and fast moves a horizontal discourse was created in which the question "what numbers are good for creating three groups" became legitimate. Like the initial vertical discourse discussion, the new horizontal discourse "story" from Nicolas' perspective seemed to make little sense. In this exchange, Nicolas' opportunities to extend his understanding of either the horizontal or the vertical discourse seemed to disappear.

Filippa herself made use of the horizontal discourse to make a connection between the story and the mathematical context. The teacher had opportunities to introduce potential mathematical obstacles to Filippa's story by suggesting that there could be as many teachers as groups. This would have indicated

that in mathematics, it is possible to make such assumptions. However, the discussion stopped and was kept within the horizontal component where Filippa seemed most comfortable. Possibilities for learning about the vertical discourse of formal mathematics never eventuated.

Finally, Lova presented her answer in such a way that it invited a discussion about numbers within the vertical discourse. The teacher subtly hinted at this by remarking that 2, 5 and 3 "is okay", but then shifted the discussion to the horizontal discourse. Thereby Lova, who of the three children, showed the most interest in the principles behind the different combinations of numbers that added to ten, lost an opportunity to extend her understanding.

The three cases reveal a group of children, which although not homogeneous, had in common the experience of not being provided with possibilities to connect their experiences to formal mathematics knowledge. The children were not confused over the knowledge needed to solve the problem. They all understood that numbers were expected. Instead confusion may have occurred when the teacher brought the solutions into the horizontal discourse of everyday knowledge, but without extending the children's social concerns, such as having sufficient teachers. If similar lessons continue to highlight the everyday knowledge, even when children such as Lova show interest in the mathematics, then there is a risk of mathematical exclusion.

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