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Structural design and active control of modular tensegrity systems

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Tensegrity systems are self stressed reticulate structures composed of a set of compressed struts assembled inside a continuum of tendons. This principle is at the origin of lightweight and transparent structures that can cover large spaces and be erected, in particular cases, by deployment. In this paper, we propose a general design and optimization procedure adapted to modular structures following this principle. An application is presented on the case of a curved deployable footbridge.

Besides, as lightweight frames, these systems are subject to deformation and vibration issues when faced to varying actions such as climatic, human, or seismic loads. Active control is as solution that allows, using actuators integrated into the structure, to attenuate these effects. On the case of a real plane modular tensegrity grid, we present a specific methodology for the active damping of the first two modes and its experimental validation.

Keywords: tensegrity, structural design, optimization, vibration control.

Subject classification codes: include these here if the journal requires them

Introduction

Tensegrity systems are selfstress space reticulate structures, composed of a set of compressed struts in stable equilibrium inside a continuum of tendons (Motro, 2003). Following this principle developed during the fifties (Fuller, 1973)(Snelson, 1973) can

emerge lightweight, large spans and transparent structures (Figure 1) that, in some cases, can be erected by deployment (Smaili & Motro, 2007)(Quirant et al., 2011).

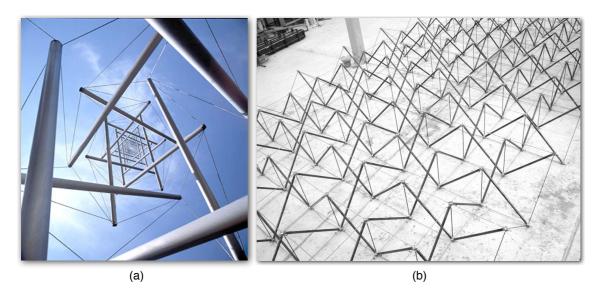


Figure 1. Tensegrity systems: (a) Needle Tower and (b) Tensarch project

Although providing many benefits, very few systems of this kind are present in structural applications, mainly because they require strong design and computation prerequisites. (Averseng & Dubé, 2012)(Tibert & Pellegrino, 2003) Indeed, their stability and rigidity imply being in a state of selfstress, which imposes a specific design approach and using not so widespread non-linear analysis techniques. Besides, as any lightweight system, their damping properties are low, which makes them sensible to dynamic actions, climatic or human, that may induce resonance and then a loss of comfort of damages in secondary fragile elements like glass panels. So, it is necessary to control the dynamic behaviour in order to attenuate these effects.

In this paper, we present the synthesis of two studies focused on the structural design and control of tensegrity systems on the case of deployable modular systems. We propose a new design methodology for determining the form, selfstress level and cross-sections characteristics of a whole structure so as to optimize the weight and flexural

rigidity. In a second part, we develop a general process for controlling the first vibration modes, using integrated actuators, and we present an application on the case of a plane tensegrity grid.

Optimal design

The design of tensegrity systems is the result of a form-finding process, which consists in optimizing the balance between form and internal forces. In addition, as for any structure, the engineer has to consider realistic project situations and justify the elements according to design rules provisions, while optimizing material cost. With these lightweight systems, self weight is not negligible, which imposes an iterative process of form-finding and design. This apparent complexity and the fact that a few structural design studies (Rhode-Barbarigos et al., 2010) (Safaei et al., 2013) exists at the time certainly explains the low development of this kind of solution. Another more fundamental aspect is related to the intrinsic low structural stiffness of these systems (Hanaor, 2012), partly of second order geometric nature. That is why a real application is possible only with structures that limit finite mechanisms, similarly to conventional space truss. In that case, optimum design involves all of its parameters: the form, selfstress level, materials and cross-section characteristics of elements. This problem can be carried out in an exploratory manner in three parts: establishing a set of different configurations with varying geometry and material, designing the selfstress level and the appropriate cross-sections characteristics for each one and evaluating its structural performances, and finally, looking for the optimum solution.

Modular tensegrity system

We demonstrate this methodology on the case of a footbridge composed of two tensegrity beams generated by replication of 4 bars tensegrity modules. Each beam is deployable in a short time and stiffened by transverse cables added in the upper layer in order to block mechanisms and then limit vertical deflection (Averseng & Dubé, 2012). A simply supported rigid deck joints the two beams. Curvature is induced by mapping the system on a horizontal axis cylindrical surface (Figure 2). The whole structure is modelled as a space reticulate system in which the deck is represented by a series of transverses bars. So as to avoid torsion in the supporting beams, they are jointed at their ends to a specific reticulate sub-system that distribute the vertical forces to the four lower nodes of each module.

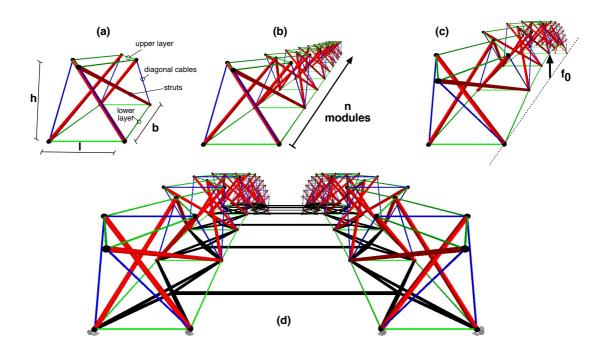


Figure 2. Geometry composition of the footbridge: (a) module, (b) beam assembly, (c) initial deflection, (d) final structure with deck

The specifications of this footbridge summarize in two parameters: the width and the span, fixed respectively in this study to 2 m and 12 m. The others fixed data are related to the deck, considered as a series of rigid plates (estimated distributed mass of 30 kg/m^2), and to the variable load, taken equal to 3 kN/m^2 . Environmental actions and accidental of seismic situations are not considered. All others parameters, defined in

Table 1, are the unknowns of this problem. Elements are classified in groups: lower and upper layer cables, struts, diagonal cables. All sections are considered full and circular, expect for steel and composite struts, made of circular hollow sections with a fixed diameter over thickness ratio.

Table 1. Material and geometrical range of parameters

number of modules	n	6 to 14
width	b	40 cm to 1.6 m
height	h	80 cm to 1.8 m
initial deflection f0	f_0	-0.5 to 1 m
material		timber (C24, $k_{mod} = 0.6$)
		steel (S235)
		carbon/epoxy composite ($f_c = 1200 \text{ MPa}$, $E_c = 125 \text{ GPa}$)

In every selfstress system, the internal forces can be represented as a state vector built by combination of fundamental states. Those vectors actually constitute a base of the kernel of the equilibrium matrix, established from the static global equilibrium equation (Quirant & al., 2003)(Pellegrino & Calladine, 1986). In modular systems, those basis states are generally localized in every module (Sanchez & al., 2007). To simplify the problem and to set a global uniform selfstress state realistically, we practice by similitude with the tensioning process proposed by Averseng & Crosnier (2004) by simulating the controlled shrinking of some elements qualified as "actives". In the presented system, those elements are the diagonal cables shared by two successive modules.

Structural design

For any given geometric and material configuration, the parameters to optimize are the selfstress level, which we define as the highest compression force among struts,

and the cross-sections of every element. This dimensioning is processed iteratively with three steps (Figure 3): calculation of cross-sections given a selfstress level, structural analysis and serviceability check (no slackening of tendons under SLS load) then Ultimate Limit State checks (resistance in tension and buckling). First natural frequencies are not considered because they mainly depend of the geometry that, given a configuration, is fixed. In ULS situation, slackening is admitted, which involves carrying a non-linear structural analysis, through dynamic relaxation in our case (Barnes, 1975) (Averseng, 2011). The global stability and resistance are checked according to the Eurocode standards. For the buckling of composite struts, provisions are derived from those applicable to steel, assuming higher imperfection and security factors. The process iterates, increasing either the selfstress level or cross-sections until all criteria are validated.

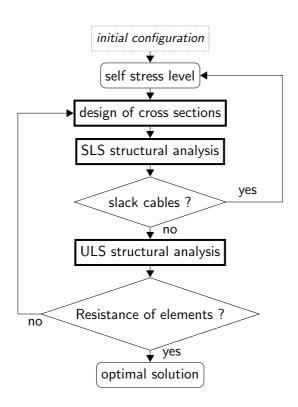


Figure 3. Structural configuration design procedure

Finding an optimum

Each optimized solution of the panel of potential configurations is evaluated through the performance index I_{perf} defined in equation (1).

$$I_{perf} = \frac{1}{f_{ELS} \cdot M} \tag{1}$$

In this expression, f_{ELS} is the ELS deflection (in meters) and m the total mass of the structure (in 10^3 kg). As we can see, a high index signifies low deflection and low mass, which means supposedly high structural performances. To identify an optimal set of parameters, the performance indexes are used to built an explicit meta-model, actually a quadratic response surface as shown in equation (2), which is a continuous and derivable representation of the performance index in function of the geometric parameters.

$$I_{perf,mod}(x) = a + \sum_{i} b_{i} x_{i} + \sum_{i} c_{ii} x_{i}^{2} + \sum_{i} \sum_{j \neq i} c_{ij} x_{j}$$

$$= a + [\cdots b_{i} \cdots] x + \frac{1}{2} x^{t} H x$$
(2)

In this expression, x is the column vector of the parameters defining the structure: the initial deflection f_0 , the width b, the height h and the number n of modules. The coefficients a, b_i and c_{ij} are obtained by least square fitting on the set of performance indexes of the designed configurations. We present in Table 2 the response surfaces obtained for the three materials considered, with correlation indices of 0.987, 0.989 and 0.938 respectively for steel, timber and composite. These functions can be written under a matrix form, in which appear the Hessian matrix H of the problem, from which an extremum can be deduced as in equation (3).

$$\nabla I_{nerf \, mod}(x) = 0 \iff b + H \, x = 0 \tag{3}$$

Table 2. Structural performance index response surfaces for steel, timber and composite tensegrity footbridges

Material	$I_{perf,mod}$					
S235	$31.3 + \begin{bmatrix} -35.1 \\ -40.8 \\ 71.5 \\ -8.35 \end{bmatrix} x + \frac{1}{2} x^t \begin{bmatrix} 26.8 & 12.4 & -6.57 & 1.76 \\ 12.4 & 19.1 & -25.8 & 4.75 \\ -6.57 & -25.8 & 9.28 & -5.07 \\ 1.76 & 4.75 & -5.07 & 0.865 \end{bmatrix} x$					
C14	$17.4 + \begin{bmatrix} -43.5 \\ -65.4 \\ 101 \\ -9.19 \end{bmatrix} x + \frac{1}{2}x^{t} \begin{bmatrix} 16.8 & 11.7 & 0.941 & 2.07 \\ 11.7 & 32.2 & -29.6 & 7.99 \\ 0.941 & -29.6 & -5.92 & -6.33 \\ 2.07 & 7.99 & -6.33 & 1.01 \end{bmatrix} x$					
composite	$35.3 + \begin{bmatrix} -102 \\ -63.6 \\ 107 \\ -9.87 \end{bmatrix} x + \frac{1}{2} x^t \begin{bmatrix} 117 & 23.3 & -2.51 & 1.23 \\ 23.3 & 75.5 & -97.2 & 11.8 \\ -2.51 & -97.2 & 75.9 & -11.5 \\ 1.23 & 11.8 & -11.5 & 1.25 \end{bmatrix} x$					

The set of parameters of each optimum, one per considered material, are presented in Table 3.

Table 3. Material and geometrical characteristics of optimized solutions

	unit	S235	C24	composite
v_0	cm	40	60	60
b	cm	120	130	140
h	cm	130	170	120
n	-	10	8	6
mass	kg	2474	2588	1807
f_0	Hz	3.26	4.22	3.7
selfstress	kN	116.5	115	110

A_{bars}	cm ²	7.6	166	16.6
$A_{up.layer}$	cm ²	1	1	1
$A_{low.layer}$	cm ²	1	1	1
A_{diag}	cm ²	2.1	1.6	1.6

As can be seen, the optimized timber and steel structures share a similar weight (110 kg/m²) that is lower than conventional solutions. Although the three solutions require a same module width, we observe that the timber footbridge needs significantly higher cross-section areas and module height, to improve the flexural inertia. Among the three solutions (Figure 4), the one based on composite differentiates clearly in lightness, due to a lower number of modules, despite longer struts. In all cases, the initial selfstress level needs to be around 115 kN, which is rather important.

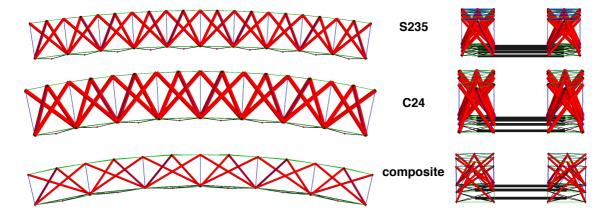


Figure 4. Rendering of the final solutions.

Finally, despite a large domain of variation (-0.5 to 1 m), the imposed curvature deflection is similar for all cases at around 50 cm. The first Eigen frequencies are under 4 Hz, which is certainly too low and implies to revise the height of the modules to gain

flexural rigidity. This suggests also improving the formulation of the performance index in order to better account for this criterion.

Active control

Active vibration control aims to lower the amplitude of vibrations induced in a structure by external actions that may excite its resonances. It consists in integrating actuators in the structure and commanding them so as to cancel vibrations. Several studies were raised on the control of tensegrity systems, using instantaneous optimal control (Djouadi & al., 1998), establishing control law from exact kinematics (Skelton, 2005), shifting soft modes through selfstress (Ali & Smith, 2010), or using robust synthesis (Averseng & al., 2005)(Tuanjie & Yujuan, 2013). In the continuity to these studies, we propose to control the first vibration modes of a plane tensegrity system with a new approach that consists in decomposing the actuation mode and control law by frequency domains, around each Eigen mode. Using H_{∞} synthesis technique, this method allows optimizing the efficiency of actuators while ensuring the robustness and stability of control law, by keeping them simple.

Experimental model

The methodology is applied to a plane modular tensegrity grid derived from the Tensarch project (Motro, 2002) showed in Figure 1. It is formed by discontinuous sets of weaved struts in equilibrium inside a continuous network of tense cables. Two hydraulic actuators are integrated in this structure, in positions inducing tension locally in the lower layer, inducing vertical deflection (Figure 5).

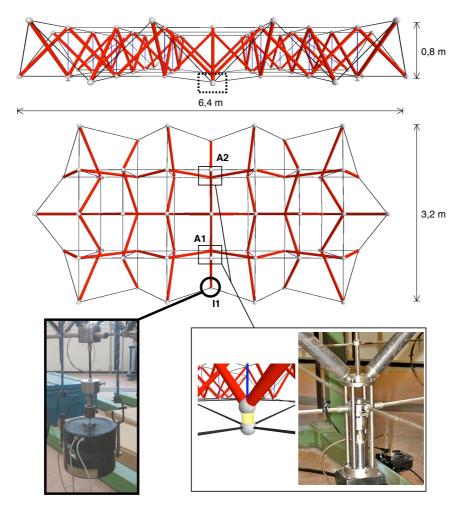


Figure 5. The active tensegrity grid: geometry, actuators (A1 & A2) and force exciter (I1)

This solution is also effective in inducing vibration up to 30 Hz, which covers the major part of the dynamic behaviour of the structure. Vertical vibration levels are measured on the upper layer, above one of the actuators (A1), allowing observing all flexion modes of even order. An electrodynamic shaker is used to introduce an external force in (I1), at a certain offset from the symmetry axis to induce flexion and also torsion.

Identification

Two aspects of the dynamic behaviour are identified: the passive part, which results from external actions, and the active part, which is the transfer between the command to

the actuator and the acceleration in output. These behaviours are measured, in terms of frequency response functions, by carrying swept sine analysis (Figure 6), which reveals resonance peaks in torsion (13.4 Hz) and flexion (17.4 Hz).

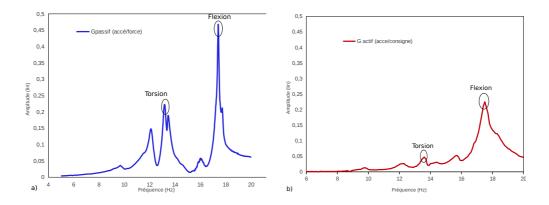


Figure 6. Frequency response function of the active grid: (a) passive part G_p and (b) active G_a .

To optimize the impact of actuators on each vibration resonance, they are coupled: they are commanded in phase for inducing flexion, and in opposition of phase to generate torsion. This means that the active part of the behaviour has to be defined for each regime. From the experimental results, identification is made by analogy with a simple rheological model (Figure 7).

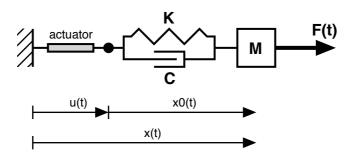


Figure 7. Equivalent rheological model

The three parameters M, C and K represent respectively the mass, the damping factor, and the stiffness of the structure. In this model, actuators are materialized by an

element of variable length put in series. From the equilibrium equation, develop equations (4)-(5):

$$M\ddot{x} = F - K(x - u) - C(\dot{x} - \dot{u}) \tag{4}$$

$$M\ddot{x} + C\dot{x} + Kx = F + Ku + C\dot{u} \tag{5}$$

The transfer function is obtained from the Laplace transform of equation (5) which leads to equations (6)-(7).

$$L[M\ddot{x} + C\dot{x} + Kx] = L[F + Ku + C\dot{u}] \tag{6}$$

$$Ms^2x(s) + Csx(s) + Kx(s) = F(s) + Ku(s) + Csu(s)$$
 (7)

We can then represent the behaviour of the structure by the two frequency response functions defined in equations (8)-(9), for the passive part G_p and the active part G_a .

$$G_a = \frac{s^2(C s + K)}{M s^2 + C s + K} \tag{8}$$

$$G_p = \frac{s^2}{Ms^2 + Cs + K} \tag{9}$$

Around the resonance peak of each mode, identified experimentally, we can first evaluate the parameter K by fixing M. We can then adjust the damping coefficient and the global level using a factor α , which represents the gain of the acquisition chain. The results are presented in Table 4.

Table 4. Mechanical parameters of equivalent models, for each mode

frequency	mode	M (kg)	K(N/m)	C (Ns/m)	α
13.42	torsion	100 kg	$280.63 \ 10^3$	300	4 10 ⁻⁷
17.42	flexion	100 kg	1198 10 ⁶	350	$3.4 \cdot 10^{-7}$

Robust active control

Among the existing modern control methods, we are interested in the robust synthesis algorithms (LQG, PRLQG, H_{∞} , μ) because they consider the uncertainties and external disturbances that affect the model or the signals. These uncertainties vary a lot and can involve the real behaviour of joints, selfstress, materials, and loading (snow for instance).

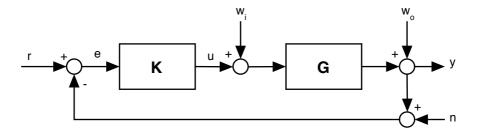


Figure 8. Typical closed-loop

In the H_{∞} approach (Fezans & el., 2008), the problem consists in minimizing the norm of the transfer between disturbances and criteria outputs. In a typical closed-loop (Figure 8.a), we note the system to control as G, the controller as K, the measured output Y and the command Y. The uncertainties are modelized as external signals Y0 or Y0 added to the in the loop. The relationship between the output Y1 and the other signals is developed in equation (10).

$$y = (1 + GK)^{-1}w_0 + (1 + GK)^{-1}Gw_i + (1 + GK)^{-1}GK(r - n)$$
 (10)

We introduce the following notations: $S = (1 + GK)^{-1}$ the output sensitivity, and $T = GK(1 + GK)^{-1}$ the complementary output sensitivity. These functions are used to build specifications for the controller. In our case, the loop takes the form presented in Figure 9.a.

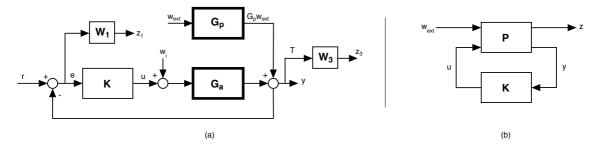


Figure 9. (a) Closed loop including G_p and G_a parts of the system and (b) standard form.

The command u is in input of the active part G_a of the system. T must be high in the bandwidth of the system, and low beyond to eliminate measurement noise. The external actions excite the Eigen modes of G_p and can be assimilated to an external noise w_o , so we need a low sensitivity S. Finally, the uncertainties of the system may be viewed as perturbations w_i on the input, so robustness requires a low KS function. In the H_∞ method, these conditions writes under a "standard" form (Figure 9.b), centered on the controller K, where P, presented in a matrix form in equation (11), describes the other part of the closed loop, the connections between external signals (w_{ext} and z) and internal ones (y and u).

$$P = \begin{pmatrix} -W_1 G_p & -G_a W_1 \\ W_3 G_p & G_a W_1 \\ -G_p & -G_a \end{pmatrix}$$
 (11)

The z_i signals are criteria output, images of S and T shaped by W_i the functions that express performance requirements. The synthesis problem consists then in finding K respecting the closed loop conditions in equation (12) that summarize all the performance requirements.

$$|W_1 S|_{\infty} < 1$$

$$|W_3 T|_{\infty} < 1$$
(12)

Experimental results

Applying this method, we synthetized two controllers, one for each resonance mode in torsion and flexion. Each controller is implemented as a z filter in a control program under LabViewTM. The dominant frequency of the acceleration signal is used to switch, between the two modes at 15 Hz, the controller and the actuation mode from one regime to the other.

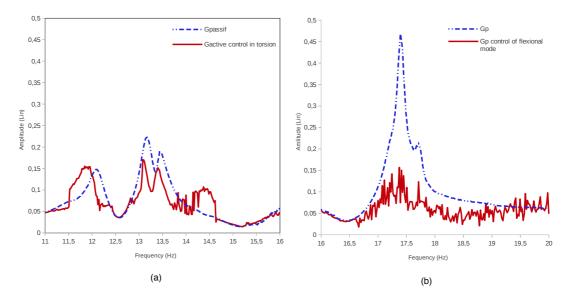


Figure 10. Comparison of the passive and controlled response (a) in torsion and (b) in flexion.

We present in figure 10 a comparison of the behaviour acceleration over external force between the passive and the controlled system, measured by performing swept sine analysis. While the attenuation is modest in the domain corresponding to the torsion mode, we observe a more interesting impact on the flexion mode. In figure 11, we present the comparison of the spectrums of the response under random force, with and without control. Under this more realistic loading, obtained for each controller in torsion and flexion, we confirm a good attenuation of the peak in flexion and an encouraging result on torsion.

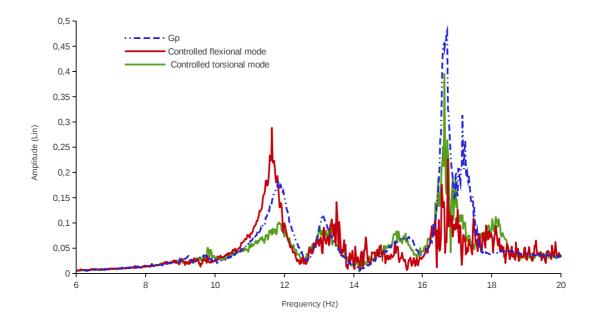


Figure 11. Comparison of the passive and controlled response for each controller, identified under random excitation.

Conclusion

Tensegrity systems are little represented in construction, mainly because of a lack of specific analysis and design methods. In this paper, we presented a design methodology associated to the search of an optimum solution. The case is a whole footbridge composed of two curved modular tensegrity beams that can be erected by deployment. Several materials are proposed and a large set of geometric parameters is explored. Using the surface response technique on a set of designed configurations, a solution can be found that optimizes the structural performances, minimizing the mass and maximizing the flexural rigidity. Among three materials, similarities appear on the value of the initial deflection and the shape of the modules, confirming the relevance and equilibrium of the proposed solutions. Although they may not appear as competitive compared to conventional solutions, it is mainly the erection process, by deployment, that gives a strong advantage to these systems.

In addition to structural design, we developed a control strategy for attenuating first vibrations modes. The principle is to synthetize a closed loop controller using robust H_{∞} synthesis technique for each vibration mode. To manage this, the behaviour of the structure is identified in several frequency domains by similitude with a springmass system. The implemented controller is then adapted for each control regime, in torsion or flexion. This original methodology gives encouraging results on the attenuation of the first resonances. It is being extended to other modes and other structures, using simulations, in order to propose new active tensegrity structures.

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