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# Language use, mathematical visualizations, and children with language impairments

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*This article presents a first approach and theoretical foundation for a new research project. It focuses on the role of language in the process of communication about structures and relations in mathematical visualizations. Mathematical knowledge is abstract in most cases. Using means of visualization is indispensable for speaking with children about the abstract mathematical concepts. Against this background language is seen as an important tool in the construction of knowledge. But this raises the question, which features of language in children with specific language impairments (SLI) pose a challenge for verbalizing abstract structures. The interest in research relates to the question how students with SLI manage to communicate about the embodied structures in mathematical means of visualization.*

**Keywords:** Means of visualization, communication, structures, language impairments.

## EPISTEMOLOGICAL VIEW ON MATHEMATICAL KNOWLEDGE

For a better understanding of the focus on means of visualization in this paper, it is useful to think at first about the special character of mathematical knowledge and the special epistemological conditions involved in processes of learning mathematics.

Mathematics as a science of pattern, structures and relations deals with essentially abstract concepts. From the beginning of the learning of mathematics the young child is confronted with this challenge: Even dealing with elementary mathematical objects like numbers, operations and later the concept of the place value system the child is confronted in a first way with the abstract ideas of mathematical knowledge. Therefore the child has to develop awareness that mathematical concepts are not empirical objects. For example the child has to learn that numbers rep-

resent more than only an amount of objects. Rather, it is the theoretical relation between these objects that constitutes the mathematical concept of number.

In the 'world of objects', '0' (zero or null) means 'no object': and in this world there is no principal difference between the removal of '5 apples and 5 pears' or of '5 black and red chips'. If, in the model with black and red chips, the same number of black and red chips is given to mean '0', this theoretical relation has to be established 'by one's own and independent activities of thinking' and, only in this way, a difference is constructed between the chips configurations, which symbolizes a number aspect, and the pears and apples, which belong to the world of things. (Steinbring, 2005, p. 20).

This example points out, that an *amount of ten chips* (five black ones and five grey ones) - that could be seen, counted and manipulated by the child - can also represent the *number zero* (see Figure 1). But this interpretation is only possible if the learning child does not only focus on the real objects, the concrete properties of the objects (the colour for example) but focuses on the *relations that are represented by the different coloured objects*. This particularity is an important basis of the understanding of numbers in general.

In his reflection about the character of numbers, Benacerraf (1983) points out:

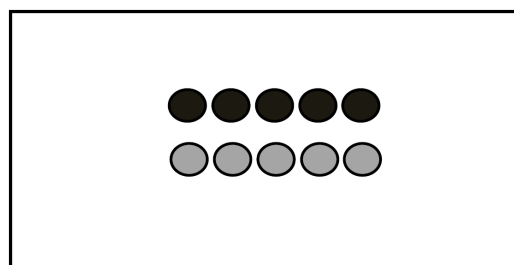


Figure 1: Number 'zero' as a relation between two amounts of five

Therefore, numbers are not objects at all, because in giving the properties (that is, necessary and sufficient) of numbers you merely characterize an abstract structure - and the distinction lies in the fact that the elements of the structure have no properties other than those relating them to other elements of the same structure. (...) To be the number 3 is no more and no less than to be preceded by 2, 1, and possibly 0, and to be followed by 4, 5, and so forth. And to be the number 4 is no more and no less than to be preceded by 3, 2, 1, and possibly 0, and to be followed by (...). Any object can play the role of 3; that is, any object can be the third element in some progression. What is peculiar to 3 is that it defines that role - not by being a paradigm of any object which plays it, but by representing the relation that any third member of a progression bears to the rest of the progression. (Benacerraf, 1983, p. 291)

This means that even the numbers which often seem to be understood in everyday experience as an amount of concrete objects, or as objects with concrete properties have to be understood in the progression and relation to the preceding and following objects. More generally:

So what matters, really, is not any condition on the objects (...) but rather in the relation under which they form a progression. To put the point differently - and that is the crux of the matter - that any recursive sequence whatever would do suggests that what is important is in not the individuality of each element but the structure which they jointly exhibit. This is an extremely striking feature. (Benacerraf, 1983, p. 290)

Mathematics is thus as shown above a science, whose concepts are used to describe and analyse abstract patterns and structures. But in contrast to this, in „non-mathematical“ everyday experiences mathematics is often understood as a collection of rules, procedures and algorithms. Against this background, the mathematical signs and symbols are often used and (mis-)understood as important „anchor points“, which have to be memorized and which are often confused with the underlying mathematical concept. However, the peculiarity about mathematics is - which Duval (2000, p. 61) describes as the “*paradoxical character of mathematical knowledge*” - that just these signs and symbols are not the mathematical concept in itself,

they only refer to it, they represent it. Presmeg (2008) describes *signs* as *interpreted relationships* between a *representative* and an *object*.

I shall take a sign to be the interpreted relationship between some representamen or signifier called the sign vehicle and an object that it represents or stands for in some way. In mathematics, the objects we talk about cannot be apprehended directly through the senses: for instance, “point”, “line”, and “plane” in Euclidean geometry refer to abstract entities that we can never see, strictly speaking, as in Sfard’s (2000) virtual reality. We apprehend these objects, “see” them, and communicate with others about them, in a mediated way through their sign vehicles, which may be drawn or written by hand or through dynamic geometry software, labeled in conventional ways, moved and manipulated for multiple purposes. We work with these sign vehicles as though we were working with their objects. (...) It is this interpreted relationship between a sign vehicle and its object that constitutes the sign. (Presmeg, 2008, p. 3)

The result is a particular challenge for the development and insight of mathematical concepts: If mathematical signs are not the mathematical concept, but only a symbol of a relationship, and if mathematical concepts have to be understood as abstract relations, how is it possible to speak and to reflect about them at all - especially with young children in elementary school?

#### **EPISTEMOLOGICAL VIEW ON MEANS OF VISUALIZATION**

For these processes of thinking about abstract structures and teaching mathematics, the use of means of visualization is an important foundation to help young students in building adequate internal representations of mathematical ideas. The mathematical ideas, as

theoretical ideas, are not things which could be conveyed as completed products. The mathematical subjects consist of relations between things and not in the objects and properties. Therefore, mathematical thinking (...) has to be visualized, in order to represent such relations. (Otte, 1983, p. 190, translated by author)

Goldin and Shteingold describe “the development of efficient (internal) systems of representation in students, that correspond coherently to, and interact well with, the (external) conventionally established systems of mathematics” as a fundamental aim for the process of mathematics teaching (Goldin & Shteingold, 2001, p. 3). But this aim implies a difficult and challenging task for the teacher: As different studies in mathematics education have shown, the intended way from external to internal representations is not straight, easy or clear (Söbbeke, 2005a, 2005b).

For a better understanding of this difficult requirement, it is helpful to consider the elementary learning processes in mathematics education in a more detailed way. For this purpose we will look at the use of means of visualization from different perspectives. In German mathematics lessons chips are a common means of visualization. In the following, the example chips illustrates in which way the interpretation of the material must be differentiated: an interpretation of the chips as concrete to an interpretation of the chips in a first systemic and relational way.

*First Perspective:* Especially in teaching processes with young learners the counting of concrete objects is an essential activity to develop a first concept of numbers. With this background, objects like “chips” have to be understood as an explanatory background for the understanding of the new number symbols and number words. The concrete materials seem to deliver an explanation for the new mathematical symbols (Steinbring, 2014). The material-based and concrete interpretation of each single object („one chip means ‘1’”) is helpful and important.

*2nd Perspective:* The interpretation of the ten chips undergoes a first conversion / change if they are sorted in the place value system. A new interpretation of the material is necessary (see Figure 3).

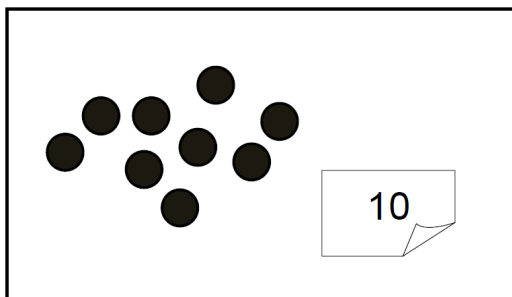


Figure 2: Counting ten chips to develop a first concept of numbers

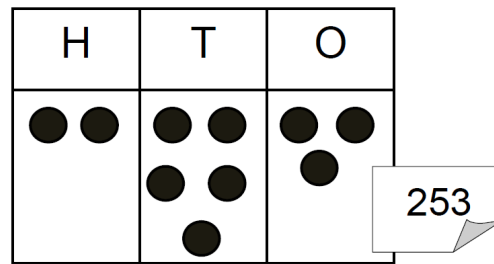


Figure 3: Ten chips in the place value table

The 10 coloured chips should be seen in their structural relationship to each other and to the whole system: Sorted in the place value table, each single chip is given a new meaning: It no longer represents „one“, but also „10“ and „100“ (Söbbeke, 2005).

*3rd Perspective:* We continue the consideration of the 10 chips a bit further. The 10 chips are arranged now in a rectangular shape (Steinbring, 2014). The relationship of each single element to the overall structure is important. The learning child has to be aware of the “new” structure(-units): e.g. “twos” and “fives”. This is an important modification and at the same time a more sophisticated approach to the material: for mathematics learning, the child has to see that the collection of these 10 chips does not only stand for the amount of ten, but also represents essential arithmetical ideas: e.g. “two times five” ( $2 \times 5$ ) or “five times two” ( $5 \times 2$ ). This view has to become even more differentiated, for example to see the possibility of a distributive decomposition of the “chips field” like “ $1 \times 5 + 1 \times 5 = 2 \times 5$ ” (see Figure 4).

These examples show that the first perspective on the material, using it to count objects (“one chip means ‘1’”), successively undergoes a major evolution. By putting the chips into the place value table or in the shape of the rectangle, the children can be encouraged to think about the first systemic aspects of the mean of visualization.

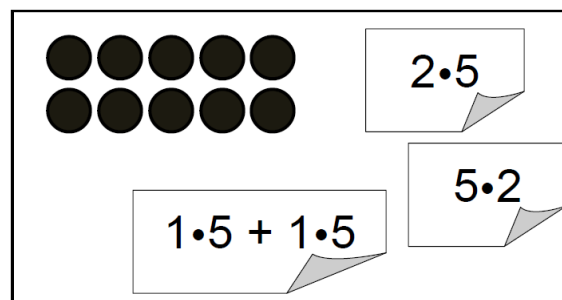


Figure 4: Ten chips structured in a rectangular shape

Against this background, it becomes clear that the children have to learn a new approach to the use and interpretation of means of visualizations (Söbbeke, 2005a): To build new mathematical knowledge it is necessary to disregard the concrete properties of the objects and to understand these objects as elements of the overall system. Therefore the child has to learn to examine the relations, structures and the theoretical ambivalence, which the means of visualization contain. The author's main interest in research belongs to this difficult and challenging requirement for the learning child (Söbbeke, 2005a, 2005b). For further research, it seems relevant to investigate, in how far it is possible for elementary learners not only to develop this systemic view on mathematical visualizations, but also to *communicate about it*.

## **THE ROLE OF LANGUAGE IN EXPLORING STRUCTURES IN MEANS OF VISUALIZATION**

### **Language and the learning of mathematics**

Language is important for the learning of mathematics in two ways: on the one hand language has a communicative function that serves the exchange among the children and with the teacher. On the other hand, it has a cognitive function that advances an increase of new mathematical knowledge (Maier & Schweiger, 1999).

### **The communicative function of language**

The importance of language is highlighted in German mathematics curricula and the German "Bildungsstandards" (educational standards) by emphasizing the activities and processes of "communication" and "reasoning" as prominent skills for mathematics learning. The guiding principle on "patterns & structures" requests the children to describe mathematical relations and structures in general to all mathematical topics. Against this background the communicative exchange can be seen as a necessary component of a stimulating and challenging teaching and learning culture.

### **The cognitive function of language**

Regarding different research traditions, today the learning of mathematics is no longer seen as a purely individual, mental process of the single child, but as a learning process in which the social interaction and communication with others is an essential basis for the development of new mathematical concepts. In this context, the sociologist Miller (2006, p. 200) dis-

tinguishes different types of knowledge (cumulative knowledge versus structural or fundamental knowledge) and justifies the importance of social discourse as an important factor for learning.

While cumulative knowledge can be developed quite individually on the basis of experiences by a subject on its own (e.g. a poem or foreign language vocabulary), structural or fundamental knowledge can only increase in the social debate and reasoning of learners with others (Miller, 2006, p. 200). As shown above, mathematical concepts are assigned to the structural knowledge. They arose historically and therefore they require cultural processes - processes of communication and interaction - to be developed and learned. Miller also highlights that new knowledge cannot be completely derived from existing knowledge, because new knowledge exceeds the acquired knowledge. That is, in processes of collective argumentation new findings, beliefs and concepts can be developed.

But children in elementary school are still on their way to become mathematically communicating people and thus „autonomous learners“ of new structural knowledge. They still have to learn to derive new structural knowledge from their empirical experiences or activities with concrete materials or means of visualization (Steinbring, 2014). Finally it becomes clear, that the communicative function of language strongly supports and requires the cognitive function. This point shows the reference for the presented interest in research: The teacher has to *reveal an (seemingly) empirical world* to the children, in which they have to make *theoretically significant experiences* and in which they *learn to verbalize* them.

### **Mathematical communication with children with language impairments**

At least in the German discussion about mathematics and language, it is quite popular to discuss primarily the influence of language impairments on performance in mathematics or to develop concrete actions (or aids) to support the learners (e.g., Donlan, 2007; Fazio, 1999; Jordan, Levine, & Huttenlocher, 1995). In contrast to that, the aim of the author's research interest is to better understand the theoretical and epistemological conditions of language use, and analyse the special features of language in the context of the interpretation of means of visualization. As shown above means of visualization deliver an important access to the theoretical and abstract "world" of mathematics.



The language plays a fundamental role to this access: Children learn to derive the theoretical and systemic aspects from their empirical experiences with the concrete materials *only by a special communication about the structures and relations in these means of visualization*. This aspect fits the author's main interest in research: Which kind of linguistic particularities affect the communication about mathematical means of visualization, especially the conceptual and systemic interpretation of the means of visualization? In the following, two essential aspects are described which seem to be relevant for a deeper understanding of the described research interest.

### **Symbol competence**

Studies in cognitive sciences emphasize the importance of an „image and symbol competence“ (DeLoache & Burns, 1994) for the cognitive development and learning of young children. They also show that this competence is a challenging skill, which has to be developed: “There are several reasons to suspect that recognition of a depicted object is not equivalent to understanding the nature of pictures or the relation between a picture and its referent” (DeLoache & Burns, 1994, p. 85).

Other studies point out that images are iconic symbols that are associated with a specific content. This “association” is based on structural similarities either on a concrete or abstract level (Elia, Gagatsis, & Demetriou, 2007). In processes of mathematical communication the children have to develop this understanding of symbols, and furthermore the competence to verbalize this. Several observations indicate that children with SLI show a delayed development of such a symbol understanding competence: Understanding signs which establish a relationship between signifier and signified is delayed. This delay can have an effect on the symbol competence in the context of interpretation and verbalisation of means of visualization in primary school education (Lorenz, 2005, p. 4).

### **Linguistic means to describe relations and generality**

In the German discussion about language impairments and learning mathematics, different technical terms have been described, that affect the communication about mathematics: For example problems of seriality include difficulties to verbalize linguistic sequences, to differentiate and interpret them. An example is the major effects that arise in number

words by interchanging the individual positions of the digits: 163, 631, 316, etc. (Lorenz, 2005). It is clear that difficulties in the context of seriality not only exchange the names of the numbers (factual knowledge) but can lead to problems for the *structural concept* of numbers.

It is important to understand the spatial relations in diagrams. The use of prepositions, which describe such spatial relationships, are important linguistic means for the development of a concept. In addition, the children have to verbalize relational concepts which combine different objects in a comparative sense and be aware of causal constructions (Nolte, 2000). These linguistic expressions are a key basis to describe first ideas of generalization; they are technical terms that need to be learned.

However, taking into account the epistemological perspective shown in the previous sections, *more* seems to be important than to examine the use of these *technical* terms. My current research aim is to develop a first theoretical framework to analyze the *children's linguistic means to express relations and generality*. The epistemological framework clarifies the special *content* of communication. But it does not clarify *in which way* the language use of children with SLI should be theoretically described. It is necessary to integrate certain theoretical concepts into the epistemological framework in order to describe the linguistic means of children with SLI adequately. A first approach to this could be the distinction of different levels and linguistic means by Akinwunmi (2012).

Mathematicians use algebraic expressions such as variables, terms or equations to express generality. But primary learners have no knowledge about variables to describe mathematical patterns and something universal. Akinwunmi (2012) examined processes of generalization. Through the analysis of clinical interviews with fourth graders, she reconstructed different types of verbal means, which the children (in this context without SLI) used to discover and describe mathematical patterns. Akinwunmi worked out five categories for generalization that could deliver a first access to the presented interest and further work: use of *a representative example*; use of *several examples*, development of *quasi-variables*, *conditionals* and *variables*. These categories embrace a range of linguistic expressions from relating to concrete objects and examples to describing generality.

## CONCLUSION FOR RESEARCH

This paper is a first theoretical foundation of a research project. Based on an epistemologically oriented view of mathematical knowledge and the role of means of visualisation, the importance of communication about structures in mathematical representations had been worked out. In the following process of research a solid framework has to be developed. This work includes to integrate certain theoretical concepts (in the field of communication and language) into the epistemological framework in order to describe the linguistic means of children with SLI.

The communication with children with SLI about means of visualisation is of current interest. For mathematics teaching in Germany this requirement constitutes currently a special challenge. Because of a new law on inclusion, children with and without SLI have to be taught together. In order to develop for this kind of teaching not merely superficial recipes, it is important to investigate processes of challenging communication.

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